GOVT. POLYTECHNIC, JAGATSINGHPUR

CIVIL ENGINEERING DEPARTMENT

LEARNING MATERIAL OF **HYDRAULICS & IRRIGATION ENGINEERING**

4^{TH} SEMESTER

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Fluid:— A fluid is a ~~sbstss~~ substance with deforms continously under the Influence of shearing force no matter how small the force may be
eg:- liquid, gas, vapour etc.

Fluid mechanics:—
→ Fluid mechanics is the branch of science which deals with study of fluid at rest or in motion
→ It has been applied in areas such as the design of canal, dam system and pumps etc.

Newton 1st law:—
Object at rest remains at rest object at motion remain in motion in a staight line unless acted upo byan an unbalance force.

2nd law:—
Force equal to mass time acelaration.
$F = MA$

3rd law:—
Every action is equals an oposite reaction

Properties of fluid:—
Mass density (ρ) row:—
It is defined as the ratio of mass of fluid to its volume.
density unit $= kg/m^3 = \frac{mass}{V}$
* density of water $= 1000\,kg/m^3$

Specific volume:—
→ It is defined as the volume to the mass. It is denoted by '~~v~~' (v)

$$\boxed{v = \frac{1}{\rho}}$$

Specific weight:— or unit weight:—
Specific weight (also called weight density) of a fluid is the wt it possess for unit volume it is denoted by symbal (γ) (γ)
$W\ it = \frac{weight}{volume} = N/m^3$

$W = M \cdot g$

$= kg \times M/sec^2$

$= N$

specific gravity :- (S)

→ specific gravity is the ratio of specific weight of fluid to the specific weight of a standard fluid.

* the standard fluid chosene for sp.g comparision in pure water. @ 4°C

$$\boxed{S = \frac{\text{Sp. wt of fluid}}{\text{Sp wt of standard fluid}}}$$

or

$$\boxed{S = \frac{\text{S.p wt of fluid}}{\text{S.p wt of water @ } 4^\circ C}}$$

$$S = \frac{\gamma_{fluid}}{\gamma_{water}} = \frac{\rho_{fluid} \cdot g}{\rho_{water} \cdot g}$$

$$\rho = \frac{mass}{Volume}$$

$$w = \frac{weight}{Volume} = \frac{mass \times g}{Volume}$$

* $\boxed{w = \rho \cdot g}$

$$S = \frac{\gamma_{fluid}}{\gamma_{water}}$$

$$= \frac{\rho_f \cdot \cancel{g}}{\rho_w \cdot \cancel{g}}$$

$$= \frac{\rho_f}{\rho_w} \qquad S = \frac{N/m^3}{N/m^3}$$

* It is a unitless quantity

specific gravity of water :-

water density = $1000\ kg/m^3$

$$S = \frac{\gamma_f}{\gamma_w}$$

or

$$\frac{\rho_f}{\rho_w} = \frac{1000\ kg/m^3}{1000\ kg/m^3} = 1$$

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$S = 1$
ρ water $= 1000 \, kg/m^3$
γ water $= \rho \cdot g$
$= 9.81 \times 1000$
$= 9810 \, N/m^2$
$= 9.81 \, kN/m^3$

→ unit weight

mercury
ρ mercury $=$ ~~10000~~ ~~13.6~~ kg/m^3 $13600 \, kg/m^3$
S mercury $= 13.6$

mercury is 13.6 time havier in water

perporties :-

relative density :-

It is defined as the ratio of density of a 1 substance with respect to other substance.

$$\boxed{\rho_{1/2} = \frac{\rho_1}{\rho_2}}$$

where, $\rho_{1/2}$ = relation density of substance 1 with respect to substance 2

Question :-

3 liter of petrol weight 23.7 N calculate the mass density, specific weight, specific volume and specific gravity of petrol.

$Wt = 23.7 \, N$
volume $= 3L$ $\quad 1 \, m^3 = 1000 \, L$
$= 3 \times 10^{-3} \, m^3$ $\quad \Rightarrow 1L = 10^{-3} \, m^3$

(2) specific weight $= \dfrac{23.7}{3 \times 10^{-3}} = 7900 \, N/m^3$

(1) $\rho = \dfrac{mass}{vol} = \dfrac{Wt \times g}{volume}$

$= \dfrac{\frac{23.7}{9.81}}{3 \times 10^{-3}} = 805.3007136 \, kg/m^3$

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(3) $\frac{1}{805.3007136} = 1.241772152 \times 10^{-3} m^3/kg$

(4) $S = \frac{\text{density of petrol}}{\text{density of water}}$

$= \frac{805.3007136}{1000} = 0.805.3007136.$

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ρ = Ro = Density

γ - gama = unit weight

μ = mu = Co-efficient of dynamic viscosity

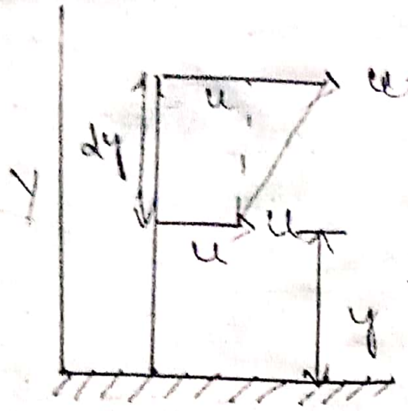
ν -> ~~coef~~ nu = coefficient of ~~kinatics~~ kinematics viscosity

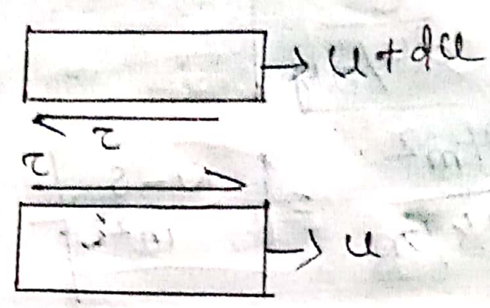
viscosity - It is a measure of resistance of fluid to defformation. It is due to internal fixonal forces that developed between ~~layer~~ different layar of fluid when they are forced to move relative to each other.

Suppose,

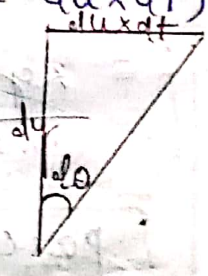
1 layer of fluid is moving with respect to other layer by velocity du and vertical gap between 2 layers be dy.

-> uper layer which is moving faster tryies to draww the lower slowly moving layer along with it. similarly as the reaction to this lower layers, tries to retaied the uper word. does the exit a shear between the two layer as shows in





In time dt the top layer will move with respect to the bottom layer by a distance da ($da = du \times dt$)



Shear strain = $\tan d\theta = \frac{da}{dy}$

(for small value of $\tan d\theta$ take $\tan d\theta = d\theta$)

$d\theta = \frac{du \cdot dt}{dy}$

$\Rightarrow \frac{d\theta}{dt} = \frac{du}{dy}$

change in Shear strain with respect to time — velocity Gradient.

The fluid for which rate of deformation is linearly perposnal to the shear stress is called Newtonian fluid.

$\tau \propto \frac{d\theta}{dt}$

τ = shear stress of opposing the movement of fluid

$\tau \propto \frac{du}{dy}$

du = change in velocity of fluid
dy = vertical distance betw fluid layer

$$\boxed{\tau = \mu \frac{du}{dy}}$$

where μ = ~~consta~~ coefficient of dynamic viscosity or viscosity

Neutron laws of viscocity

Shear stress in a fluid he directly perposnal to velocity gradient or rate of change of shear strain.

unit of viscocity :-

$$\boxed{\mu = \frac{\tau}{du/dy}}$$

$$= \frac{N/m^2}{m/s/m} = \boxed{\frac{N-s}{m^2}} \text{ — SI unit}$$

$$N = kg - m/s^2$$

$$= \frac{kg - m/s^2 \times s}{m^2}$$

$$\boxed{= \frac{kg}{m \cdot s}} \text{ — M.K.S unit}$$

$\boxed{\text{pascal} - s}$ — C.G.S unit

$\boxed{\text{poise}}$ — C.G.S

$\boxed{10 \text{ poise} = \frac{1 N-s}{m^2}}$ — C.G.S

$\boxed{1 \text{ centipoise} = \frac{1}{100} \text{ poise}}$ — C.G.S.

Kinametic viscosity is defined as the ratio of dynamic viscocity divided by density ~~.....~~ $u = \frac{\text{dynamic viscocity}}{\text{density}}$

$$u = \frac{\mu}{\rho}$$

Kanametic viscosity :-

Kanametic viscosity is defined as the ratio of dynamic viscocity divided by density · $u = \frac{\text{dynamic viscocity}}{\text{density}}$

$$\boxed{u = \frac{\mu}{\rho}}$$

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$$= \frac{\frac{N-s}{m^2}}{kg/m^3}$$

OR $= \frac{kg/m\text{-}s}{kg/m^3}$

$\boxed{\frac{m^2}{s}}$ or $\boxed{\frac{cm^2}{sec}}$

$\boxed{1 \text{ stoke} = 10^{-4} m^2/s}$

note :—

kinametics viscocity of water at 20°C equal to 0.01 stoke.

kinametics viscosity of air at 20°C equals to 0.152 stoke.

doubt :— kinametics viscosity of air is 15.2 time that of water.

question :—

A fluid flows between 2 parallel plates separated at a distance at 20mm. 1 of the plate is fixed other one is moving with a velocity of 2 m/s. the viscocity of this fluid is 0.1 poise. find the shear stress on the fixed plate.

$\mu = 0.1$ poise

$dy = 0.02 m$

$du = 2-0 = 2 m/s$

$$10 \text{ poise} = \frac{1 N-s}{m^2}$$

$1 \text{ poise} = 0.1 \frac{1 N\text{-}s}{m^2}$

$= 0.1 \times 0.1 \frac{N\text{-}s}{m^2}$

$= 0.01 \frac{N\text{-}s}{m^2}$

$\tau = \mu \frac{du}{dy}$

$\tau = 0.01 \frac{N\text{-}s}{m^2} \times \frac{2}{0.02}$

$\tau = 1 \; N/m^2$

Question no - 2 :—

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viscosity of a fluid with specific gravity 1.3 is measured to be $0.0034 \frac{N\text{-}s}{m^2}$ find out the kinematic viscosity of fluid.

Ans :—

$\nu = \frac{\mu}{\rho}$

$S = 1.3$

$\mu = 0.0034 \frac{N\text{-}s}{m^2}$

$S = \frac{\gamma_{fluid}}{\gamma_{water}}$

$\gamma_{fluid} = S \times \gamma_{water}$

$= 1.3 \times 1000$

$= 1300 \; kg/m^3$

$\nu = \frac{0.0034}{1300}$

$= 2.615384615 \times 10^{-6} \; m^2/s$

UNIT OF VISCOSITY

Dynamic Velocity :-

$$\tau = \mu \frac{du}{dy}$$

$$\Rightarrow \boxed{\mu = \frac{\tau \cdot dy}{du}}$$

$$\mu = \frac{F}{A} \times \frac{y}{u} = \frac{N}{m^2} \times \frac{m}{m} \times s = \boxed{\frac{N-s}{m^2}} \quad -(1)$$

$$1N = 1kg - m/s^2 \quad \left\{ \frac{N-s}{m^2} = kg - \frac{m}{s^2} \times \frac{s}{m^2} = \boxed{\frac{kg}{m-s}} \quad -(2) \right.$$

$$1N = 1Pa \quad \left\{ 1 \frac{N-s}{m^2} = \boxed{1Pa-s} \quad -(3) \right.$$

Dynamic viscosity in CGS system :-

$$dyne = 10^{-5} N$$
$$1N = 10^5 dynes$$

$$10 \text{ poise} = 1 \frac{N-s}{m^2} = Pa \cdot s = \frac{10^5 dynes-sec}{(100)^2 \cdot cm^2} = 10 \boxed{\frac{dyne-sec}{cm^2}}$$

$$\frac{1 \, dyne-sec}{cm^2} = \boxed{1 \text{ poise}}$$

$$\frac{1kg}{m-s} = \frac{1000 gm}{100 cm-sec} = 10 \boxed{\frac{gm}{cm-sec}}$$

$$\boxed{\frac{dyne-sec}{cm^2} = \frac{gm}{cm-sec} = poise} \text{ in CGS system}$$

unit of Dynamic viscosity $\rightarrow \frac{N-s}{m^2}, \frac{kg}{m-s}$, Pa.s (SI unit)

$\rightarrow \frac{dyne-sec}{cm^2}, \frac{gm}{cm-sec}$, Poise (CGS system)

Kinemetic viscosity :-

$$\boxed{v = \frac{\mu}{\rho}}$$

$$v = \frac{\mu}{\rho} = \frac{kg}{m-s} \times \frac{m^3}{kg}$$

$$= \boxed{\frac{m^2}{s}} \text{ SI system}$$

Kinemetic viscosity in CGS system :-

$$\frac{m^2}{s} = \frac{(100)^2 cm^2}{sec} = 10^4 \boxed{\frac{cm^2}{sec}} = \boxed{10^4 \text{ stokes}}$$

$$\boxed{1\ \frac{cm^2}{sec} = 1\ \text{stokes}}$$

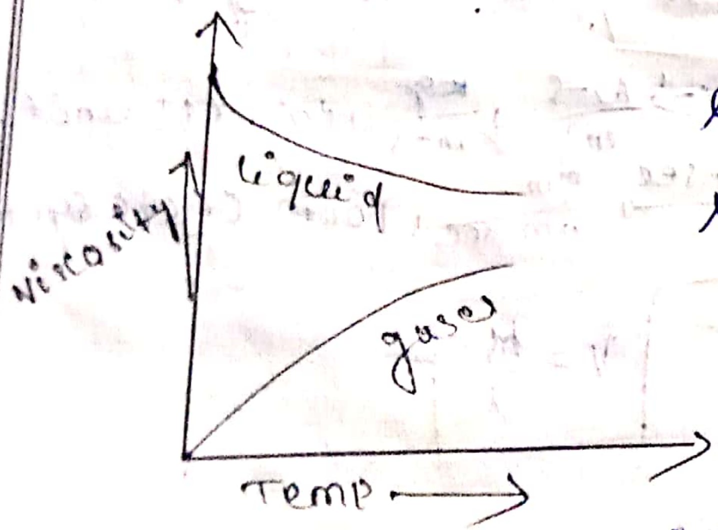
$$\boxed{1\ \text{stokes} = 10^{-4}\ m^2/s}$$

unit of kinematic viscosity is

$\boxed{\frac{m^2}{s}}$ SI system

$\boxed{\frac{cm^2}{sec}} = \boxed{\text{stokes}}$ CGS system

Effect of tempreture on viscosity of fluid



$\mu \propto \frac{1}{T^\circ c}$ (liquid)

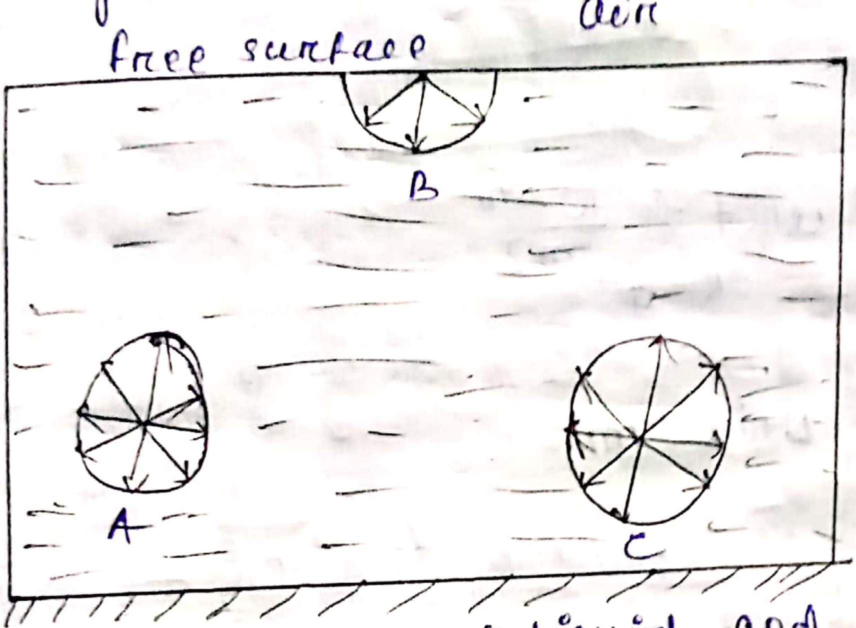
$\mu \propto T^\circ c$ (gases

In case of liquid raising temprature reduces viscosity.

→ In case of gases raising tempresture increases viscosity.

* Surface tension :— (σ)

It is defined as the tensile force acts on the surface of a liquid when it is exposes to any air or gas



→ at the interface of liquid and gas molicul or between 2 immisciable. liquid a ~~film or~~ file or special layer file from apparatly woing to the attraction of liquid moliecul on surface from below the surface.

→ As a result of net down ~~woor~~ ward force surface will be ~~co~~ pulled down causing a curvature to the surface.

→ this in turn devlope a tension in a strace ~~...~~ membrane this is known as surface tension.

$$\sigma = \frac{\text{force}}{\text{Length of the surface}}$$

unit = N/m

~~Eg~~/Energy

or work done = force × displacement

= force × dx

= unit → N.m

$\sigma = \frac{\text{force}}{\text{Length}} \times \frac{dx}{dx} = \frac{\text{Energy or work done}}{\text{Area}}$

$\frac{N-m}{m^2}$ = SI unit

$\Rightarrow J/m^2$

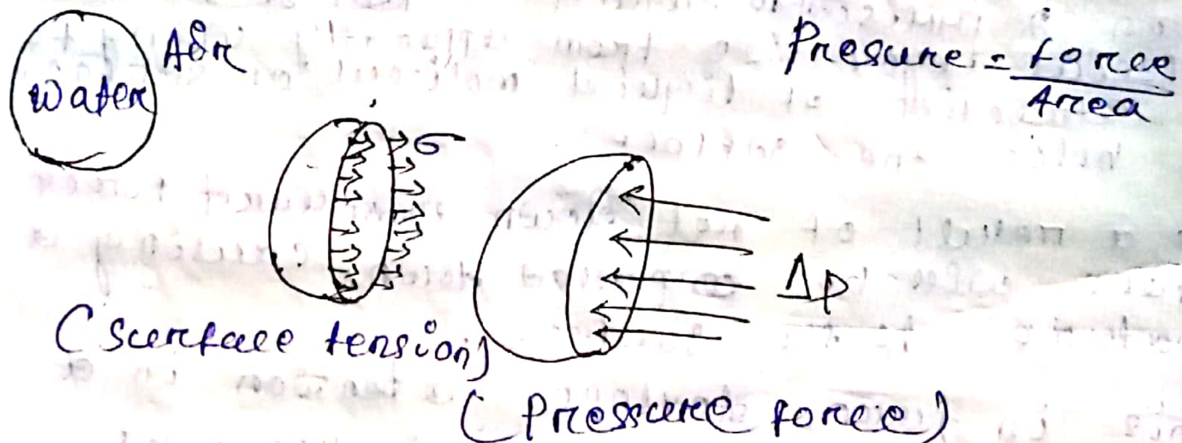
$\sigma = \frac{\text{Energy}}{\text{Area}}$ unit $\rightarrow \frac{N-m}{m^2}$ OR J/m^2

OR

$= \frac{\text{force}}{\text{length}}$ unit N/m

Application of surface tension :-

Droplet of liquid in Air :-



for due to surface

Tension $= \sigma . L$

$= \sigma . 2\pi r$

force due to Δp

$= \Delta P . A$

$= \Delta p . \pi r^2$

$\therefore \sigma . 2\pi r = \Delta P . \pi r^2$

$\Rightarrow \cancel{\Delta P = \frac{2\sigma}{r}}$

$$\frac{\sigma\, 2\pi r}{\pi r^2}$$

$$\boxed{\Delta p = \frac{2\sigma}{r}}$$

where, Δp = pressure deference of pressure intensity inside the droplet (in excess of the outside pressure intensity)

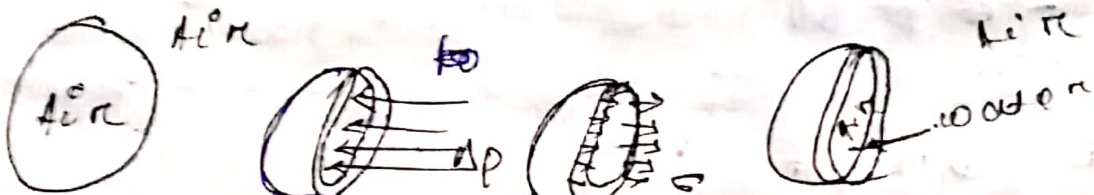
σ = surface tension.

r = radius

d = diameter of droplet

$$\boxed{\Delta p = \frac{2\sigma}{r} \text{ OR } \frac{4\sigma}{D}}$$

Hollow bubble :-



Force due to inside pressure difference $= \left(\frac{\pi}{4} d^2\right) \times \Delta p$ or $= \pi r^2 \times \Delta p$

Force due to surface tension.

$= 2 \times (2\pi r \times \sigma)$

$= 2 \times (2\pi r \times \sigma) = 4\pi r \times \sigma$

$$= \frac{2 \times (2\pi r \times \sigma)}{\pi r^2}$$

$$= \frac{4\cancel{\pi}\sigma}{r} = \frac{4\sigma}{r}$$

$$\Delta p = \frac{4\sigma}{r}$$

$$\text{or } \frac{8\sigma}{D}$$

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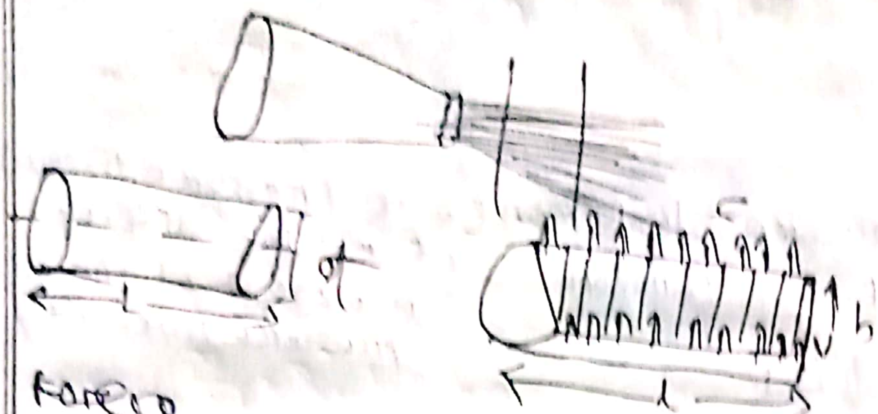
Liquid Jet :-



Force

Force due to pressure difference $(\Delta p) = \Delta p \times \text{Area}$

$= \Delta p \times (b \times L)$

$= \Delta p \, bL$

Force due to surface tension

$= \sigma \times (2l)$

$= 2\sigma L$

Force due to surface tension = Force due to Δp

$\Rightarrow 2\sigma \cancel{l} = \Delta p \cdot b \cdot \cancel{l}$

$\Rightarrow 2\sigma = \Delta p \cdot D \Rightarrow \boxed{\Delta p = \frac{2\sigma}{D}}$ ($\because b = D$)

$\Delta p = \frac{2\sigma}{2r}$

$\boxed{\Delta p = \frac{\sigma}{r}}$

$\Delta p = \frac{\sigma}{r} \rightarrow$ liquid jet

$\Delta p = \frac{2\sigma}{r} \rightarrow$ Liquid Droplet or Air Bubble

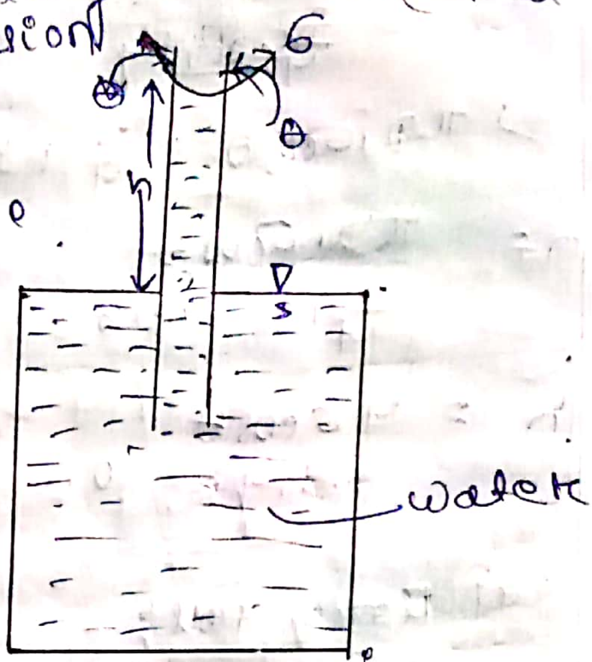
$\Delta p = \frac{4\sigma}{r} \rightarrow$ Soap Bubble.

Capilarity :-

> capilarity effect is a consiquences of a surface tension and adhesion

→ it is defined as the rise or fall of a liquid in a small diameter tube instrated into liquid.
→ The rise is called capilary rise and fall is called capilary depression.

$\theta < 90°$



Let

d = diameter of glass tube open at both end.

h = hight of the liquid in the tube

σ = surface tension of the liquid. ~~contact between liquid~~

θ = angle of contact between liquid and gas.

capillary Rise

wt of liquid of ~~hight~~ height h in the tube

= volume × unit weight of liquid

$= \underbrace{\frac{\pi}{4} \times d^2 \times h}_{} \times \rho g$

$= \frac{\pi}{4} \cdot d^2 \cdot \rho g h$

under the state of ~~equilibrium~~ equilibrium the wt of the liquid of hight (h) in the tube is balance by the force at the surface of the liquid in the ~~liq.~~ tube due to surface tension.

Cohesion < Adhesion

force due to surface tension (vertical component)

$= \sigma \cos\theta \times \pi d$

$= \pi \cdot d \cdot \sigma \cos\theta$ —— ①

$\Rightarrow \frac{\pi}{4} \cdot d^2 \cdot \rho \cdot g \cdot h$ —— ②

$\Rightarrow \pi D \cdot \sigma \cos\theta = \frac{\pi}{4} \cdot D^2 \cdot \rho g h$

$h = \frac{\pi D \sigma \cos\theta}{\frac{\pi}{4} d^2 \rho g}$

$\Rightarrow \pi D \cdot \sigma \cos\theta = \frac{\pi}{4} \cdot D^2 \cdot \rho \cdot g \cdot h$

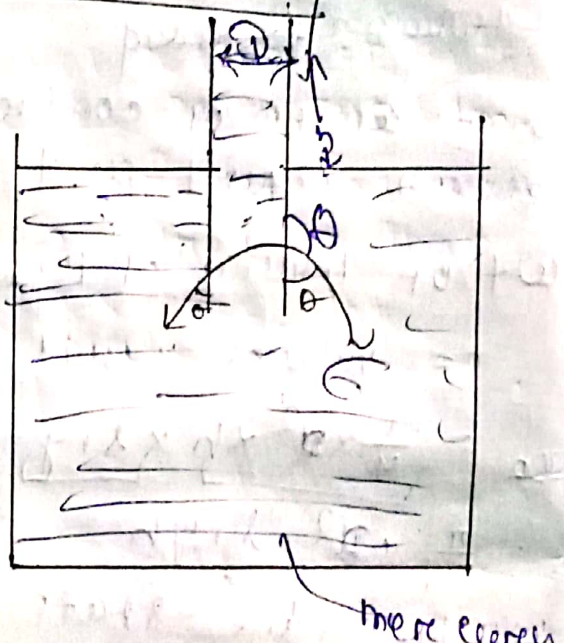
$h = \frac{\pi D \cdot \sigma \cos\theta}{\frac{\pi}{4} \cdot D^2 \cdot \rho \cdot g}$

$\boxed{h = \frac{4 \sigma \cos\theta}{D \cdot \rho g}}$ OR $\boxed{h = \frac{4 \sigma \cos\theta}{\gamma \cdot D}}$

Capillary fall –

$\theta > 90^\circ$

Cohesion > Adhesion.



capillary depression

$\Rightarrow$ when the glass tube dipped in mercury 2 forces act on the mercury inside the tube.

1. force due to surface tension acting in the downward direction.

$= \pi D \times \sigma \cos\theta$

2. force due to hydrostatic pressure acting upward $= \frac{\pi}{4} D^2 \cdot p$

$= \frac{\pi}{4} \cdot D^2 \cdot (\rho \cdot g \cdot h)$ $\boxed{p = \rho \cdot g \cdot h}$

$\pi D \times \sigma \cos\theta = \frac{\pi}{4} \cdot D^2 \cdot (\rho \cdot g \cdot h)$

$$h = \frac{\pi D \times \sigma\cos\theta}{\frac{\pi}{4} \cdot D^2 \cdot (\rho g)}$$

$$= \frac{4\sigma\cos\theta}{D \cdot \rho \cdot g}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D} \quad (\text{when } \rho \cdot g = \gamma)$$

Note:-

capilary rise is inversly perposnal to the diameter of pipe.

→ the capilary effect of water is easly negligible in tubes which diameter is greater 1 cm.

question:-

determing the viscocity of a liquid having kinametics viscocity 6 stokes and specific gravity 1.9.

given data:-

$S = 1.9$

$\nu = 6$ stokes 1 stokes $= 10^{-4}\ m^2/s$

$\nu = 6 \times 10^{-4}$

$S = \frac{\gamma_{fluid}}{\gamma_{water}}$

$1.9 = \frac{\gamma_{fluid}}{1000}$

$1.9 \times 1000 = \gamma_{fluid}$

γ fluid = 1.9 × 1000 = 1900 kg/m³

$\nu = \dfrac{\text{dynamic viscosity}}{\text{density}}$

$6 \times 10^{-4} = \dfrac{\text{dynamic viscosity}}{1900}$

dynamic viscosity = $6 \times 10^{-4} \times 1900$

= 1.14 Ns/m2

or 11.4 poise.

Question 6:—

The surface tension of water in contact with air at 20°c is 0.0725 N/m. The pressure inside droplate of water is to be 0.02 N/cm² greater than outside pressure. Calculate the diameter of the droplate of water.

Given data :—

σ = 0.0725 N/m

Δp = 0.02 N/cm²

= 0.02×10^4 N/m²

$\dfrac{2\sigma}{r}$ or $\dfrac{4\sigma}{D}$

$D = \dfrac{4\sigma}{\Delta p} = \dfrac{4 \times 0.0725}{0.02 \times 10^4}$

= 1.45×10^{-3}

= 1.45 mm

Dt - 10/04/2020

Question :-

Find the surface tension in a sope bubble in a 40 mm diameter in a inside pressure is 2.5 N/m² above atmosperic ~~equation~~ pressure.

Given data :-

$d = 40mm = 0.04m$

$\Delta p = 2.5 \, N/m^2$

$$\Delta p = \frac{8\sigma}{D}$$

$$2.5 = \frac{8\sigma}{0.04}$$

$$2.5 \times 0.04 = 8\sigma$$

$$\sigma = \frac{2.5 \times 0.04}{8} = 0.0125 \, N/m$$

Question :-

The pressure outside the droplate of water of Daimeter 0.04 mm is 10.32 N/cm² (at Atmosperic pressure) calculate the pressure with in the drouplate if surface tension is given as 0.0725 N/m of water.

given data :-

$D = 0.04mm = 0.04 \times 10^{-3} m$

$P_{atm} = 10.32 \, N/cm^2 = 10.32 \times 10^4 \, N/m^2$

$\sigma = 0.725 \, N/m$

Pressure within the Droplate $p = P_{atm} + \Delta p$

$$\Delta p = \frac{4\sigma}{D}$$

$$\Delta p = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \, N/m^2$$

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$= 10.33 \times 10^4 * 7250$

$= 110450 \ N/m^2$

$= 11.045 \ N/cm^2$

Question :-

The capillary rise in glass tube is not ~~exceeded~~ 0.2mm of water. determine its
exceed

minimum size given that surface tension of water in contact with air is equal to 0.0725 N/m

Given data :-

$h = 0.2mm = 0.2 \times 10^{-3} m$

$\sigma = 0.0725 \ N/m$

$\rho_{water} = 1000 \ kg/m^2$

$g = 9.81 \ m/s^2$

$\theta = 0^\circ$

$$h = \frac{4\sigma\cos\theta}{\gamma \cdot d}$$

$$h = \frac{4\sigma\cos\theta}{\rho \cdot g \cdot d}$$

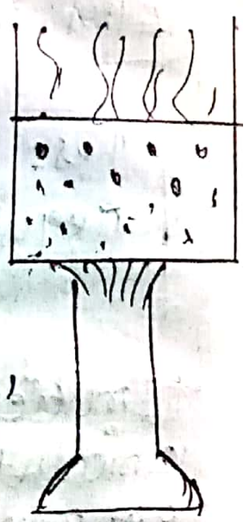
$$0.2 \times 10^{-3} = \frac{4 \times 0.0725 \times 1}{1000 \times 9.81 \times D}$$

$0.2 \times 1000 \times 9.81 \times D = 4 \times 0.0725 \times 1$

$$D = \frac{4 \times 0.0725 \times 1}{0.2 \times 1000 \times 9.81} = 0.1478 \ M$$

$= 0.1478 \times 10^3 = 147.8$ mm

Questions :-



Vapour pressure :-

Boiling :- when water start creating bubble inside it

cause - when Temperature increases, pressure decreases.

When tempreture increase :-

-> energy of a molecul increases.

-> malicular bonding breakes.

-> liquid molecul turn into steam or gas molecul.

-> bubble start forming.

What is vapour pressure :-

-> Vapour pressure occurre when the pressure exerted by vapour liquid.

-> when air get fully saturated in a closed in a closed contener at that point the pressure exerted by vapour is called vapour pressure.

dt :- 13.07.2020

Atm. Pressure :-

-> Atm. Pressure = 101325 N/m²

-> Water vapours are 100°c

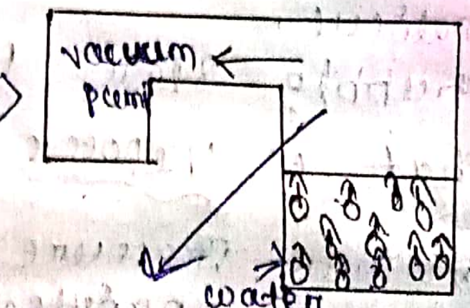
Atm. Pressure ↓ → Less heat required

Vapour Pressure ↓ → Less Energy → Less heat required

* Molecules of water always have some amount of energy but not enough energy to surpass that Atm. pressure so extra energy is provided in the form of heat to overcome Atm. pressure

When pressure is decrease :-

→ Atm. pressure is decreased of a liquid using a vacuum pump and taken nearly zero



When Atm. pressure is taken to zero water will feel no pressure above it and will start converting into $\boxed{\text{steam}}$ for any point inside its mass

Cavitation :-

→ Now consider a flowing liquid in a system if the pressure at any point in this flowing liquids becomes equals to or less than the vapour pressure

The vaporisation of liquid starts. The bubble of this vapour are carried by the flowing liquid in to the region of high pressure where then collaps giving rise to high impact pressure. The pressure devloped by the collapsing bubble is so high that the materials from the adjoining boundries get eroded and cavities are from on them. this phenomenon are called cavitation.

Question :-

Calculate the capilary rise in a glass tube of 2.5 mm diameter when a immerded vertically in a (a) water and (b) mercury.
Takes surface tension $\sigma = 0.0725$ N/m for water, $\sigma = 0.52$ N/m for mercury in contact with air. this specific gravity for mercury is given as 13.6 and angle of contact is equal to 130°.

For water :-

given data :-

$D = 2.5\,mm = 2.5 \times 10^{-3}\,m$

$\sigma = 0.0725$ $\quad \rho_w = 1000$

$\theta = 0$ $\quad g = 9.81$

$h = ?$

$$h = \frac{4\sigma\cos\theta}{\rho_w \cdot g \cdot D} = \frac{4 \times 0.0725 \times \cos 0}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$= 0.01182\,m = 11.82\,mm$

For mercury :-

given data :-

$D = 2.5\,mm = 2.5 \times 10^{-3}\,m$

$g = 13.6$, $\sigma = 0.52$

$\theta = 130°$, $\rho_{Hg} = 13546$

$$h = \frac{4\sigma\cos\theta}{\rho_{Hg} \cdot g \cdot D} = \frac{4 \times 0.52 \times \cos 130}{13546 \times 13.6 \times 2.5 \times 10^{-3}}$$

$= -2.90 \times 10^{-3}\,m = -2.90\,mm$

Question :-

Calculate the capilary effect of mm is a glass tube of 4mm diameter, when imorse (i) water, (ii) mercury the temprature of liquid is 20°c and values of the surface tension of water and mercury at 20°c in contact with air are 0.0735 N/m and 0.51 N/m respectively the angle of contact for water is 0 and that from mercury is 130°. take density of water at 20°c as equal to 998 kg/m³

(i) for water :-

given data :-

$D = 4mm = 4 \times 10^{-3} m$

$\sigma = 0.073575$

$\theta = 0$

$\delta_w = 998$

$g = 9.81$

$$h = \frac{4\sigma\cos\theta}{\delta_w \cdot g \cdot D}$$

$$= \frac{4 \times 0.073575 \times \cos 0}{998 \times 9.81 \times 4 \times 10^{-3}}$$

$$= 2.071 \times 10^{-3} m = 2.071 mm.$$

for mercury :-

given data :-

$D = 4 \times 10^{-3} m$

$\sigma = 0.51 \quad \delta_{Hg} = 13546$

$\theta = 130° \quad g = 9.81$

$$h = \frac{4\sigma\cos\theta}{\delta_{Hg} \cdot g \cdot D}$$

$$h = \frac{4 \times 0.51 \times \cos 130}{13546 \times 9.81 \times 4 \times 10^{-3}}$$

$$= -2.466 \times 10^{-3} m$$

$$= -2.466 mm$$

Question :-

If the velocity distribution of a fluid over a plate is given by $u = \frac{3}{4}y - y^2$

where u is the velocity in metres per second at a distance of y metres above the plate. Determine the shear stress at $y = 0.15$ metre. Take dynamic viscosity of the fluid as $8.34 \times 10^{-4} \frac{N\text{-}s}{m^2}$

given data:-

$u = \frac{3}{4}y - y^2$

$Y = 0.15 m$

$\mu = 8.34 \times 10^{-4} \frac{N\text{-}s}{m^2}$

$\tau = \mu . \frac{du}{dy}$

$\frac{du}{dy} = \frac{d}{dy}(\frac{3}{4}y - y^2)$

$\Rightarrow \frac{du}{dy} = \frac{3}{4}1 - 2y$

$\Rightarrow 3/4 - 2y$

$\Rightarrow \frac{3}{4} - 2 \times 0.15 \quad (\because Y = 0.15)$

$= 0.45$

$\tau = \mu . \frac{du}{dy}$

$= 8.34 \times 10^{-4} \times 0.45$

$= 3.753 \times 10^{-4} N/m^2$

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Pressure and its measurement :-

Pressure :- [11-14/01/2020]

Pressure is defined as the normal force exerted by a fluid per unit Area.

Unit of pressure :-

$1 Pa = 1 N/m^2$

$1 MPa = 1 N/mm^2$

$1 bar = 10^5 Pa = 0.1 N/mm^2$

$1 Atm = 101.325 kPa = 0.101325 MPa$

$1 Atm = 1.01325 bar$

1 torr = 1 mm of Hg in barometer

[Hg = mercury]

Atm. pressure :-

$$\boxed{\text{Pressure} = \frac{\text{Normal force}}{\text{Area}}}$$

→ It is the pressure exerted by Atm. Its value is taken as 1.013 bar at meansea level. It is measured by by a 12m.

→ At meansea level it is equal to 10.3m head of water or 76cm head mercury.

Note :-

h meter head of water is equivalent to $(\gamma_w h)$

Pressure and h meter head of Hg (mercury) is equivalent to

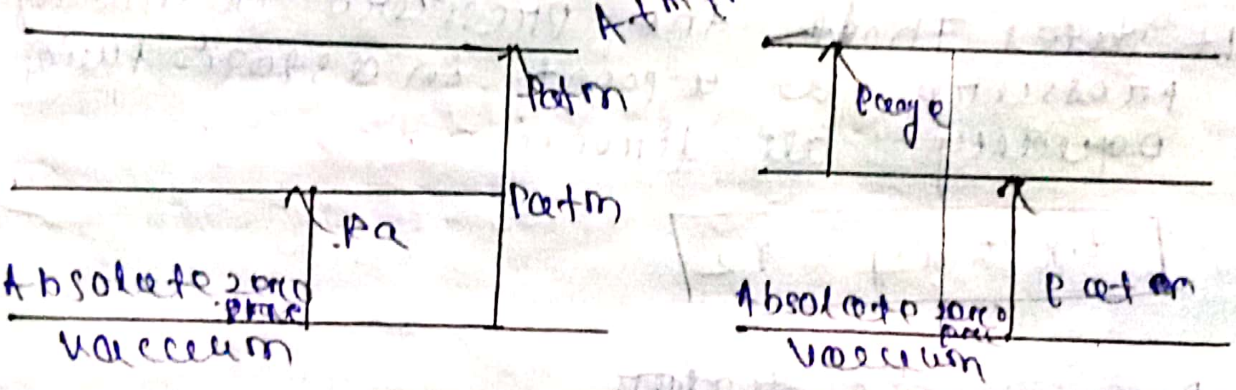
Kllg PL Tllg h

Apsolate Pressure :-

Apsolate pressure measured to respect to Absolute zero complete vaccum is called Apsolate pressure. ~~It is also called at the given position~~. It is also called the actual pressure at the given position. It is measured by an analoid barometer.

gauge pressure :-

it is pressure measured with respect to local Atm. pressure as the datum. it is measured by the using manometer or bourdon coise.



Relation between Apsolate pressure, gauge pressure and Atm. pressure :-

$$\boxed{P_{Absolate} = P_{atm} + P_{gauge}}$$

$$P_{absolate} = P_{atm} - \text{vaccume pressure}$$

$$\Rightarrow \boxed{P_{vaccume} = P_{atm} - P_{absolate}.}$$

Varions of pressure in vertical direction fluid for fluid in rest :-

for formula :-

$$\boxed{P_{gauge} = \rho.g.h \text{ OR } \gamma.h \quad (\because \gamma = \rho.g)}$$

→ The pressure at any point in a fluid at rest is obtanid by the hydrostatic law wo states that the rate of increase of pressure in a vertical vertically down ward direction must be equal to the specific wet or unit wet of the fluid at a point.

→ This is known as hydrostatic law.

Pascal :-

→ It states that the pressure or intensity of pressure at a point in a static fluid is equally all direction.

$$\boxed{P_x = P_y = P_z}$$

→ ~~Pressure~~

Question :-

calculate the pressure due to a columb of 0.3 m of (a) water, (b) an oil of specific gravity 0.8 and (c) mercury of specific gravity 13.6 take density of water 1000 kg/m³

given data :-

$h = 0.3 m$

$\rho_{oil} = 0.8$

$S_{Hg} = 13.6$

$S_W = 1000 \, kg/m^3$

$P = S \cdot g \cdot h$

(a) for ~~water~~ oil

$$S = \frac{S_F}{S_W}$$

$$= 0.8 \cdot \frac{S_F}{1000}$$

$= 800 \, kg/m^3$

$P_{gauge} = 800 \times 9.81 \times 0.3$

$= 2354.4$

for mercury,—

$S \cdot 13.6 \cdot \frac{S_F}{1000}$

$= 13600$

$P_{gauge} = 13600 \times 9.81 \times 0.3$

$= 40024.8$

for water,—

$S_W = 1000$

$P_{gauge} = 1000 \times 9.81 \times 0.3$

$= 2943.$

The pressure intensity at a point in fluid is given by 3.942 N/cm² find the corresponding hight of fluid when the fluid is (a) water (b) oil specific gravity 0.9.

given data :-

$S_{oil} = 0.9$

$P = 3.942 \times 10^{4} = \cancel{39420}$

$39420 = 0.9 \times 9.81 \times h$

$h = \cancel{\dfrac{0.9 \times 9.81}{3.942 \times 10^{4}}}$ $\qquad \dfrac{3.942 \times 10^{4}}{1000 \times 9.81} = 4.01$

$= \cancel{2023 \times 10}$

water :-

$h = \cancel{\dfrac{1000 \times 9.81}{39420}}$ $\qquad \cancel{\dfrac{3.942 \times 10^{4}}{100}}$

for water :-

$h = \dfrac{3.942 \times 10^{4}}{1000 \times 9.81} = 4.01$

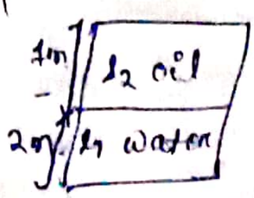
for oil $= \dfrac{3.942 \times 10^{4}}{900 \times 9.81} = 4.46$

Question :-

an open tank content water upto a depth of 2m an oil specific gravity 0.9 for a depth of 1m. find the pressure intensity.

(1) at the interface of a liquid

(2) At the bottom of the tank



(i) $P@A = S_2 \cdot g \cdot h_2$

$= 0.9 \times 9.81 \times 1 \times 10^{3}$

$= 8829 \, N/mm^2$

(ii) $P@B = \rho_1 g h_1 + \rho_2 g h_2$

$= \cancel{0.9 \times 9.81 \times 2 + \rho_2 g h_2}$

$= 1000 \times 9.81 \times 2 + \rho_2 g h_2$

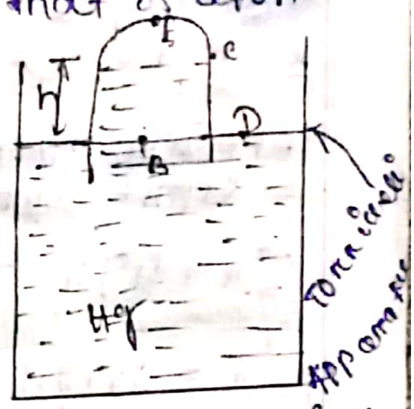
$= 19620 + 8829$

$= 28449$

Pressure measurement devices :-

Barometre :-

To measure Atm pressure a mercury filled tube is inverted into a mercury container that is open to Atm.



$P@D = P@B = P_{atm}$

$P_{atm} = \rho_{Hg} \cdot g \cdot h$

pressure @ D is Atm

Measurement of pressure in a fluid :-

Measurement of pressure is done by
(1) manometre
(2) mechanical gauge

manometre :-

manometre are based on the principle of balance in a column of fluid by the same or other column of fluid.

→ manometre are classified as
(1) simple manometre (pressure measure at a point)
(2) differential manometre used to measure different of pressure between 2 points.

Simple manometre :-

simple manometre are classified as
(a) piezo metre
(b) u-tube manometre
(c) single column manometre

Piezo metre :-

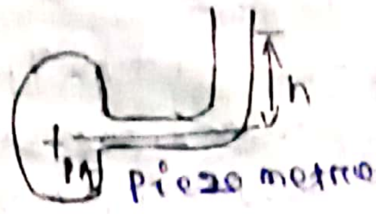
$P_A = \rho \cdot g \cdot h$

or $\gamma \cdot h$

where P_A = gauge pressure at A

→ negative pressure or vacuum pressure cannot be measure by piezo metre.

→ for large pressure non-column of piezo metre is required.



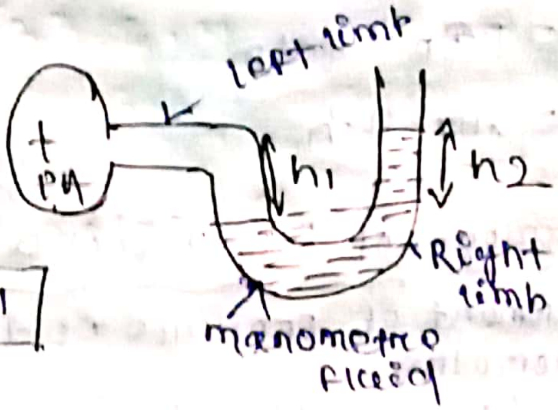
→ to avoid the effect of capillarity pipe diameter of piezo metre tube should not less than 12mm

→ piezo metre ~~does~~ doesnot measured of gas pressure. because it doesn't have free surface.

→ It is generally not measured high pressure

Dt - 16/11/2020

U-tube manometre :-



left limb pressure = Right limb pressure

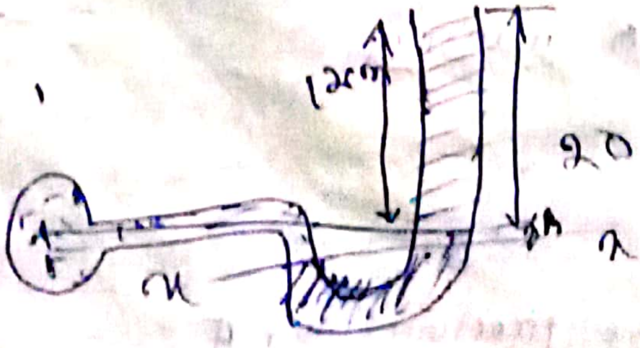
$= P_A + s_1 g h_1 = s_2 g h_2$

$$\boxed{P_A = s_2 \cdot g \cdot h_2 - s_1 g h_1}$$

Problem :-

The write lane of a simple U-tube manometre containing mercury is open to the Atm. where the left limb is conected to a pipe in which a fluid of specific gravity 0.9 is flowing the centre of pipe is 12cm below the ~~wright limb~~ level of mercury in the write limb. find the pressure of ~~pipe~~ fluid in the pipe in the defference of the mercury level in ~~20cm~~ 2 limb 20cm.

Ans :-



$S.G = 0.9$

$S_f = 900\ kg/m^3$

$S_{Hg} = 136000\ kg/m^3$

Left limb pressure $= P_A + S_f . g . h$

$= P_A + 900 . 9.81 \times 0.08$

$= 706.32$

Right limb pr. $= S_{Hg} \times g . h_2 = 13600 \times 9.81 \times 0.20$

$= \cancel{2660}\ 26683.2$

$P_A =$ left limb pressure $=$ right limb pressure

$= \cancel{25976.88\ N/m^2}$

$26683.2 - 706.32$

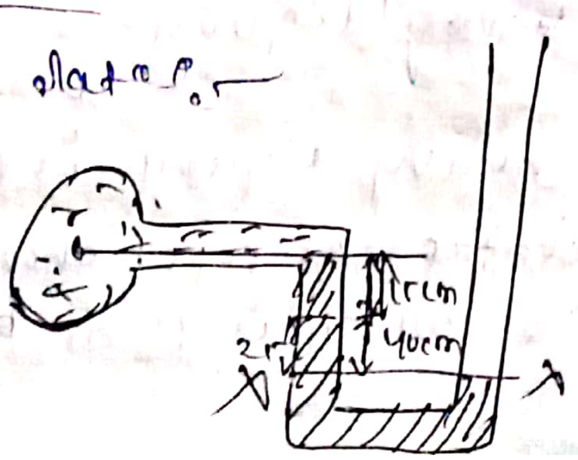
$= 25976.88\ N/m^2$

problem :—

A simple U-tube manometre is contening mercury is connected to a pipe, in which a fluid of specific gravity 0.8 and having a vaccum pressure is flowing The other end manometre is open to ~~atm~~ atm, find the vaccum pressure of pipe is the defforent of mercury leves in two limb is 40 c and the hight of fluid in the left from the centre of pipe is 15 c below

Ans :—

given data :—



$Sp = 0.8$

$\rho_f = 0.8 \times 1000$

$= 800 \, kg/m^3$

$\rho_{Hg} = 13600 \, kg/m^3$

left limb pressure $= P_A + \rho_f \cdot g \cdot h$

~~$P_A + 800 \times 9.81 \times ...$~~

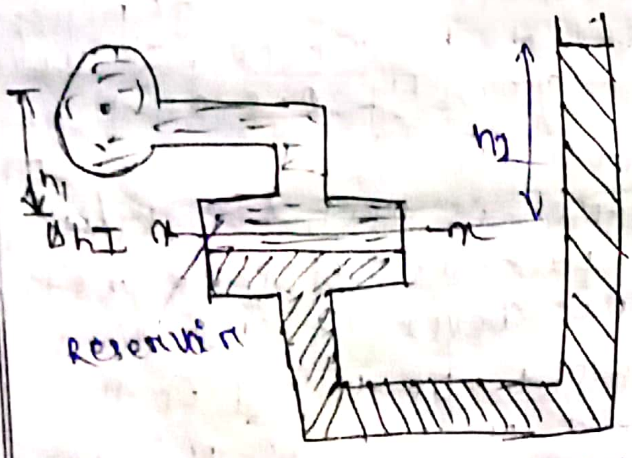
~~$= 117720 \, N/m^2$~~

$P_A + 800 \times 9.81 \times 0.15 + 13600 \times 9.81 \times 0.40$

$P_A + 1177.2 + 53366.4$

$= 54543.6 \, N/m^2$

vertical
single column manometre :-



let Δh = fall of heavy liquid in Reservoir.

h_2 = Rise of heavy liquid in Rightly

h_1 = hight of centre of pipe above x-x

P_A = pressure at A which is to be measured

A = crossectional area of the reservoir
a = crossectional area of the right limb
x_1 = Specific gravity of liquid in pipe
x_2 = specific gravity of heavy liquid in reservoir and Right limb.
δ_1 = density of liquid in pipe
δ_2 = density of liquid in Reservoir

→ fall of heavy liquid is reservoir will caused a rise of heavy liquid level is Rightly

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \boxed{\Delta h_1 = \frac{a \cdot h_2}{A}}$$

know consider the ~~daton~~ datum line Y-Y as shown in figure.

Pressure intensity in the Right limb above ~~datton~~ datum Line = $\boxed{\delta_2 \cdot g \cdot (h_2 + \Delta h)}$

pressure intensity in left limb above datum line = $\boxed{\delta_1 \cdot g(h_1 + \Delta h) + P_A}$

left limb pressure = $\delta_2 \cdot g \cdot (h_2 + \Delta h)$
Right limb pressure = $\delta_1 g(h_1 + \Delta h) + P_A$

$$\delta_2 \cdot g\,(h_2 + \Delta h) = \delta_1 g(h_1 + \Delta h) P_A$$

$$P_A = \delta_2 g\,(h_2 + \Delta h) - \delta_1 g(h_1 + \Delta h)$$

$$= \delta_2 g \cdot h_2 + \delta_2 g \cdot \Delta h - \delta_1 \cdot g \cdot h_1 - \delta_1 g\,\Delta h$$

$$P_A = \delta_2 \cdot g \cdot h_2 - \delta_1 g h_1 + \delta_2 g \cdot \Delta h - \delta_1 g \cdot \Delta h.$$

$$= s_2 \cdot g \cdot h_2 - s_1 \cdot g \cdot h_1 + \Delta h (s_2 \cdot g - s_1 \cdot g)$$

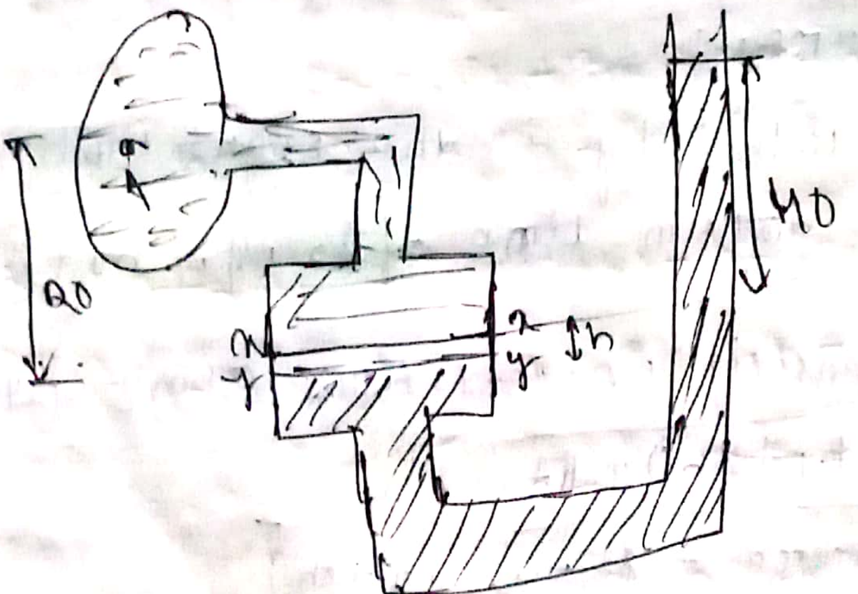
$$\boxed{P_A = \frac{a h_2}{A} \times (s_2 \cdot g - s_1 \cdot g) + s_2 \cdot g \cdot h_2 - s_1 \cdot g \cdot h_1}$$

negleting $\frac{a h^2}{A} \cdot (s_2 \cdot g - s_1 g)$ value as the area A is very large as compare to small a hence ratio $\frac{a}{A}$ become very small.

$$\boxed{P_A = s_2 \cdot g \cdot h_2 - s_1 \cdot g \cdot h_1}$$

Question:-

A single column manometre is conected to a pipe containing a liquid of specific gravity 0.9 as shown in figure. Find the pressure in the pipe the area of the reservoir is 100 times area of the tube for the manometre reading shown in figure. The specific gravity of mercury.



$$P_A = \frac{a h_2}{A} (s_2 g - s_1 g) + s_2 \cdot g \cdot h_2 - s_1 \cdot g \cdot h_1$$

$s_1 = 900 \, kg/m^3$ $\quad s_2 = 13600 \, kg/m^3$

$h_1 = 0.20 \, m$ $\quad h_2 = 0.40 \, m$

$$\frac{a}{A} = \frac{1}{100}$$

$\frac{1}{100} \times 0.40 (13600 \times 9.81 - 900 \times 9.81) + 13600 \times 9.81 \times 0.40 - 900 \times 9.81 \times 0.20$

$= 52098.948 \ kg/m^3$

Inclinde single column :-

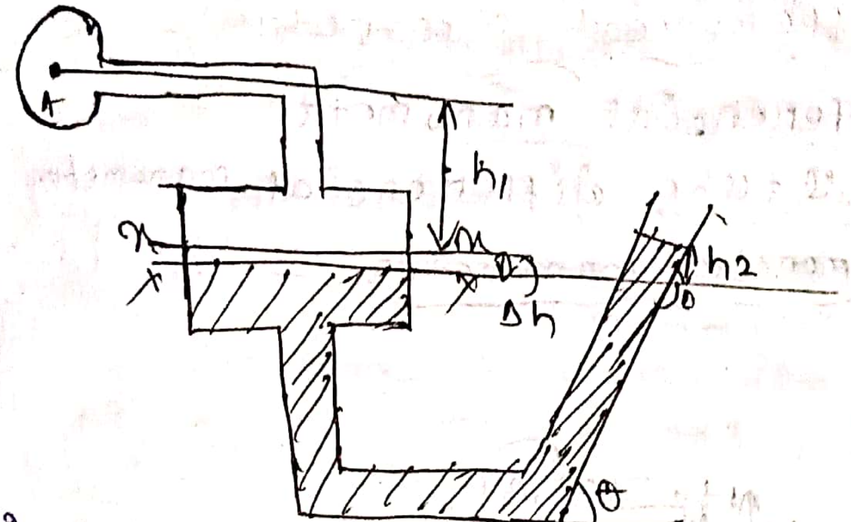


figure shows inclinde single column manometre. This manometre is more sensity due to inclination the distance move by the heavey liquid is right limb is more.

Let,

L = length of heavy liquid moved in righty from $x-x$.

θ = Inclindention of right limb with horizental.

h_2 = vertical rise of heavy liquid in right limb from $x-x$.

$h_2 = h \sin\theta$

$P_A = \sin\theta \, s_2 \cdot g \cdot h - s_1 g_1 h_1$

differencial manometre :-

> differencial manometre is devices are used for measuring the difference of pressure between two point in a pipe or in to two difference pipe

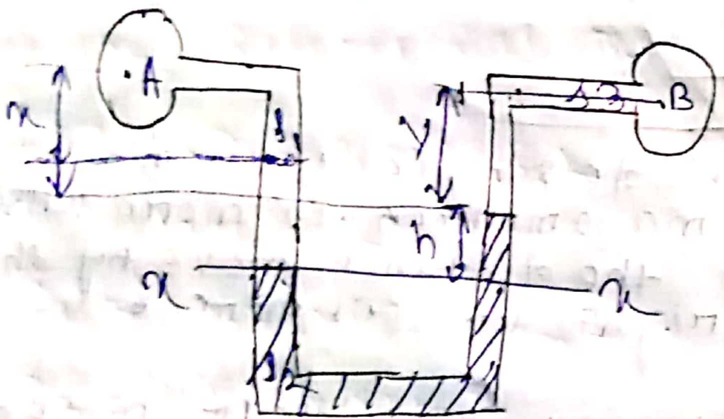
A differencial manometre is consist of a utube contening a heavy liquid whose to end as conected to the point whose difference of pressure is to be measure.

Types of differencial manometre :-

1-> utube differencial manometre

2-> Inverted utube differencial manometre

utube differencial manometre :-



Left limb $P_r = P_A + s_1 \cdot g(x+h)$

Right limb $P_r = P_B + s_3 \cdot g \cdot y + s_2 \cdot g \cdot h$

$P_A - P_B = P_A + s_1 \cdot g(x+h) - P_B + s_3 \cdot g \cdot y + s_2 \cdot g \cdot h$

$\Rightarrow P_A - P_B = s_3 \cdot g \cdot y + s_2 \cdot g \cdot h - s_1 \cdot g(x+h)$

$\Rightarrow s_3 \cdot g \cdot y + s_2 \cdot g \cdot h - s_1 \cdot g \cdot x + s_1 \cdot g \cdot h$

$P_A - P_B \Rightarrow s_3 \cdot g \cdot y - s_1 g \cdot x + gh(s_2 - s_1)$

h = difference of mercury level in the utube.

y = ~~discharge~~ distance of the centre of B from the mercury level

in the Right limb.
x = distance of the centre of a from the mercury level in the right limb

S_1 = density of liquid at A
S_2 = density of heavy liquid or mercury
S_3 = density of liquid at B

Problem :-

A pipe contenee an oil of specific gravity is 0.9, differaneial manometre connected at the two point A and B shows as difference in mercury level. as 15 cm. find the difference of pressure at the 2 point

given data :-

oil specific gravity $= 0.9 \times 1000 = 900\ kg/m^3$

$x = y$

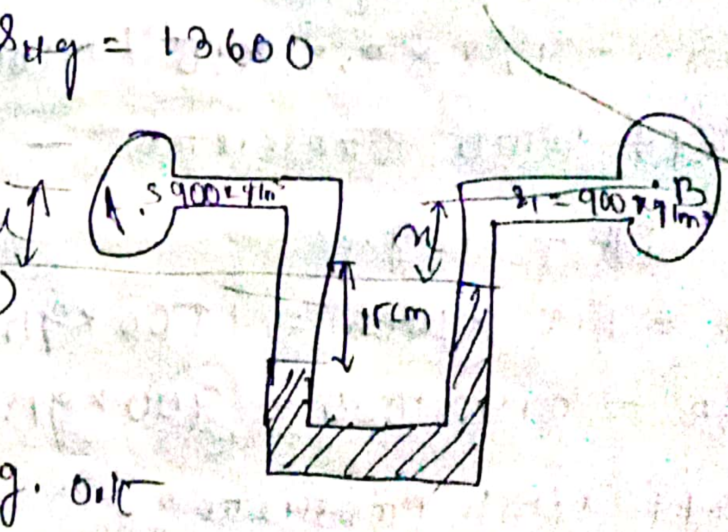
$S_3 . g . y - S_1 . g . y + gh(S_2 - S_1)$

$g . y(S_3 - S_1) + gh(S_2 - S_1)$ $h = 15cm = 0.15$

$S_{Hg} = 13.6$, $S_{Hg} = 13600$

Left limb pr :-

$= P_A + S_1 g(x + 0.15)$



Right limb pr :-

$P_B + S_1 g . x . S_{Hg} . g . 0.15$

$= P_A + 900 \times 9.81 (x + 0.15)$

$\Rightarrow P_B + 900 \times 9.81 . x . 13600 . 9.81 . 0.15$

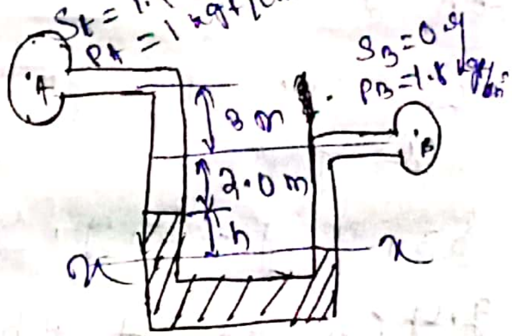
$P_A - P_B = 900 \times 9.81 (x + 0.15) - 900 \times 9.81 . x . 13600 \times 9.81 \times 0.15$

$P_A - P_B = 13600 \times 9.81 \times 0.15 - 900 \times 9.81 \times 0.15$

$= 18688.05$

Problem :-

A differencial manometre is connected are the 2 point A and B of 2 pipes as shown in figure. The pipe A contence a liquid of specific gravity equal to 1.5 pipe B contence liquid of specific gravity equals to 0.9 the pressure at A and B are $1\ kgf/cm^2$ and $1.80\ kgf/cm^2$ respectively find the difference in merccery level in the differencial manometre.



$1\ kgf/cm^2 = 9.81\ N/cm^2$

$= 9.81 \times 10^4\ N/m^2$

$P_A = 9.81 \times 10^4\ N/cm^2$

$P_B = 1.8\ kgf/cm^2$

$= 1.8 \times 9.81 \times 10^4\ N/cm^2 = 176580\ N/cm^2$

~~Left limb pressure :-~~

~~$P_A + S_1 \cdot g$~~

$S_A = 1.5 \times 1000 = 1500\ kg/m^3$

$S_B = 0.9 \times 1000 = 900\ kg/m^3$

Left limb pressure :-

$P_A + S_A \cdot g \cdot 5 + 13600 \times g \times h$

$\Rightarrow 9.81 \times 10^4 + 1500 \times 9.81 \times 5 + 13600 \times 9.81 \times h$

$\Rightarrow 171600 + 133416 \times h$

$\Rightarrow h = 305016$

Light limb pressure:-

$= P_B + \rho_m \cdot g \cdot (h+z)$

$= 1.8 \times 9.81 \times 10^4 + 900 \times 9.81(h+2)$

$= \cancel{1.8 \times 9.81 \times 10^3}$

$= \cancel{1...}$

$176580 + 8829h + 17658$

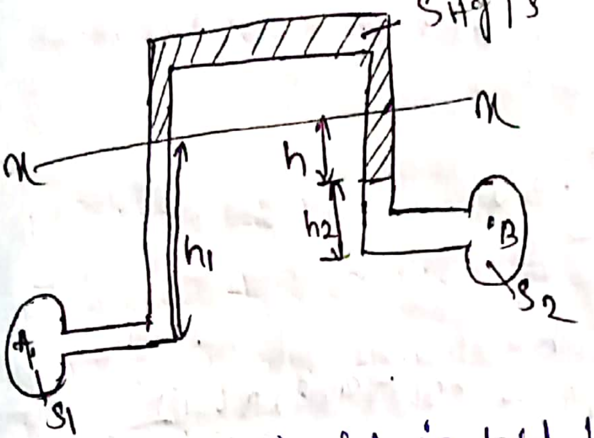
$= 185409h + 17658$

$h = 36198$

$h = \frac{36198}{305016}$

$h = \cancel{0.118m}\ 0.118m$

Inverted U-tube Differential manometre :-



h_1 = height of liquid in left limb below the datum x-x line

h_2 = hight of liquid in Right limb

h = difference of the light liquid.

δ_1 = density of liquid at A

δ_2 = density of liquid at B

δ_s = density of light liquid

P_A = Pressure at A

P_B = pressure at B

Left limb Pressure :-

$P_A - \delta_1 . g . h_1$

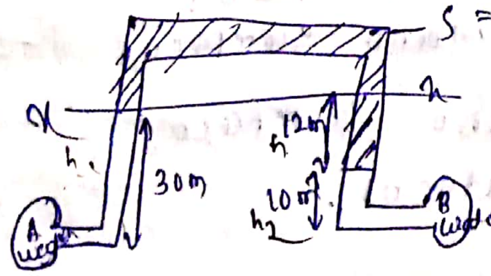
Right limb pressure :-

$P_B - \delta_2 . g . h_2 - \delta_s . g . h$

$P_A - P_B = \delta_1 . g . h_1 - \delta_2 . g . h_2 - \delta_s . g . h$

Problem :-

water is flowing through to difference pipes to which an inverted differential manometre having an oil of S.P gravity 0.9 is connect The pressure head in the pipe A is 2m of water. Find the pressure of pipe B for the manometre Reading as shown in figure



Pressure @ A = $\rho \cdot g \cdot h$

$= 1000 \times 9.81 \times 2$

$= 19620 \ N/m^2$

Left limb pressure :-

$P_A - \rho_1 \cdot g \cdot h_1$

$19620 - 1000 \times 9.81 \times 0.30$

$= 16677$

$P_B = \rho_2 \cdot g \cdot h_2 - \rho_3 \cdot g \cdot h$

$P_B = 1000 \times 9.81 \times 0.10 - 500 \times 9.81 \times 0.12$

dt - 18/01/2020

Hydrostatic Process on surfaces :-

Total pressure :-

-> Total pressure is defined as the force exerted by a static fluid on the surface either plane or curve when the fluid comes in contact ~~which~~ with the surfaces. This force always act normal to the surface.

Centre of pressure :-

Centre of Pressure is defined as the point of application of the total pressure on the surface.

There are 4 cases of submerged surfaces on which the total pressure forces and centre of pressure is to be determined

(1) Vertical plane surfaces

(2) horizontal plane surfaces

(3) Inclined plane surfaces

(4) curved surfaces.

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Vertical plane surface submerged in liquid :-

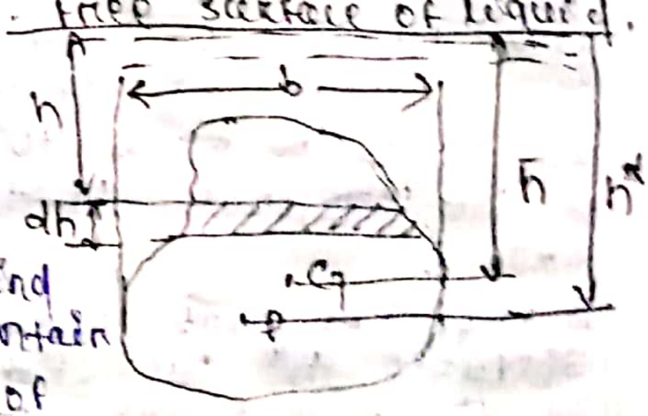
let, A = total area of the surfaces.

$\bar{h}$ = distance of C.G of the area from free surface of liquid.

G = centre of gravity of plane surface.

P = centre of pressure.

h^* = distance of centre of pressure from free surface of liquid.



imp
Total pressure :-

The total pressure on the surface may be determind by deviding the ~~tatian~~ intain surface into a number of small parallel strip

→ The force on small strip is then calculated and the total pressure force on the hale area is calculate by integrating by the force in small strip.

→ consider the ~~strip~~ strip of thickness dh and the width b and at a depth of h from free surface of liquid as shown in figure.

$$P = \rho.g.h$$

$$P = \frac{\text{force}}{\text{Area}}$$

force = P × Area

$= P \times dh \times b$

$$dF = \rho.g.h \times dh.b$$

$$\Rightarrow \int dF = \int \rho.g.h.dA \quad (\because dh.b = dA)$$

$$\Rightarrow \boxed{F = \rho g \int h.dA}$$

$\boxed{\int h \cdot dA}$ = moment of surface Area about the free surface of liquid

=> (that is Area of surface × distance of C.G in free surface)

= $\boxed{F = \rho \cdot g \cdot \bar{h} \cdot A}$

Centre of pressure P_c :-

Centre of Pressure is calculated by using the principle of moment which states that the moment of resultant force about an axis is equal to the sum of moment of the componets of the about the same axis.

-> The resultant force F is active at distance h^* from free surface of the liquid as shown in figure. ~~hence~~

hence the moment of the force dF about free surface of liquid equal to $F \times h^*$

moment of force dF, acting on strip about free surface of liquid.

$= dF \times h$

$= \rho \cdot g \cdot h \cdot dh \cdot b \times h$

Sum of moments of all such froces about free surface of liquid

$\Rightarrow \int \rho \cdot g \cdot h \cdot h \cdot dA$

$= \rho \cdot g \int dA \cdot h^2$

$\boxed{\int dA \cdot h^2}$ = moment of inertia of surface about free surface of liquid I_0

Sum of about free surface of liquid

$= \rho \cdot g \cdot I_0$

Sum of moment about free surface of liquid $= \rho . g . I_0$

$$\boxed{\begin{aligned} F \times h^* &= \rho . g . I_0 \\ h^* &= \frac{\rho g . I_0}{F} \end{aligned}}$$

$$\Rightarrow h^* = \frac{\rho . g . I_0}{\rho . g . \bar{h} A}$$

$$= h^* = \frac{I_0}{\bar{h} A} \quad \boxed{h^* = \frac{I_0}{A \bar{h}}}$$

$$I_0 = I_G + A \bar{h}^2$$

$$h^* = \frac{I_0}{A\bar{h}} = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \cancel{I_G} + \bar{h}$$

$$h^* = \frac{I_G}{A\bar{h}} + \frac{A\bar{h}^2}{A\bar{h}}$$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$ — centre ~~total~~ pressure.

$$\boxed{F = \rho . g . A\bar{h}}$$ — total ~~centre~~ of pressure

~~Problem~~:—

hence centre of pressure h^* lies below the centre of gravity of vertical surface

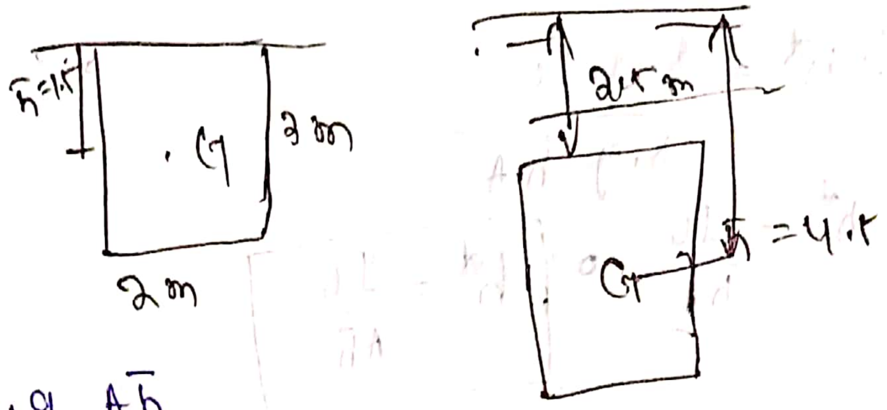
→ the distance of centre of pressure from free surface of liquid is independed of the density of the liquid.

Problem:—

A rectangular plane surface is ~~based~~ 2m wide and 3m deep it lies in vertical plan in water determind the total pressure and position of centre of pressure on the plane surface when its offer .

edge is horizontal and (A) coincide with water surface (B) 2.5 m below the free water surface.

given data:-



(a)

$$F_G = \rho . g . A\bar{h}$$

$$= 1000 \times 9.81 \times 2 \times 3 \times 1.5$$

$$= 88290$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5$$

$$h^* = \frac{4.5}{3 \times 2 \times 1.5} + 4.5$$

$$= 2 m$$

(b)

$$F = \rho . g . \bar{h} . A$$

$$= 1000 \times 9.81 \times 4 \times 6$$

$$= 235440 N$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$= \frac{4.5}{6 \times 4} + 4 = 4.18 m$$

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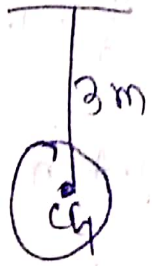
Problem :-

determine the total pressure on a circular plane on diameter 1.5m which is placed vertically in water in such away that the centre of plate is 3m below the free surface of water find the ~~area~~ centre of pressure also



$$I_G = \frac{\pi d^4}{64} = 0.24$$

$$A = \pi r^2 = 3.14 \times (0.75)^2$$
$$= 1.766$$

$$\bar{h} = 3m$$

$$F = \rho . g . \bar{h} A$$
$$= 1000 \times 9.81 \times 3 \times 1.76$$
$$= 1.1796 08 N$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$
$$= \frac{0.24}{1.76 \times 3} + 3 = 3.04 m$$

Problem :- 21.04.2020

determine the total pressure and centre of pressure on an triangle plate of base 4m and Altitude 4m when it is imorge vertically in an oil of specific gravity 0.9. the base of the plate coincide with the free surface of oil.

given data :-

$$I_G = \frac{bh^3}{36} \qquad b = 4m$$
$$\qquad h = 4m$$
$$S_o = 0.9$$

$I_G = \frac{bh^3}{36}$

$= \frac{4 \times 4^3}{36}$

$= 7.11$

$\bar{h} = \frac{h}{3} = \frac{4}{3} = 1.33$

$F = \rho . g . A . \bar{h}$

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 4$

$= 8 m$

$h^* = \frac{7.11}{8 \times 1.33} + 1.33$

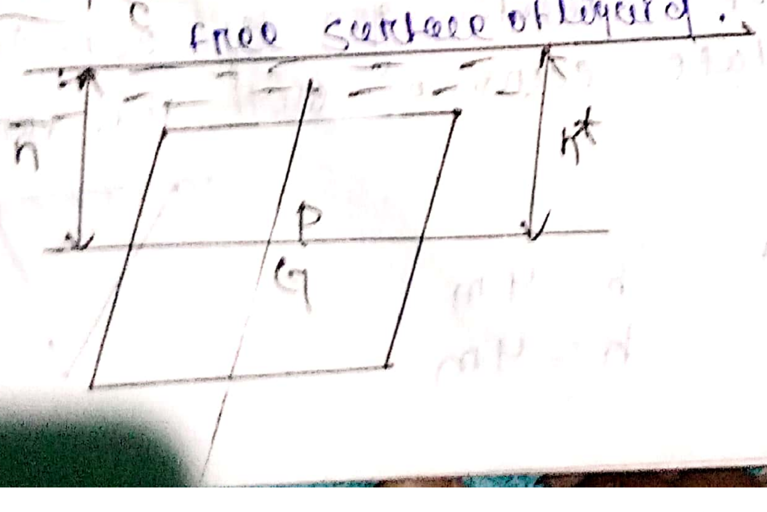
$= 1.99 m$

$F = \rho . g . A . \bar{h}$

$= 900 \times 9.81 \times 8 \times 1.33$

$= 93940.56 m$

horizontal plan surface submerged in liquid :-



A = Total Area of surface Total force or pressure

$\Rightarrow \boxed{F = P \times Area}$

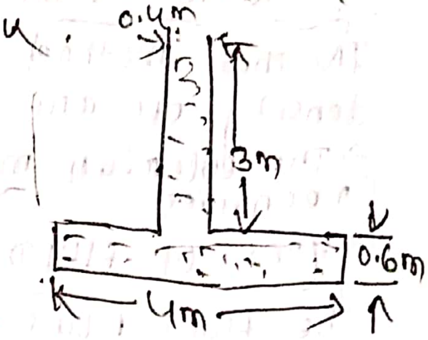
$\Rightarrow P \cdot A$

$\Rightarrow \boxed{F = \rho \cdot g \cdot h \cdot A}$

* $\boxed{h^* = \bar{h}}$

Problem:-

figure shows a tank full of water find
(i) total pressure on the bottom of the tank
(ii) weight of water in the tank.
width of tank is 2m.



given data

$A = b \times d = 4 \times 2 = 8$

$h = 3 + 0.6 = 3.6$

(i) $F = \rho \cdot g \cdot h \cdot A$

$F = 1000 \times 9.81 \times 3.6 \times 8$

$= 282528$

weight of tank $= \gamma \times volume$

$= \rho \cdot g \cdot volume$

$= 1000 \times 9.81 \times volume$

$volume = 0.4 \times 3 \times 2 + 4 \times 0.6 \times 2$

$= 7.2m$

$= \rho \cdot g \cdot volume$

$= 1000 \times 9.81 \times 7.2$

$= 70632m.$

KINAMETICS OF FLUID FLOW :-

KINAMETICS :-

Dt - 22.01.2021.

KINAMETICS is defind as that branch of science which deals with motion of particle without considering the force causing the motion.

METHOD of DESCRIBE FLUID MOTION :-

Lagrangian method :-

In this method A single fluid particle is followed during its motion and its velocity, acceleration, density etc are describe.

EULERIAN METHOD :-

IN this method the velocity, accelaration, pressure, density etc are describe at a point in flow field.

→ The eulerian method is commonly use in fluid mechanics.

TYPES OF FLUID FLOWS :-

The fluid flows is classified as

(1) Steady and unsteady flows ;

(2) uniform and non-uniform flows ;

(3) Laminar and turbulent flows ;

(4) compressible and incompressible flows ;

(5) Rotational and irrotational flows ; and

(6) one, two and three-dimensional flows.

(1) STEADY AND UNSTEADY FLOWS :-

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density respect to time. Thus, mathematically, for unsteady flow.

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc}$$

2) UNIFORM AND NON-UNIFORM FLOWS :-

~~uniform flow is defined as which the velocity at any given time does not change with respect to space (i.e. - the flow). mathematically, for uniform flow~~

Uniform flow is defined as that type of flow at which the velocity at any given time does not change with respect to space (i.e length of direction of the flow). mathematically, for uniform flow,

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}}=0$$

Where.

∂v = change of velocity

∂s = length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow.

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}}\neq 0$$

3) LAMINAR AND TURBULENT FLOWS :-

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. The type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number called the Reynold number $= \boxed{\frac{VD}{\nu}}$

where D = Diameter of pipe

V = mean velocity of flow in pipe.

and ν = kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

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(4) Compressible and incompressible flows :-

Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$\rho \neq$ constant

Incompressible flow is that type of flow in which the density is constant for the fluid flow, liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

ρ = constant

(5) Rotational and Irrotational flows :-

Rotational flow is that type of flow in which the fluid particle while flowing along stream-lines, also rotate about their own axis. And if the fluid particle while flowing along stream-line, do not rotate about their own axis, ~~And if the fluid particles while flowing along~~ then that type of flow is called irrotational flow.

(6) One-, Two- and Three-Dimensional flows :-

One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z direction respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow

$$u = f_1(x,y),\ v = f_2(x,y) \text{ and } w = 0$$

Three dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular direction. But for a steady three-dimensional flow the

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Fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z),\ v = f_2(x, y, z) \text{ and } w = f_3(x, y, z)$$

RATE OF FLUID :-

It is defined as the quantity of fluid flowing per section

→ For an incompressible fluid (liquid) the rate of flow or discharge is express as the volume of fluid flowing across the section per second

$\boxed{Q = A \times V}$ Q = discharge m^3/s or cumec or lit/sec
A = cross-section of pipe
V = average velocity

→ For compressible fluid the rate of flow is usually expressed as the weight of fluid flowing across the section

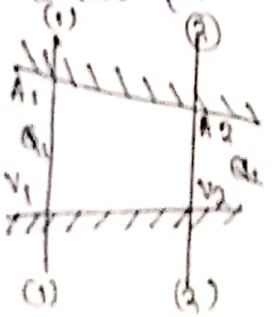
$\boxed{Q = A \times V \times Y}$ when $\boxed{s.g = Y}$

For gases unit of Q = Kgf/sec or N/s

CONTINUITY EQUATION :-

The equation based on the principle of conservation of mass is called continuity equation.

conservation mass :- This mass not be created or not be destroyed.



$A_1V_1 = A_2V_2$

$Q_1 = Q_2$

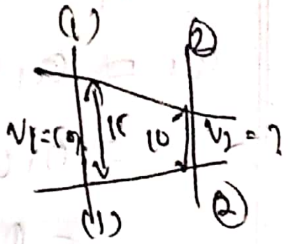
where V_1 = average velocity at cross section (1-1)
A_1 = Area of pipe at section (1-1)
and V_2, A_2 at corresponding value at section (2-2)

rate of flow at section 1-1 = A_1V_1
rate of flow at section 2-2 = A_2V_2.

Note :-
This equation for fluid which is incompressible (liquid)
for gas the modify equation $\rho_1A_1V_1 = \rho_2A_2V_2$
where, ρ_1 = density of fluid.

Problem:-

The diametre of pipe in the section 1 and 2 are 15cm and 10cm respectivily. find the discharge of the pipe if the velocity of water flowing through the pipe at the section 1 is 5 m/s and determind the velocity of the section 2.



given data is:-

$V_1 = 5\ m/s$

$d_1 = 15cm$

$d_2 = 10cm$ $\qquad d = 0.15$

Area of (1) $= \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.15^2 = 0.0176 m^2$

$V_1 = 5 m/s$

$Q = A_1 V_1$

$= 0.0176 \times 5 = 0.088\ m^3/s$

$Q_1 = Q_2$

$= Q = Q = 0.088$

$\Rightarrow A_2 \cdot V_2 = 0.088$ $\qquad A_2 = \frac{\pi}{4} \times (0.10)^2 = 7.85 \times 10^{-3}$

$\Rightarrow$ ~~0.078~~ ~~0.088~~

$\cancel{V_2 = \frac{0.088}{0.078}} = 1.128$

$A_2 = 7.85 \times 10^{-3}$

$V_2 = \frac{0.088}{7.85 \times 10^{-3}} = 11.21\ m/s$

Problem:-

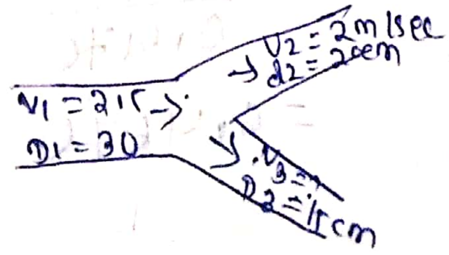
A 30 cm diametre pipe conveing water branches into 2 pipe of diameter 20cm and 15cm respectivily. If the avrage velocity in this 30cm diameter pipe is 2.5 m/s. find the discharge in this pipe. also determind the velocity in 15cm pipe if the avrage velocity in 20 cm diameter pipe in 2m/s

given data

$V_1 = 2.5$ $\qquad V_2 = 2\ m/sec$

$d_1 = 30$ $\qquad d_2 = 20m$

$d_3 = 15$ $\qquad V_3 = ?$



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$A = \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times 0.30^2 = 0.070$

$Q_1 = a_1 v_1$

$= 0.070 \times 2.5$

$= 0.17662 \ m^3/s$

$\cancel{A_3 = \frac{\pi}{4} \times 0.15^2}$

$\cancel{= 0.017}$

$\cancel{Q_2 = a_2 v_2}$

$\cancel{= 0.017 \times 2}$

$\cancel{= 0.034}$

$A_2 = \frac{\pi}{4} \times 0.20$

$= 0.031$

$Q_2 = a_2 v_2$

$= 0.03 \times 2$

$= 0.0628 \ m^3/s$

$A_3 = \frac{\pi}{4} \times 0.15^2$

$= 0.0176$

$a_1 = a_2 + a_3$

$= \cancel{0.175 = 0.0628 + a_3 \ 0.0176}$

$Q_3 = Q_1 - Q_2$

$= 0.17662 - 0.0628 = 0.11382 \ m^3/s$

$Q \Rightarrow A_3 v_3 \Rightarrow 0.0178 \times v_3$

$V_3 = \frac{Q_3}{A_3}$

$= \frac{0.11382}{0.0176}$

$= 6.4670$

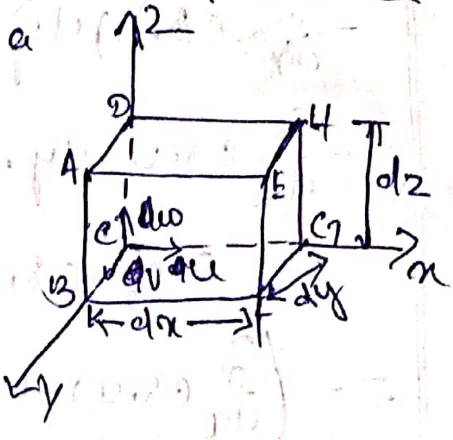
0.06

7.6

Dt - 27.01.2020

CONTINUITY EQUATION IN THREE DIMENSIONAL FLUID FLOW:-

Consider a fluid element ABCD in a fluid



Let, du, dv, dw are velocity of liquid flowing in x, y, z direction respectively.

Let the mass flow rate of liquid flowing in the element through the face ABCD.

= Density × velocity × Area

$= \rho \cdot u \times (dy \times dz)$

Then flow of liquid outward direction through the face EFGH.

$$= \rho \cdot u \cdot dy \cdot dz + \frac{\partial}{\partial x}(\rho \cdot u \cdot dy \cdot dz) \times dx$$

Inward :-

$= \rho \cdot u \cdot dy \cdot dz$

Outward flow :-

$\rho \cdot u \cdot dy \cdot dz + \frac{\partial}{\partial x}(\rho \cdot u \cdot dy \cdot dz) \times dx$

mass of liquid stored

$\Rightarrow \cancel{\rho \cdot u \cdot dy \cdot dz} - \cancel{\rho \cdot u \cdot dy \cdot dz} - \frac{\partial}{\partial x}(\rho \cdot u \cdot dy \cdot dz)\, dx$

$$\boxed{= -\frac{\partial}{\partial x}(\rho \cdot u \cdot dy \cdot dz)\, dx}$$

Similarly in y direction mass stored is $\boxed{= -\frac{\partial}{\partial y}(\rho \cdot v \cdot dx \cdot dy \cdot dz)}$

in z direction

$$\boxed{= -\frac{\partial}{\partial z}(\rho \cdot w \cdot dx \cdot dy \cdot dz)}$$

Total mass stored in the element ABCDEFGH

$$-\cancel{\frac{\partial}{\partial x}(\rho u)\,dy\,dz\,dx} - \frac{\partial}{\partial y}(\rho v$$

$$= -\frac{\partial}{\partial x}(\rho\cdot u\cdot dy\cdot dz\cdot dx) - \frac{\partial}{\partial y}(\rho\cdot v\cdot dx\cdot dy\cdot dz)$$
$$- \frac{\partial}{\partial z}(\rho\cdot w\cdot dx\cdot dy\cdot dz)$$

$$= -\left(\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w)\right) dx\cdot dy\cdot dz \quad (1)$$

Sence The mass is neither created not destroid in the fluid element. The net increase in mass or total mass store per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.

Total mass stored in the element

$$= -\left[\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w)\right] dx\cdot dy\cdot dz$$

Mass of element.

$$= \text{Density} \times \text{Volume}$$
$$= \rho \times dx\cdot dy\cdot dz$$

Rate of variation of mass of liquid per sec.

$$= \frac{\partial}{\partial t}(\rho\cdot dx\cdot dy\cdot dz) \quad (2)$$

$$= \frac{\partial}{\partial t}(\rho)\cdot dx\cdot dy\cdot dz$$

Equality equation (1) & (2)

$$-\left[\frac{\partial(\rho\cdot u)}{\partial x} + \frac{\partial(\rho\cdot v)}{\partial y} + \frac{\partial(\rho\cdot w)}{\partial z}\right]\cdot dx\cdot \cancel{dy\cdot dz}$$

$$= \frac{\partial}{\partial t}(\rho)\cdot dx\cdot \cancel{dy\cdot dz}.$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho.u)}{\partial x} + \frac{\partial(\rho.v)}{\partial y} + \frac{\partial(\rho.w)}{\partial z} = 0}$$ — contineauty equation

equation 3 is generalised from of contineacuty equation in cartesian ~~coordinat~~ coordinet. This equation is applicable to

(1) steady and unsteady flow

(2) unifrom and ununifrom flow

(3) compressible and incompressible fluid.

For steady flow :—

$$\frac{\partial \rho}{\partial t} = 0$$

so, equation will be

$$\boxed{\frac{\partial(\rho.u)}{\partial x} + \frac{\partial(\rho.v)}{\partial y} + \frac{\partial(\rho.w)}{\partial z}} \quad -(4)$$

If the fluid is incompressible the

ρ = constant

$$\boxed{\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0} \quad -(5)$$

For 2 diamensional fluid flow :—

for 2 diamensional fluid flow velocity componet

$w = 0$

so, the equation = $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$

Types of flowline :—

dt - 28.01.2020

Path Line :—

The path flower by fluid praticle in motion is called path line.

→ Thus path line shows the direction of praticle for certain period of time or between to given seection.

STREAM Line :—

The imagninary line drawn in the fluid in such a way that the changent to any point gives the direction of motion at the point is called stream line.

→ Thus the stream line shows the direction of motion of a number of paaricles at the same times.

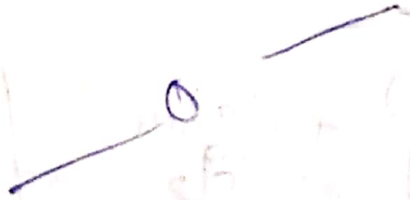
an element of fluid, bonded by a number of stream lines - which confine the flow is called stream Tube. Are there is known since moment of fluid equals a stream line, therefore no fluid equal can be entre of leave the stream tube except at the each it is thus obvious that a stream tube behaves like a solid tube

STREAK LINE :—

The instantaneous picture of a position of all the fluid particles, which have passed through given point at some previous time are called streak line.

The steady and uniform flow :—

The steam, steak line and path line are lies in same line.



DYNAMICS OF FLUID FLOW :—

In this chapter we shall discuss the following.

(1) motion of the fluid or liquid and the force causing the flow which is known as hydrodynamics.

ENERGY OF LIQUID IN MOTION :—

(1) potential energy – Z - Meters

(2) kinetic energy – $\frac{V^2}{2g}$ meters of liquid

(3) pressure energy – $\frac{P}{\gamma}$ meters of liquid

Total energy $= Z + \frac{V^2}{2g} + \frac{P}{\gamma}$

POTENTIAL ENERGY :—

It is the energy expressed by the liquid particles by vertue of its position. If a fluid ~~par~~ or liquid particles is Z meter above the horizontal datum, the potential energy of the particle will be Z meter into killogram per ~~[illegible]~~ kg of liquid.

$(m - kg/kg)$

the potential head of the liquid at the point will be Z m of the liquid.

(2) KINETICS ENERGY :—

kinetics energy of a liquid particle in motion posess by a liquid particle by ~~ver~~ vertue of its motion or velocity. If a liquid particle is flowing with a main velocity v m/s then the kinetics energy of the particle will be $\frac{V^2}{2g}\left(\frac{m-kg}{kg}\right)$ of liquid.

(3) PRESSURE ENERGY OF LIQUID PARTICLE IN MOTION :—

It is the energy posess by a liquid particle by vertue of its existing pressure. If a liquid particle is under pressure @ P - kN/m^2 then the pressure energy of the particle will be $\frac{P}{\gamma}$

(4) TOTAL ENERGY :—

A total energy of a liquid in motion is the some of its potential energy kinetics energy and pressure energy. mathematically.

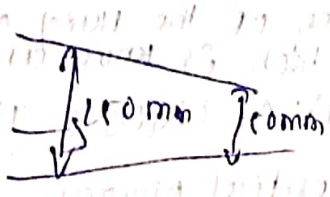
total pressure $= Z + \frac{V^2}{2g} + \frac{P}{\gamma}$

Whater is flowing through tampped pipe having end diameter of 150mm and 70mm respectively find the discharge of the large end at the velocity head at the smaller end, If the the velocity of water at larger end is 2 m/s.

Given data

$d_1 = 150mm = 0.15m$, $v_1 = 2m/s$

$d_2 = 50mm = 0.50m$



$Q_1 = a_1 v_1$

$A = \frac{\pi}{4}\times 0.150^2 = 0.0176$

$Q = 0.0176\times 2$
$= 0.0352$

$Q = a_2 v_2$

$A_2 = \frac{\pi}{4}\times 0.05^2 = 1.96\times 10^{-3}$

$Q = a_2 \cdot v_2$

continuity eqn

$a_1 v_1 = a_2 v_2$

$\cancel{0.0176\times 2 = 1.96}\, v_2 \Rightarrow \frac{0.0352}{1.96\times 10^{-3}}$

$\cancel{v_2 = \frac{0.0352}{1.9}}$

$= 17.95$

$\cancel{Q_1 = Q_2}$

$\cancel{0.0352 = 1.96\, v_2}$

velocity head $= \frac{v_2^2}{2g}$

$= \frac{(17.95)^2}{2\times 9.81}$

$= 16.42 m$

dt:- 04/02/2020

Bernoulls Therems :-

"for a perfect incompressible liquid flowing in a continous stream the total energy of a particle remains the Same, while the particle moves from one point to anothere this statement based on the assumption that there are no loss due to friction in pip

$$\boxed{Z + \frac{V^2}{2g} + \frac{P}{\gamma g} = \text{constant}}$$

Z = potential energy

$\frac{V^2}{2g}$ = kinetic energy per unit wt

Bernoulls Equation :-

The Berenoulls equation has been derived under the assumption that the velocity of every liquid particle across any crossection of a pipe is uniform.

But, in actual practicle it is not so. the velocity of liquide particle in the center of pipe is maxm and gradually decriese to wards. The wall of the pipe due to the pipe friction.

Thus while using the bernoulls equation only the mean velocity of the liquid should be taken into account.

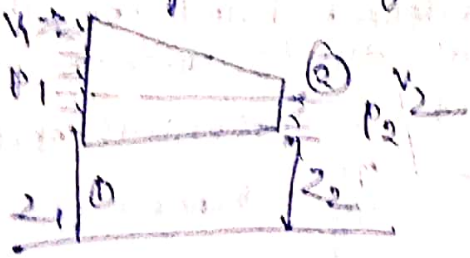
→ The Bernoulls equation has been derived under the assuption that no external force expect the gravity force is acting on the liquid But in actual particle it is not so there are always some external force (such as pipe friction etc) Acting on the liquid which effect the flow of the liquid thus while using the berenoulls equation all such external forces should be neglected; But if some energy supplyied to or extrected from the flow the some should be taken into account.

→ the bernoulls equation has been derived and the assption that there is no less of energy of the liquid particle while flowing.

But it actual practicle it is reardly in a turbulent flow some kinatic energy is converted into heat energy and in viscocity flow some energy is less due to sheare forcess.

Thus, while using Bernoulli's equation all such losses should be neglected.

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}.$$



Q- The water flowing taper pipe of length 100m having diameter 600mm at the upper end & 300mm diameter at the lower end at the rate of 50 l/sec. The pipe has a slope 1 in 50 find the pressure at the lower end if the pressure at higher level is 19.62 N/cm²

Given data :-

$D_1 = 600mm$

$D_2 = 300mm$

$l = 100m$

Slope = 1 in 50

$Q = 50$ l/s
$= 50 \times 10^{-3}$

$A_1 = \frac{\pi}{4} D_1^2 = 0.28 m^2$

$A_2 = \frac{\pi}{4} D_2^2 = 0.07 m^2$

$P_1 = 19.62 N/$
$= 19.62 \times 10^4 N/m^2$

$Q = V_1 A_1$

$$V_1 = \frac{Q}{A_1} = \frac{50 \times 10^{-3}}{0.28} = 0.17$$

$$V_2 = \frac{Q}{A_2} = 0.71$$

$$z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = z_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

$$z_1 - z_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + \frac{P_2}{\rho g} - \frac{P_1}{\rho g}$$

$$= \frac{(0.71)^2}{2 \times 9.81} - \frac{(0.17)^2}{2 \times 9.81} + \frac{19.62 \times 10^4}{1000 \times 9.81} - \frac{P_2}{1000 \times 9.81}$$

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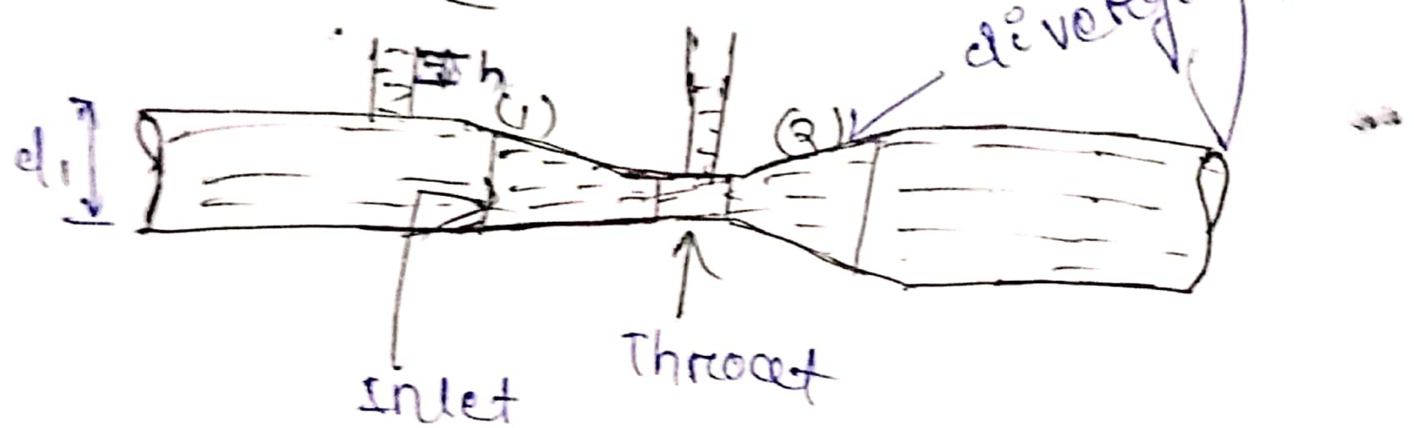
dt 06/07/2020

Application of Bernoulli's Equation :-

(1) venturi meter
(2) orifice meter
(3) pitot - tube.

Venturimeter :-



(Venturimeter)

Let d_1 = Diameter at inlet
P_1 = Pressure at section (1)
V_1 = velocity of fluid at section (1)
a_1 = Area at section (1)

Similarly, d_2, P_2, V_2, a_2 corresponding velocity @ (2)

Venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.

It consist of 3 parts
- A short converging part
- throat
- Diverging part

applying Bernoull's equation ^{n}-

$$\cancel{z_1} + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \cancel{z_2} + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}.$$

As Venturimeter is in Horizontal.

$$z_1 = z_2$$

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$$

$$\Rightarrow \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \cdot \left(\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = h\right)$$

$$= h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

~~Applying~~.

~~applying~~ applying continuity eqn to section 1 and (2)

$$a_1 V_1 = a_2 V_2$$

$$\Rightarrow V_1 = \frac{a_2 V_2}{a_1}$$

Put the V_1 value in eq (1).

$$\Rightarrow h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}.$$

$$\Rightarrow h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{\frac{a_2^2 \cdot V_2^2}{a_1^2}}{2g}$$

$$\Rightarrow 2 \cdot g \cdot h = V_2^2 - \frac{a_2^2 V_2^2}{a_1^2}$$

$$= \frac{V_2^2 a_1^2 - a_2^2 V_2^2}{a_1^2}$$

$$\Rightarrow 2 \cdot g \cdot h = V_2^2 \left(\frac{a_1^2 a_2^2}{a_1^2} \right)$$

$$\Rightarrow V_2^2 = (2 \cdot g \cdot h) \cdot \left(\frac{a_1^2}{a_1^2 - a_2^2} \right)$$

$$= \boxed{V_2 = \sqrt{2 \cdot g \cdot h} \times \frac{a_1}{\sqrt{a_1^2 - a_2^2}}}$$

$$Q = Q_2 \cdot V_2 = a_1 V_1$$
$$= a_2 \cdot V_2$$

$$\boxed{Q = a_1 \times \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2 \cdot g \cdot h}}$$ → Theoretical discharge formula

$$Q_{Actual} = C_d \times a_2$$

$$\boxed{Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

C_d = co-efficent of venturimeter (value less than 1)

Value of h given by differential u tube manometer :-

Case 1 :-

Let, the differential contence's a liquid which is heavier then the liquid flowing through the pipe.

Let,

Sh = specific gravity of heavier liquid

So = Specific gravity of the liquid flowing through the pipe.

x = differance of heavier liquid column in u-tube.

$$\boxed{h = x\left[\frac{S_h}{S_0} - 1\right]}$$

Case-2 :–

If the differential manometer contance a liquid which is lighter than the liquid flowing through the pipe the value of h is given by

$$\boxed{h = x\left[1 - \frac{S_L}{S_0}\right]}$$ → where S_L = specific gravity of light liquid

Case-3 :–

Inclined venturimeter with differential u-tube manometer.

$$\boxed{\frac{P_1}{\rho g} + z_1 - \frac{P_2}{\rho g} + z_2 = h}$$

The above 2 cases are given for a horizontal venturimeter. this case is related to inclined venturimeter having differential u-tube manometer let,

the differential manometre contence heavier liquid. then h is given by

$$\boxed{h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_0} - 1\right]}$$

Case-4 :–

let, the differential manometer contance lighter liquid contence then h is given by

$$\boxed{h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_h}{S_0}\right]}$$

Q) A horizontal venturimeter with ~~ele~~ ~~enter~~ inlet and through diameter 30cm and 15cm respectively is used to measured the flow of water. Reading of diffential manometer connected to the inlet and the throught is

$$\boxed{h = x\left[\frac{S_h}{S_0} - 1\right]}$$

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→ where S_L = specific Gravity of light liquid

Case-3 :-

Inclind venturimeter with differential u-tube manometer.

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Case-4 :-

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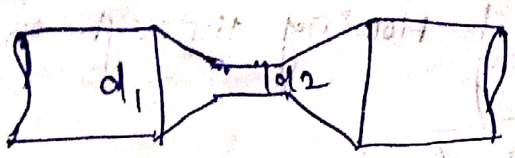
$$\boxed{h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_h}{S_0}\right]}$$

Q. A horizontal venturimeter with ~~ent~~ inlet and through diameter 30cm and 15cm respectively is used to measured the flow of water. Reading of differential manometer connected to the inlet and the throught is is 20 cm of Hg. determine the rate of flow. take Cd = 0.98

given data:—



$d_1 = 30cm$
$d_2 = 15cm$
$x = 20cm$
$cd = 0.98$

$$Q_{act} = cd \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$a_1 = \frac{\pi}{4} \times 30^2 = 0.07068$$

$$a_2 = \frac{\pi}{4} \times 15^2 = 0.0176$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

$$= 0.2\left[\frac{13.6}{1} - 1\right] = 2.52m$$

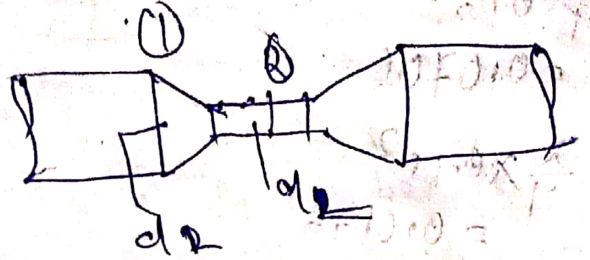
$$= 0.98 \times \frac{0.07068 \times 0.0176}{\sqrt{(0.07068)^2 - (0.0176)^2}} \times \sqrt{2 \times 9.81 \times 2.52}$$

$$= 0.1257 \ m^3/s$$

$$= 125.7 \ litre/sec$$

Q. An oil of specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat diameter 10cm. the oil Hg differential manometer shown a reading of 25cm. calculate the discharge of oil through the horizontal venturimeter. take Cd = 0.98.

Given data :-



Soil = 0.8

Sh = 13.6

d1 = 20cm

d2 = 10cm

Cd = 0.98.

x = 25 cm

$a_1 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\, m$

$a_2 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3} m$

$$Q_{act} = cd \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$h = x \left[\frac{S_h}{S_o} - 1\right]$$

$$= 0.25 \left[\frac{13.6}{0.8} - 1\right] = 4$$

$$Q_{act} = 0.98 \times \frac{0.0314 \times 7.85 \times 10^{-3}}{\sqrt{(0.0314)^2 - (7.85 \times 10^{-3})^2}} \times \sqrt{2 \times 9.81 \times 4}$$

$= 0.07038\ m^3/s.$

$=$ ~~7038 80.85~~ 70.38 l/s.

A 30cm in 15cm into venturimeter is in sected in a vertical pipe there is water flowing in the upward direction A differential Hg manometer connected to the inlet and the end gives a reading of 20cm find the discharge series cd = 0.98.

given data :-

d1 = 30

d2 = 15

cd = 0.98

sh = 13.6

x = 20cm

$a_1 = \frac{\pi}{4} \times 0.3^2$
$= 0.0706$

$a_2 = \frac{\pi}{4} \times 0.15^2$
$= 0.0176$

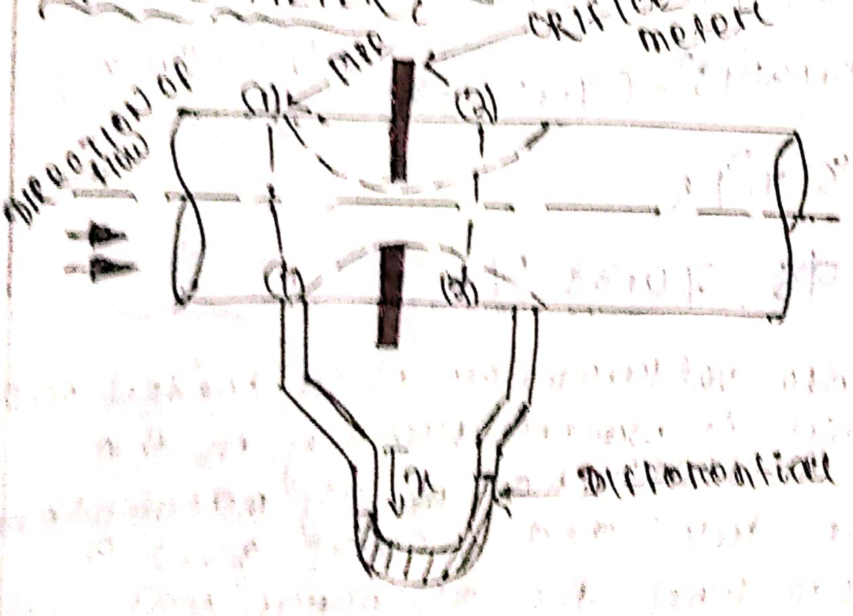
$h = x\left[\frac{S_h}{S_o} - 1\right]$
$= 0.20\left[\frac{13.6}{1} - 1\right]$
$= 2.52$

$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$= 0.98 \times \frac{0.0706 \times 0.0176}{\sqrt{(0.0706)^2 - (0.0176)^2}} \times \sqrt{2 \times 9.81 \times 2.52}$

$= 0.1252 \, m^3/s$
$= 125.2 \, L/s$

ORIFICE METER :-



$$\boxed{Q_{act} = \frac{C_d \cdot a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

for heavy $= x\left[\frac{S_h}{S_o} - 1\right]$
for light $x\left[1 - \frac{S_h}{S_o}\right]$

where C_d = coefficient of discharge ~~~~ for orifice meter

Note :-

coefficient of discharge for orifice meter is much smaller than that from a venturimeter.

A_0 = Area of orifice

a_1 = are of the pipe section one,

h = pressure head difference between section 1 & 2

Q An orifice meter with orifice diameter 10cm is inserted in a pipe of 20cm diameter the pressure gauges fitted upstream and downstream of the orifice meter gives reading of 19.62 N/cm² & 9.81 N/cm² respectively. coefficient of discharge for the orifice meter is given as 0.6. find of the discharge water through pipe.

given data :-

$d_1 = 20cm = 0.2$

$d_0 = 10cm = 0.1$

$P_1 = 19.62\ N/cm^2 = \cancel{0.1962\ N/m^2}$

$P_2 = 9.81\ N/cm^2$

$C_d = 0.6$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

$$= \frac{19.62}{1000 \times 9.81} - \frac{9.81}{1000 \times 9.81}$$

$$= 1 \times 10^{-3} \times 10^{-4}$$

$= 10m$ of water

$a_1 = \frac{\pi}{4} \times 0.2^2 = 0.031$

$a_0 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3}$

$$\frac{C_d \cdot a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$= \frac{0.6 \times 7.85 \times 10^{-3} \times 0.031 \times \sqrt{2 \times 9.81 \times 10}}{\sqrt{(0.031)^2 - (7.85 \times 10^{-3})^2}}$$

$= 0.06819\ m^3/s$

$= 68.19\ l/s$

Vena - contracta :-
The liquid of particles flows out of orifice, some flow particles take turn to enter into orifice. It has been observe that the jet after living the orifice then contracted. the maximum contraction takes place at a section, sildly on the down word stream side of the orifice is known as vena - contracta.

Q An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. the pressure difference measured by a Hg and differential manometer on the 2 side of the orifice meter gives a reading of 50 cm of Hg. find the rate of flow of oil of specific gravity 0.9 when the co-efficient of discharge of the orifice meter equal to 0.64

In given data :-

$x = 0.50$ $\quad d_1 = 30cm = 0.3$

$S_h = 13.6$ $\quad d_0 = 15 cm = 0.15$

$S_0 = 0.9$ $\quad C_d = 0.64$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

$$= 0.5\left[\frac{13.6}{0.9} - 1\right]$$

$$= 7.05 \text{ m of oil}$$

$$a_1 = \frac{\pi}{4} \times 0.3^2 = 0.0706$$

$$a_0 = \frac{\pi}{4} \times 0.15^2 = 0.0176$$

$$Q_{act} = \frac{C_d\, a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$= \frac{0.64 \times 0.0176 \times 0.0706 \sqrt{2 \times 9.81 \times 7.05}}{\sqrt{(0.0706)^2 - (0.0176)^2}}$$

$$= 0.1367 \text{ m}^3/s$$

136.7 4|s

$\boxed{V_1 \text{ actual} = C_v \times \sqrt{2 \cdot g \cdot h}}$

Probleme:-

Q A pitot static tube placed in a centre of A 300mm pipe line has one orifice pointing upstream and other perpendicular to it. the main velocity in the pipe is 0.80 of central velocity. find the discharge through the pipe if the pressure difference betⁿ the 2 orifices is 60 mm of water. take the co-efficient of water tube as $C_v = 0.98$

given data:-

$h = 0.06\,m$

$C_v = 0.98$

$V_m = 0.8\, V_1$

$d = 0.3\,m$

$(V_1)_{act} = C_v \sqrt{2 \cdot g \cdot h}$

$= 0.98 \sqrt{2 \times 9.81 \times 0.06}$

$= 1.063\ m/s$

$V_m = 0.8\, V_1$

$= 0.8 \times 1.063$

$= 0.8504\ m/s$

$Q = V_m \times A$

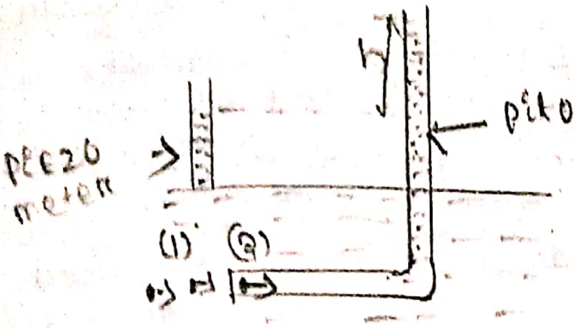
$A = \frac{\pi}{4} \times 0.3^2$

$= 0.07065$

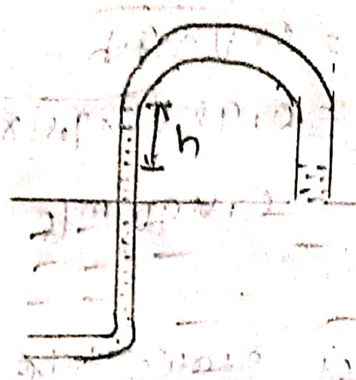
0.8504×0.07065

$= 0.0600\ m^3/s$

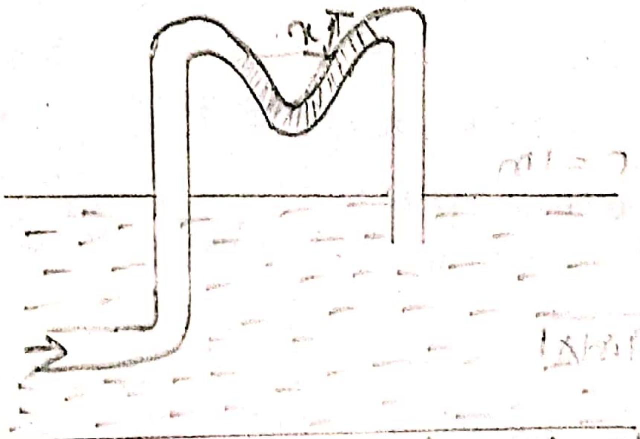
$= 60\ L/s$



pitot tube along with a vertical piezometer tube



pitot tube connected with piezometer tube



Pitot tube and vertical piezometer tube connected with U-tube manometer

Q. Find the velocity of the flow of an oil through a pipe when the difference of Hg level in a differential U-tube manometer connected to the ~~tubing~~ two tapping on the pitot tube is 100mm take co-efficient of pitot tube 0.98 and specific gravity of oil 0.8.

given data :-

$x = 100mm$

$C_v = 0.98$

$S_{oil} = 0.8$

$V_{act} = C_v \sqrt{2 \cdot g \cdot h}$

$h = x \cdot \left[\frac{S_{Hg}}{S_o} - 1\right]$

$= 0.1 \left[\frac{13.6}{0.8} - 1\right] = 1.6m$ of oil

$$V_{act} = C_V\sqrt{2 \cdot g \cdot h}$$

$$= 0.98\sqrt{2 \times 9.81 \times 01.5}$$

$$= 5.490 \text{ m/s}$$

Q a pitot static tube used to measure the velocity of water in a pipe the stagnation pressure head is 6m and static pressure head is 5m calculate the velocity of flow assuming the co-efficient of pitot tube equal to 0.98.

given data P_c —

$C_V = 0.98$

$h_1 = 5m$

$h_2 = 6m$

$$h = h_2 - h_1 = 6 - 5 = 1m$$

$$V_{act} = C_V\sqrt{2gh}$$

$$= 0.98\sqrt{2 \times 9.81 \times 1}$$

$$= 4.340 \text{ m/s}$$

FLOW THROUGH NOTCHES AND WEIRS :-

dt.-13/02/2020

Notches :-

A Notches is a deviced use for measuring the rate of flow of a liquid through a small channel or a tank.

→ It may be defined as and an opening in the side of a tank or a small channel is such a way that the liquid surface in the tank or channel is below the top edge of the opening.

→ the Notches is of small size.

Weirs :-

A weirs is a concrete and masonry structure placed on a open channel over which the flow occurs.

→ It is generally in the from of vertical wall is a sharp edge at the top running all the way across open channel.

→ The weirs is of a bigger size.

→ The Notches is generally made of metallic plat while weir is made of concrete of masonary structure.

classification of notches :-

(1) according to the shape of ~~oping~~ opening :-

a) rectangular Notches

b) triangular Notches

c) trapezium Notches.

d) stepped Notches.

according to the effect of the sides on the Nappe :-

a) Notches width end constraction

b) Notches with in contraction / suppresed Notches

(II) classification of weirs :-

according to the shape of the opening :-

(a) rectangular weirs

(b) triangular weirs

(c) Trapezium ~~we~~ weirs

According to the shape of the crest :-

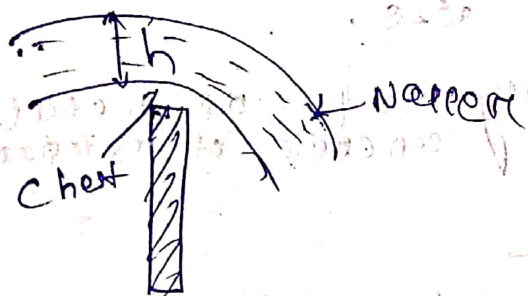
(a) Sharp weirs (b) broad weirs

(c) Narrow weirs (d) ogee shaped weirs

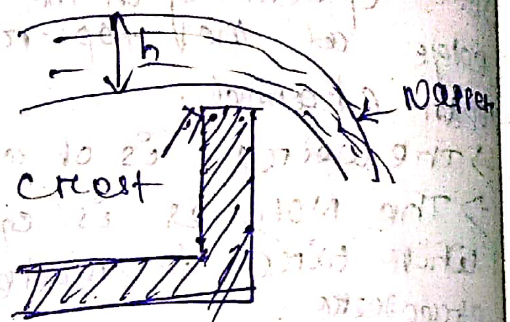
According to the effect of sides on the imerging Nappe

(a) weir with end contraction.
(b) weir without end contraction

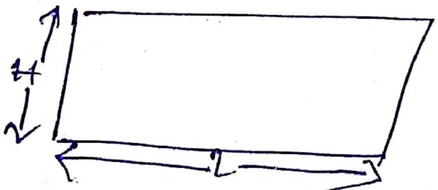
discharge over A rectangular Notches or weir :-



(Rectangular weir)



(rectangular Notches)



(Section at chest)

$$\boxed{Q = \frac{2}{3} \times Cd \times L \times \sqrt{2g} \cdot [H]^{3/2} \ M/s}$$

Where,

Cd = co-efficient of discharge

L = length of the weir OR notch

H = Head of water flowing over crest.

Scanned with CamScanner

find the discharge of water flowing over a rectangular Notch of 2m length when the constant head over the Notch is 300mm take $C_d = 0.60$

given data :-

$L = 2m$

$h = 300mm = 0.3m$

$C_d = 0.60$

$$\frac{2}{3} \times C_d \times L \times \sqrt{2g} \cdot [H]^{3/2}$$

$$= \frac{2}{3} \times 0.60 \times 2 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2}$$

$$= 0.582 \ m^3/s$$

Q determine the height of the rectangular weir of length 6m. to bild across a rectangular channel. the maximum depth of water on the upstream side of the ~~orifice~~ weir is 1.8 m and discharge is 2000 l/s take $C_d = 0.6$ and neglet end contractions

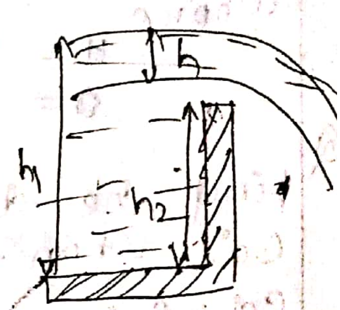
given data :-

$L = 6 \ m$

$Q = 2000 \ l/s = 2000 \times 10^{-3} = 2. \ m^3/s$

$C_d = 0.6$

$h = 1.8m$



$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2 \times g} \ [H]^{3/2}$$

$$2 = \frac{2}{3} \times 0.6 \times 6 \times \sqrt{2 \times 9.81} \times [H]^{3/2}$$

$$2 = 10.6306 \ [H]^{3/2}$$

$$[H^{3/2}] = \frac{2}{10.6307} = 0.18813$$

$$H = \sqrt[3/2]{0.18813} = 0.328m$$

$$H_2 = H_1 - H$$

$$= 1.8 - 0.328 = 1.472 \ m$$

Q The head of water over a rectangular notch is 900 mm the discharge is 300 L/s find the length of the notch when C_d = 0.62

given data :-

$Q = 300\ L/s = 300 \times 10^{-3} = 0.3\ m^3/s$

$C_d = 0.62$

$h = 900\ mm = 0.9\ m$

$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2 \times g}\ [H]^{3/2}$

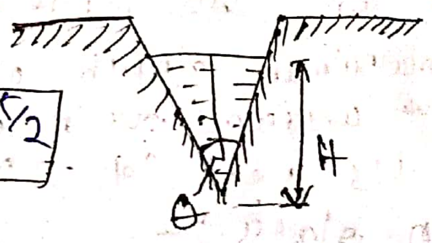
$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times [0.9]^{3/2}$

$0.3 = 1.5631\ L$

$L = \frac{0.3}{1.5631} = 0.191926\ m$

discharge over a triangular notch or weir :-

$$\boxed{Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}}$$



$\theta = 90°,\ C_d = 0.6$

$Q = 1.417\ H^{5/2}$

where

H = head of water above V Notch

θ = Angle of gauge

Q find the discharge over a triangular notch of angle 60° when the head over the V Notch is 0.3 m $C_d = 0.6$.

given data :-

$\theta = 60°$ $C_d = 0.6$ $h = 0.3\ m$

$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$

$= \frac{8}{15} \times 0.6 \times \tan\frac{60}{2} \times \sqrt{2 \times 9.81} \times 0.3^{5/2}$

$= 0.0403\ m^3/s$

dt - 14/02/2020

Q water flows over a rectangular weir 1m wide at a depth of 150mm and after words passes through triangular right angle weir. taking Cd for rectangular and triangular weir 0.62 & 0.59 respectively find the depth over the triangular weir.

given data :-

for rectangular $C_d = 0.62$

for triangular $C_d = 0.59$

for rectangular $b = 1m$

$H = 150\,mm = 0.15m$

for rectangle :-

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2 \times g} \cdot [H]^{3/2}$$

$$Q = \frac{2}{3} \times 0.62 \times 1 \times \sqrt{2 \times 9.81} \times [0.15]^{3/2}$$

$$= 0.1063\ m^3/sec$$

for triangular :-

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2 \cdot g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.59 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81} \times H^{5/2}$$

we know that

$$Q_1 = Q_2$$

$$0.1063 = \frac{8}{15} \times 0.59 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$0.1063 = 1.3937 \times H^{5/2}$$

$$H^{5/2} = 0.0762$$

$$H = \sqrt[5/2]{0.0762}$$

$$= 0.357\ m$$

Q water flow through a triangular right angle weir 1st and they over a rectangular weir of 1m width. the discharge coefficent of the triangular and rectangular weir are 0.6 and 0.7 respectively

If the depth of water over the triangular weir is 360mm find the depth of water over the rectangular weir.

given data :—

$L = 1m$

~~CD~~ rectangle $C_d = 0.\cancel{7}$

triangle $C_d = 0.6$

H for triangle = 0.36

for rectangle :—

$$\cancel{Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \cdot [H]^{3/2}}$$

for triangle :—

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times [H]^{\frac{5}{2}}$$

$$= \frac{8}{15} \times 0.6 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81} \times [0.36]^{5/2}$$

$$= 0.1102 \ m^3/s$$

for rectangle :—

$$Q = \frac{2}{3} \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times [H]^{3/2}$$

We know that

$$Q_1 = Q_2$$

$$0.1102 = \frac{2}{3} \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times [H]^{3/2}$$

$$= 0.1102 = 2.0670$$

$$[H]^{3/2} = \frac{0.1102}{2.0670}$$

$$= 0.0533$$

$$H = \sqrt[3/2]{0.0533}$$

$$= 0.1416 \ m$$

Q. A rectangular channel 2.00 m wide has a discharge of 250 L/s which is measure by a Right angle V notch weir find the position of the apex of the notch from the bed of the channel if maximum depth of water is not exceided 1.3 m. Take $C_d = 0.62$

Given data i.e. —

$Q = 250\ L/s = 0.25\ m^3/s$

$L = 2\ m$

$C_d = 0.62$

$H_1 = 1.3\ m$

$H_2 = ?$

$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\,[H]^{3/2}$

$0.25 = \frac{2}{3} \times 0.62 \times 2 \times \sqrt{2 \times 9.81} \times [H]^{3/2}$

$0.25 = 3.6616 \times [H]^{3/2}$

$[H]^{3/2} = \frac{0.25}{3.6616}$

$= 0.0682$

$H = \sqrt[3/2]{0.0682}$

$H = 0.1669$

$H_2 = H_1 - H$

$= 1.3 - 0.1669$

$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2 \times g} \times [H]^{\frac{5}{2}}$

$0.25 = \frac{8}{15} \times 0.62 \times \tan 45 \times \sqrt{2 \times 9.81} \times [H]^{\frac{5}{2}}$

$0.25 = 1.4646 \times [H]^{5/2}$

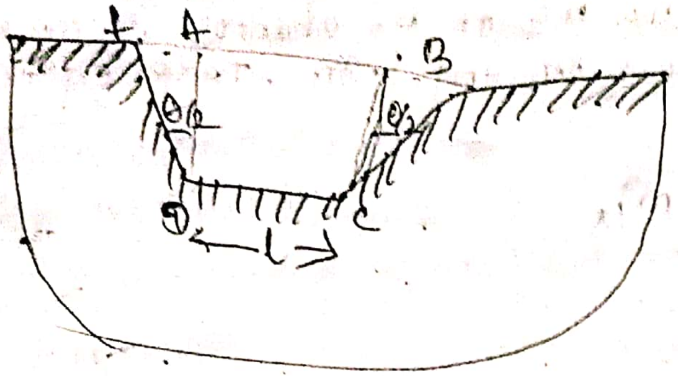
$[H]^{5/2} = \frac{0.25}{1.4646} = 0.1706$

$H = \sqrt[5/2]{0.1706} = 0.4929$

$H_2 = H_1 - H = 1.3 - 0.4929 = 0.8071\ M$

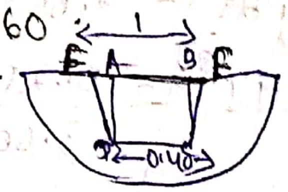
~~Discharge over a trapezoidal~~

Discharge over a trapezoidal Notch or weir :-



$$\boxed{Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g}\,[H]^{3/2} + \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times [H]^{5/2}}$$

Q Find the discharge through a trapezoidal Notch which is 1m wide at the top and 0.40m at the bottom and its 30 cm in height. the head of water on the notch is 20cm. assume C_d for rectangular portion = 0.62 and file from triangular portion = 0.60



Given data :-

$FE = 1m \quad DC = 0.4m$

$FA = BE = FE - DC$

$= 1 - 0.40 = 0.6\,m$

$AA = BE = \frac{0.6}{2} = 0.3m$

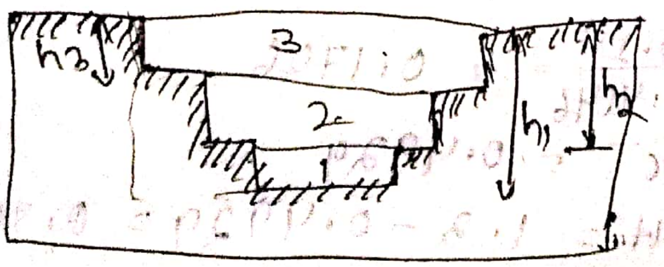
$\tan\frac{\theta}{2} = \frac{P}{B} = \frac{FA}{AD} = \frac{0.3}{0.3} = 1.$

$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times [0.3]^{3/2} + \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times [0.2]^{5/2}$

$= 0.0908\ m^3/s$

$= 90.8\ L/s$

Discharge over a stepped Notch :-



$Q = Q_1 + Q_2 + Q_3$

$$= \boxed{\frac{2}{3} C_d \times L_1 \times \sqrt{2g} \times \left[H_1^{3/2} - H_2^{3/2}\right] + \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \cdot \left[H_2^{3/2} - H_3^{3/2}\right] + \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \cdot \left[H_3\right]^{3/2}}$$

Q where h_1 = hight of water above the crest of notch 1 similarly 2 and 3

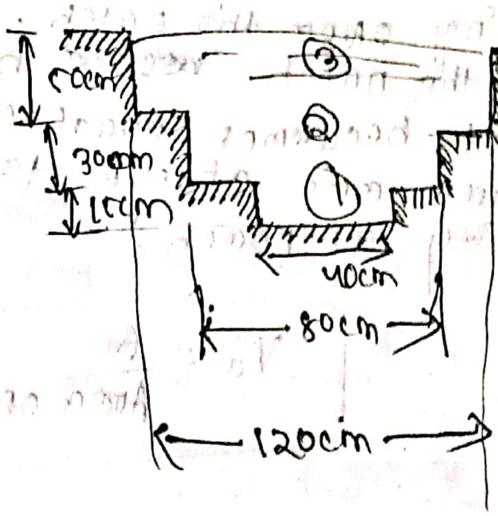
L_1 = length of notch 1

$h_2 L_2$ and $L_3 h_3$ are corresponding for notches 2 and 3 respectively.

C_d = coefficent of discharge for all notches

dt - 18/02/2020

Q figure shows a stated notch. Find the discharge through the notch. If C_d for all section = 0.62.



given data:—

$C_d = 0.62$

$L_1 = 40cm$

$L_2 = 80cm$

$L_3 = 120cm$

$h_1 =$ ~~cool~~ $15cm + 30 + 50 = 95cm$

$h_2 = 30cm + 50 = 80cm$

$h_3 =$ ~~50cm~~ $50cm$

$Q = Q_1 + Q_2 + Q_3$

$$\Rightarrow \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g} \times \left[H_1^{3/2} - H_2^{3/2}\right] + \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \cdot \left[H_2^{3/2} - H_3^{3/2}\right] + \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \cdot \left[H_3\right]^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 9.81} \times \left[95^{3/2} - 80^{3/2}\right] + \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 9.81} \times \left[80^{3/2} - 50^{3/2}\right] + \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 9.81} \times [50]^{3/2}$$

$\Rightarrow$ ~~15405.6046 + 96288.23196564 + 102163.41907~~

$= 15408.6048 + 53019.36565 + 77675.88043$

$= 146103.8509\ cm^3/s$

$= 0.146\ m^3/s$

$= 146\ l/s$

Velocity Approach P_c —

Velocity of Approach is defined as the velocity of with which the water approaches or reaches the weir or notch before it flowes over it. Thus if V_a is the velocity of Approach, than additional head ha equal to $\frac{V_a^2}{2g}$.

$$\left(h = \frac{V_a^2}{2g}\right)$$

due to velocity of approach is acting on the water flowing over the notch. Than intial hight of water over the notch ~~recide~~ becumes $(H + h_a)$ and final hight becaomes equal to h_a. then all the formula are change taking into consideration of velocity approach.

$$\boxed{V_a = \frac{Q}{\text{Area of channel}}}$$

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\left[(H_1 + h_a)^{3/2} - h_a^{3/2}\right]}$$

Q water is flowing in a rectangular channel of 1m wide and 0.75m deep. find the discharge over a rectangular weir or creast length 60cm if the head of water over the creast of weir is 20cm and water from channel flows over the weir. take $C_d = 0.62$ neglet end contraction take velocity approach in to consideration.

given data

$C_d = 0.62$ $\quad h = 20cm$

$b = 1m$

$d = 0.75m$

$L = 60cm$

$\cancel{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\,[(H_1 + h_a)}$

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \times (0.20)^{3/2}$$

$$= 0.09820 \ m^3/s$$

$$V_a = \frac{Q}{\text{Area of channel}}$$

$$= \frac{0.0982}{1 \times 0.75} = 0.130933 \ m/s$$

$$h_a = \frac{V_a^2}{2g} = \frac{0.130933}{2 \times 9.81}$$

$$= 8.73 \times 10^{-4} = 0.0008735 \ m$$

Then discharge velocity approach is given by

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \cdot [(H_1 - h_a)^{3/2} - h_a^{3/2}]$$

$$Q = \frac{2}{3} \times 0.62 \times 0.6 \times \sqrt{2 \times 9.81} \times [(0.20 - 0.0008735)^{3/2} - 0.0008735^{3/2}]$$

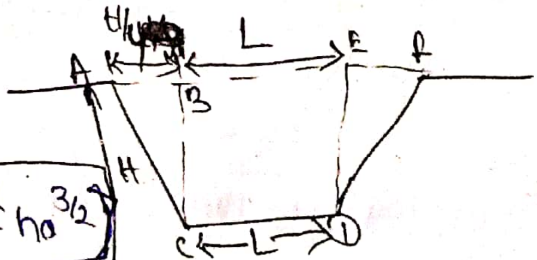
$$= 0.098869 \ m^3/s$$

CIPOLLETI WEIR OR NOTCH :-

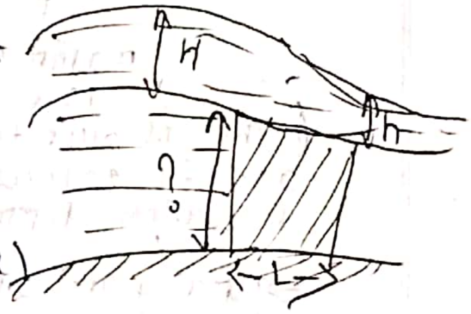
Cipolleti weir is a trapezoidal weir which has side slope of 1 horizontal to 4 vertical, as shown in figure.

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2 \times g} \cdot H^{3/2}}$$

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g}\,[(H + h_a)^{3/2} - h_a^{3/2}]}$$



discharge over a broad crested weir :-



where,

H = hight of water above the creast

L = Length of creast.

If 2L > H - BCW (broad crested weir

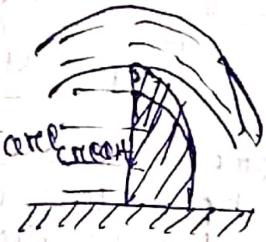
2L < H - NCW (Narrow creast weir)

$$\boxed{Q_{max} = 1.705 \times C_d \times L \times H^{3/2}}$$

NARROW CREASTED WEIR :-

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times [H]^{3/2}}$$

DISCHARGE OVER OGEE WEIR :-



$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}}$$

Flow Through pipes :-

dt 20/02/2020

Reynolds Number :-

$$Re = \frac{\rho . V . D}{\mu} \text{ OR } \boxed{\frac{VD}{\nu}}$$

$$\nu = \frac{\mu}{\rho} \Rightarrow \frac{1}{\nu} = \frac{\rho}{\mu}$$

Re < 2000 → Laminar

Re > 4000 → Turbulent

2000 - 4000

(flow changing from lam. to turbulent)

LOSS OF ENERGY IN PIPES :-

Energy loss

major energy ↓ loss

minor energy loss ↓

→ This is due to friction and it is calculated by the following formula energy loss

major energy loss
(This is due to friction and it calculated the following formula)
- Dracy and weisbach formula
- Chezy's formula

minor energy loss
(This is due to)
- Sudden expansion of pipe
- Sudden contraction of pipe
- bend in pipe
- pipe fetting etc.
- And abstraction in pipe

Loss of energy or (head) due to friction :—

Dracy and weisbach formula :—

$$\boxed{h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}}$$

where,
h_f = loss of head due to friction.
f = co-efficent of friction which is a function of Reynolds number.

$$\boxed{f = \frac{16}{Re} \text{ for } Re < 2000}$$

$$\boxed{= \frac{0.079}{Re^{1/4}}} \text{ for Re varying from 4000 to } 10^6$$

L = length of pipe
V = mein velocity of flow
d = diameter of pipe.

... the friction in a pipe

2. Find the head loss due to the friction in a pipe of diameter 300mm. & length 50 m, through which water is flowing at a velocity of 3 m/sec using Darcy's formula. Take ν for water 0.01 stoke

Given data

$d = 300\,mm = 0.3\,m$

$L = 50\,m$

$V = 3\,m/sec$

$\nu = 0.01$ stoke

$= 0.01 \times 10^{-4}\,m^2/s$

$$R_e = \frac{\rho . v . d}{\mu}$$

$$= \frac{3 \times 0.3}{0.01 \times 10^{-4}}$$

$$= 9 \times 10^5$$

As reynold's number value in between 4000 to 10^6 so $f = \frac{0.79}{R_e^{1/4}}$

$$\frac{0.079}{(9\times10^5)^{1/4}} = 0.00256$$

$$h_f = \frac{4\times f\times L\times V^2}{d\times 2g}$$

$$= \frac{4\times 0.00256\times 50\times 5^2}{0.3\times 2\times 9.81}$$

$$= 0.7828\ m$$

Chezy's formula for loss of head due to friction in pipe :-

$$\boxed{V = C\sqrt{Mi}}$$

$$\boxed{i = \frac{h_f}{L}}$$

$$\boxed{C = \sqrt{\frac{\rho . g}{f'}}}$$

Where,

$i = \frac{h_f}{L}$ (Loss of head for unit length of pipe)

$C = \sqrt{\frac{\rho . g}{f'}}$ (chezy's constant)

f' = frictional resistance for unit wetted area for unit velocity

$M = \frac{A}{P}$ (Hydraulic mean depth or hydraulic radius)

Scanned with CamScanner

P = Wetted perimeter of pipe

A = Area of Cross section of pipe

V = mean velocity of pipe.

Q As the previous question all data are same chezy's constant $C = 60$ calculate the head loss.

Given data

$d = 300mm = 0.3\ m$
$L = 50\ m$
$V = 3\ m/sec$
$\nu = 0.01 \times 10^{-4}\ m^2/s$

$$M = \frac{A}{P} = \frac{\pi/4\, d^2}{\pi d} = \frac{d}{4}$$

$$= \frac{0.3}{4} = 0.075$$

$$i =$$

$$= \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075}$$

$$= 0.0333$$

$$\Rightarrow \frac{h_f}{L} = 0.033 \quad h_f = 0.0333 \times 50$$

$v = C\sqrt{mi}$

$3 = 60\sqrt{0.075 \times 0.333}$

$3 = 2.998$

-o-

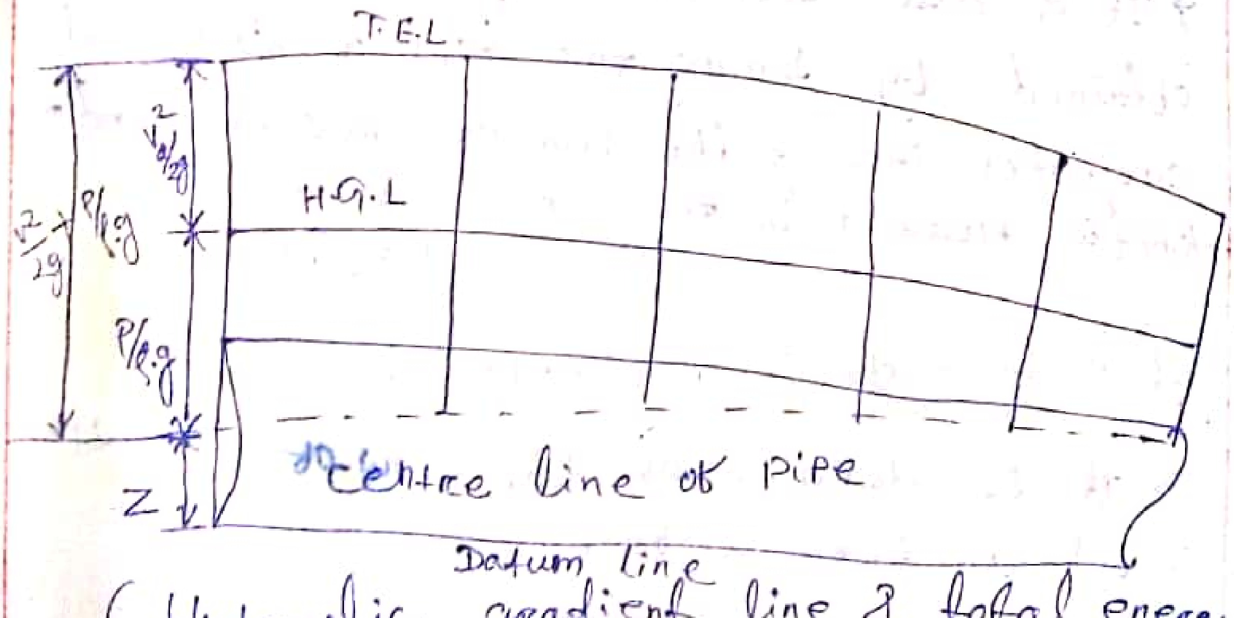
D-03.03.2020.

Hydraulic gradient line:-

→ It is a defined as ~~various~~ the line joining points representing piezometric head $\left(Z + \frac{P}{\rho g}\right)$ at various crossection of a pipe.

Total energy line:-

→ It is defined as the line joining points representing total energy $\left(\frac{P}{\rho g} + \frac{V^2}{2g} + Z\right)$ at various crossection of a pipe.



(Hydraulic gradient line & total energy line)

Note :- Hydraulic gradient

→ Hydraulic gradient line is defined as the line which gives sum of pressure head & datum head of a flowing fluid in a pipe with respect to some difference line.

OR

→ It is a ~~vertical~~ line which is optained by joining the top of all vertical ordinates showing the pressure head of a flowing fluid in a pipe from the centre of the pipe.

Note:- Total energy line

→ It is defined as a line which gives the sum of pressure head, datum head & kinetic head of a flowing fluid in a pipe with respect to some difference line.

OR

→ It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure heads & kinetic head from the centre of the pipe.

D-6.03.2020

Flow through open channel :—

→ It is defined as the flow of a liquid with a free surface.

→ A freesurface is the surface having constant pressure such as atmospheric pressure. Th

→ Thus a liquid flowing at atmospheric pressure through a passage is known as flow in open channel.

Classification of flow in channels:—

1→ Steady flow & unsteady flow

2→ Uniform & non uniform flow.

3→ Laminar & turbulent flow

4→ subcritical, critical & supercritical flow.

1. Steady flow & unsteady flow:—

Steady flow	Unsteady flow
1- $\frac{dy}{dt} = 0$	1- $\frac{dy}{dt} \neq 0$
2- $\frac{dQ}{dt} = 0$	2- $\frac{dQ}{dt} \neq 0$
3= $\frac{dv}{dt} = 0$	3- $\frac{dv}{dt} \neq 0$

v= velocity of ~~fluid of~~ flowing through open channel

y = Depth "

Q = Discharge "

2- Uniform & non uniform flow:—

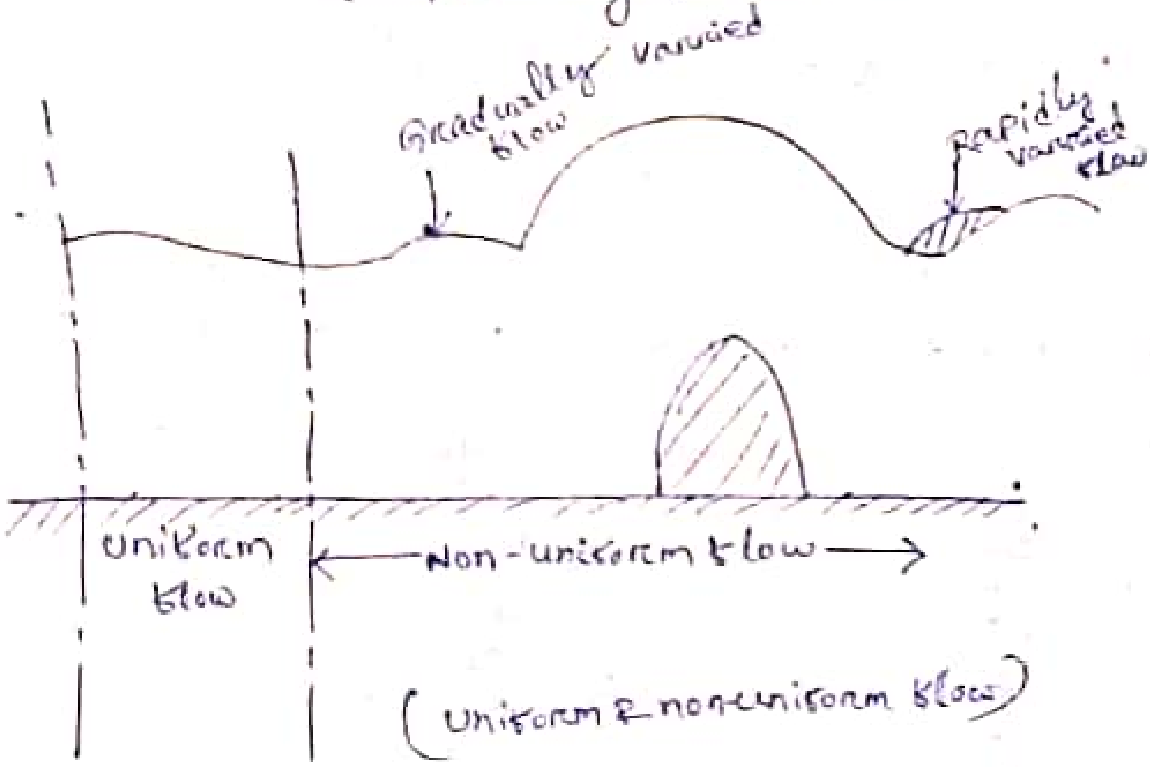
Uniform	Non uniform
1- $\frac{dy}{ds} = 0$	1- $\frac{dy}{ds} \neq 0$
2- $\frac{dv}{ds} = 0$	2- $\frac{dv}{ds} \neq 0$

$S = f(x, y, z)$ = space

→ Non-uniform flow in open channels is also called varried flow (which is classified as)

1- Rappidly varried flow

2- Gradually varried flow



1- Rapidly varried flow:—

→ It is defined as that flow in which depth of flow changes abruptly over a small length of the channel.

2. Gradually varried flow:-

→ If the depth of flow in the channel changes gradually over a long length of the channel. The flow is said to be gradually varried flow.

3. Laminar & turbulent flow:—

→ The flow in open channel is said to be laminar if the reynold's number (R_e) is less than 500 or 600

$R_e < 500$ or 600

→ The flow is said to be turbulent is open channel flow if the reynold's number is more than 2000

→ $\boxed{500 \text{ to } 2000}$ → Transition zone

$$R_e = \frac{\rho \cdot V \cdot R}{\mu} \text{ or } \frac{V \cdot R}{\nu}$$

R = Hydraulic mean depth or radius

$$R = \frac{A}{P} = \frac{\text{Area of crossection}}{\text{wetted perimeter}}$$

4. Sub critical, critical & super critical flow:—

→ If flow in open channel is said to be sub critical if froude number (F_e) is less than 1. * $\boxed{F_e \neq 1.0}$

Formula:—

→ $$\boxed{F_e = \frac{V}{\sqrt{gD}}}$$

⇒ $$\boxed{D = \frac{A}{T}}$$

where,

V = mean velocity of flow

D = Hydraulic depth of the channel & is equal to the ratio of wetted area to the top width of the channel.

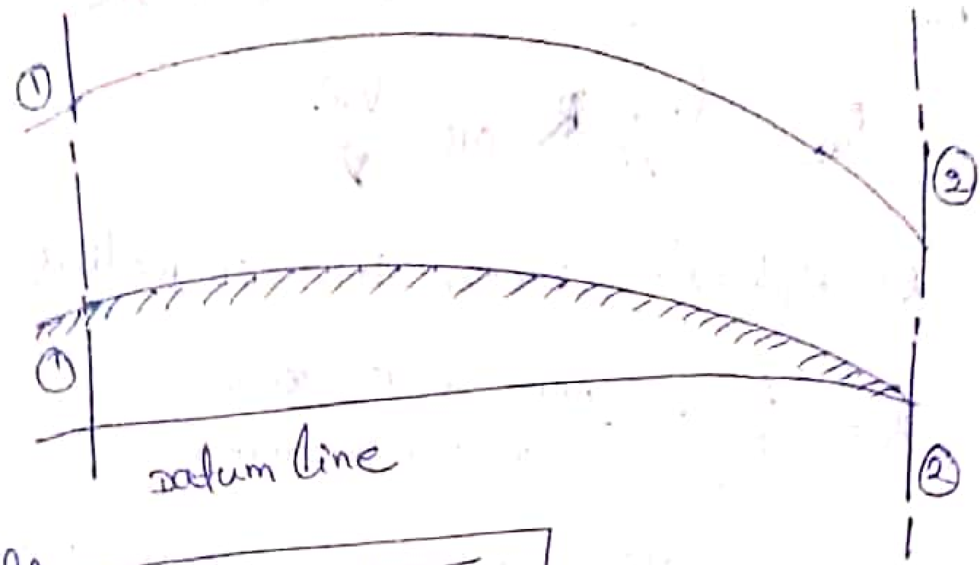
Sub-critical flow:—

→ It is also called Tranquial flow or streaming flow. * $\boxed{F_e < 1.0}$

Super critical :—

$\boxed{F_e > 1.0}$

Discharge through open channel by Chezy's formula



Formula $\boxed{Q = A \times C\sqrt{Mi}}$

Where,

C = Chezy's constant.

M = Hydraulic Mean depth of pipe or Hydraulic radius

$\boxed{M = \frac{A}{b}}$

I = Slope of the bed

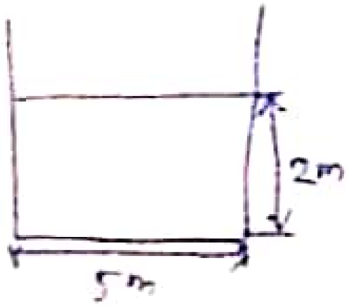
Q Find the slope of the bed of a rectangular channel of width 5 m. when the depth of water is 2 m. & the rate of flow is given as 20 m³/sec. take C = 50 given data

width $= b = 5m$

$d = 2m$

$Q = 20\ m^3/sec$

$C = 50$



sol^n

$$\dot{M} = \frac{A}{P} = \frac{5\times 2}{2\times 2+5} = \frac{10}{9} = 1.11$$

$$Q = A \times C\sqrt{MI}$$

$$\Rightarrow 20 = 10 \times 50\sqrt{1.11 \times I}$$

$$\Rightarrow \left(\frac{\cancel{20}}{10\times \cancel{50}_{25}}\right)^2 = \left(\sqrt{1.11 \times I}\right)^2$$

$$\Rightarrow \left(\frac{1}{25}\right)^2 = 1.11 \times I$$

$$\Rightarrow \frac{1}{625 \times 1.11} = I$$

$$\Rightarrow I = \frac{9}{6250} = \frac{1}{694.14}$$

$$I = \frac{1}{694.14}$$

Q:-

Discharge through open Channel By Chezy's formula

$$Q = \text{Area} \times \text{Velocity}$$
$$= A \times V$$
$$\boxed{Q = A \times C\sqrt{Mi}}$$

where, A = Area of the flow of water

C = Chezy's Constant

M = hydraulic mean depth or hydraulic Radious

i = slope of the bed.

Empirical formula for the value of chezy's Constant

1. Bazin formula (in MKS units)

$$\boxed{C = \frac{157.6}{1.81 + \frac{K}{\sqrt{M}}}}$$

where, K = Bazin Constant and depends upon Roughness of the Surface.

M = hydraulic mean depth

2. Ganguillet - Kutter formula.

$$\boxed{C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right)\frac{N}{\sqrt{m}}}}$$

N = Roughness Co-efficient which is known as kutter's Constant.

i = Slope of the bed.

m = hydraulic mean depth.

3. Manning's formula :-

$$C = \frac{1}{N} m^{1/6}$$

where N = Manning's Constant.

MOST ECONOMICAL SECTIONS OF CHANNELS

→ A section of a channel is said to be most economical when the cost of construction of the channel is minimum, But the cost of construction of a channel depends upon the excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum.

→ Most economical section is also called the best section or most efficient section. as the discharge, passing through a most economical section of channel for a given cross-sectional area (A), slope of the bed (i) and a resistance co-efficient, is maximum.

$$\text{Discharge, } \boxed{Q = AC\sqrt{Mi}} = AC\sqrt{\frac{A \times i}{P}}$$

Where, C = Resistance Co-efficient.

$M = \frac{A}{P}$

Q will be maximum when the wetted perimeter P is minimum.

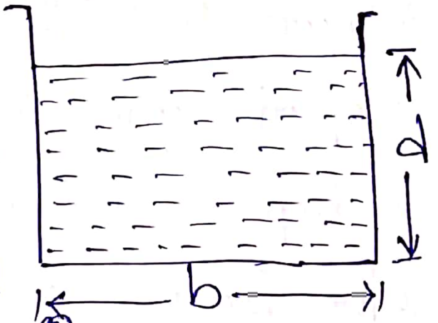
① Rectangular Channel.
② Trapezoidal Channel.
③ Circular Channel.

MOST ECONOMICAL RECTANGULAR CHANNEL

Let,

b = width of channel

d = depth of the flow



∴ Area of flow $= A = b \times d$

$\Rightarrow \boxed{b = A/d}$ — (i)

Wetted perimeter, $P = d + b + d = b + 2d$ — (ii)

$\cancel{(b = A/d)}$

Substituting the value of b in (ii)

$\Rightarrow$

$$P = b + 2d$$

$$\Rightarrow P = A/d + 2d \quad \text{— (iii)}$$

for most economical section, P should be minimum for a given area.

$$\frac{dP}{d(d)} = 0$$

$$\frac{d}{d(d)}\left[A/d + 2d\right] = 0$$

$$\Rightarrow -\frac{A}{d^2} + 2 = 0$$

$$\Rightarrow \boxed{A = 2d^2}$$

$$A = b \times d$$

So,

$$b \times d = 2d^2$$

$$\Rightarrow \boxed{b = 2d} \quad \text{— eq (1)}$$

Now, Hydraulic mean depth,

$$m = \frac{A}{P} = \frac{b \times d}{b + 2d}$$

$$= \frac{2d^2}{4d}$$

$$\boxed{m = d/2} \quad \text{— eq (2)}$$

~~from eq~~

∴ from eq ① and eq ②, it is clear that the rectangular channel will be most economical when

(i) Either $b = 2d$ means width is two times depth of flow.

(ii) Or $m = d/2$ means hydraulic depth is half the depth of flow.

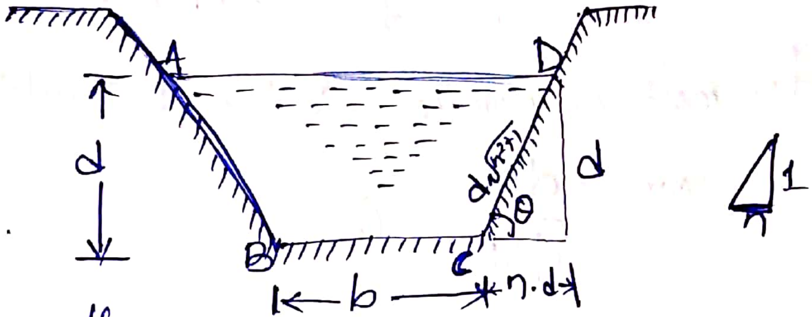
Most Economical Trapezoidal Channel

Let,

b = width of channel at bottom.

d = depth of flow.

θ = angle made by the sides with horizontal.



(Trapezoidal Section)

(i) The side slope is given as 1 vertical to n horizontal

$$\therefore \text{Area of flow, } A = \frac{BC + AD}{2} \times d$$

$$= \frac{b + (b + 2nd)}{2} \times d$$

$$= \frac{2b + 2nd}{2} \times d \qquad (\because AD = b + 2nd)$$

$$= (b + nd) \times d \qquad \text{—— (i)}$$

$$\therefore \frac{A}{d} = b + nd$$

$$\therefore b = \frac{A}{d} - nd$$

Now wetted perimeter, $P = AB + BC + CD = BC + 2CD$

$(\because AB = CD)$

$$= b + 2\sqrt{CE^2 + DE^2}$$

$$= b + 2\sqrt{n^2d^2 + d^2}$$

$$= b + 2d\sqrt{n^2 + 1} \quad \text{.. (ii)}$$

Substituting the value of b from equation (ii)

we get, $P = \frac{A}{d} - n.d + 2.d\sqrt{n^2+1}$ — (iii)

for most economical section, P should be minimum

or $\frac{dP}{d(d)} = 0$

∴ Differentiating equation (iii) with respect to d and equating it equal to zero, we get

$$\frac{d}{d(d)}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right] = 0$$

or $-\frac{A}{d^2} - n + 2\sqrt{n^2+1} = 0$ [∵ n = constant]

$$\Rightarrow \frac{A}{d^2} + n = 2\sqrt{n^2+1}$$

Substituting the value of A from equation (i) in the above equation

$$\frac{(b+nd)d}{d^2} + n = 2\sqrt{n^2+1}$$

or $\frac{b+nd}{d} + n = 2\sqrt{n^2+1}$

$$\Rightarrow \frac{b+nd+nd}{d} = \frac{b+2nd}{d} = 2\sqrt{n^2+1}$$

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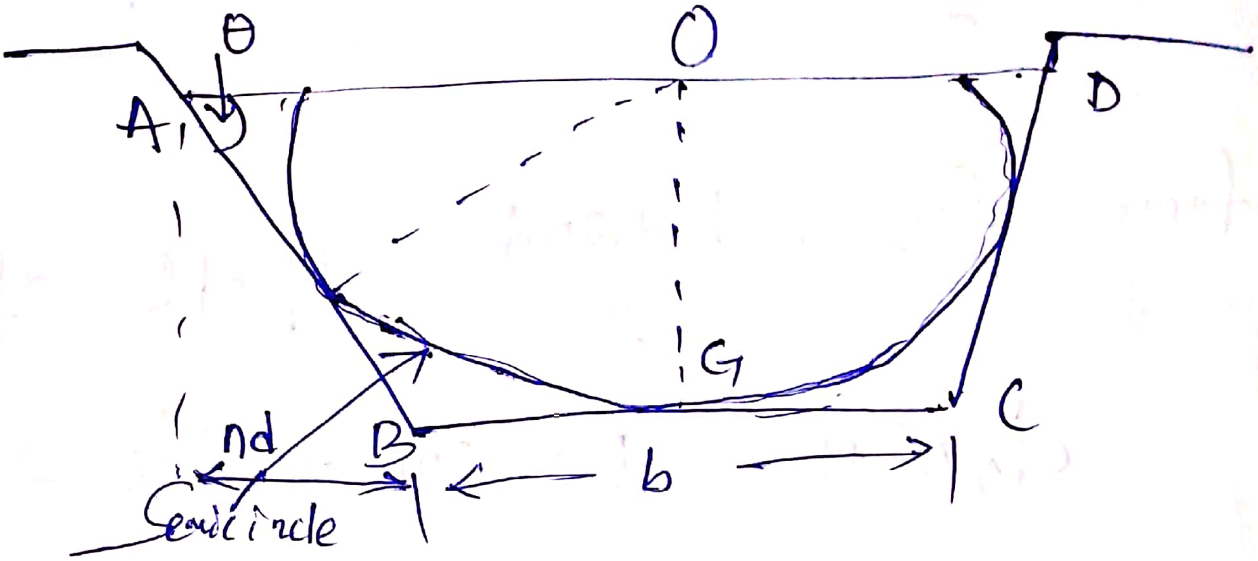


Hence, the Condition for the Most economical Section for trapezoidal Section are

1. $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

2. $M = \frac{d}{2}$

3. A Semi-Circle drawn from O with radius equal to depth of flow will touch the three Sides of the channel.

$$\boxed{\frac{b+2nd}{2} = d\sqrt{n^2+1}} \quad \text{— eq(3)}$$

But from fig, $\frac{b+2nd}{2}$ = Half of top width

$d\sqrt{n^2+1}$ = CD = one of the sloping side

Eq(3) is the required condition for a trapezoidal section to be most economical which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

(ii) Hydraulic Mean depth.

$$M = \frac{A}{P}$$

$A = (b+nd) \times d$ (from eq i)

$P = b + 2d\sqrt{n^2+1} = b + (b+2nd)$ (from eq ii)

$= 2b + 2nd$

$= 2(b+nd)$

$$M = \frac{A}{P} = \frac{(b+nd)d}{2(b+nd)} = d/2 \quad \text{— eq (4).}$$

Hence, for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow.

(iii) The three sides of the trapezoidal section of most economical section are tangential to the semicircle

CENTRIFUGAL PUMP

The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy.

→ If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

→ The flow in centrifugal pumps is in the radial outward directions.

The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place.

The rise in pressure head at any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point.
(i.e. rise in pressure head $= \frac{V^2}{2g}$ or $\frac{\omega^2 r^2}{2g}$).

thus,

At the outlet of the impeller, where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head.

Due to this high pressure head, the liquid can be lifted to a high level.

Main Part of a Centrifugal pump

1. impeller
2. Casing
3. Suction pipe with foot valve and a strainer.
4. Delivery pipe.

1. **Impeller:-** The rotating part of a centrifugal pump is called ~~of~~ impeller. It consists of series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

2. **Casing:-** It is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe.

3 types of casing
(a) Volute Casing.
(b) Vortex casing.
(c) Casing with guide blades.

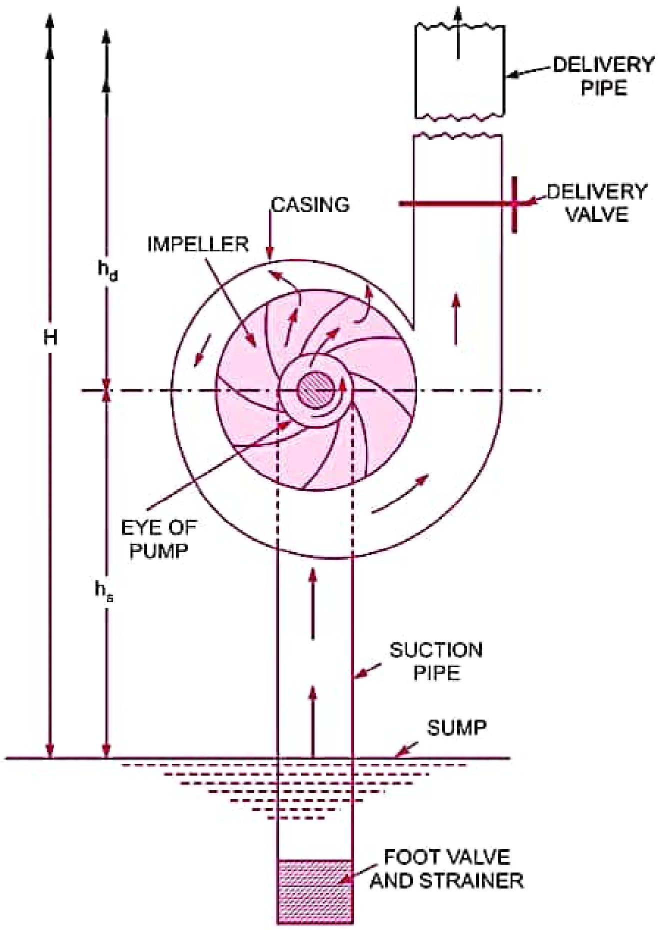


Fig. 19.1 *Main parts of a centrifugal pump.*

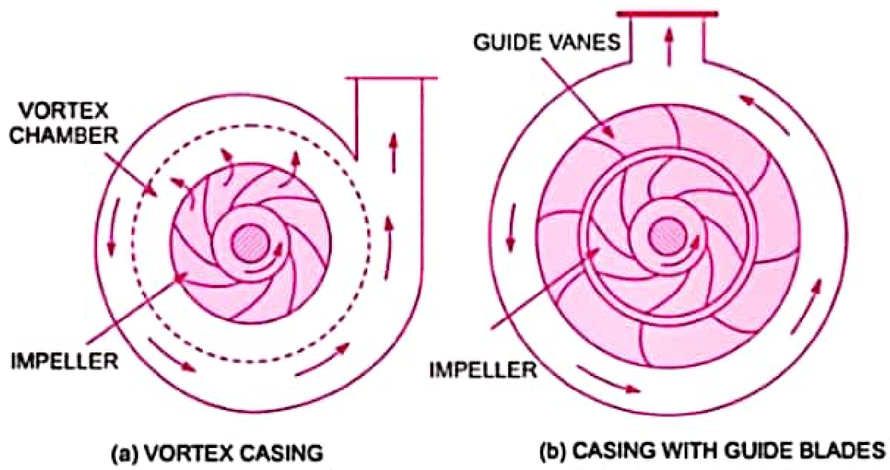


Fig. 19.2 *Different types of casing.*

3. Suction Pipe with a Foot Valve and a Strainer:-

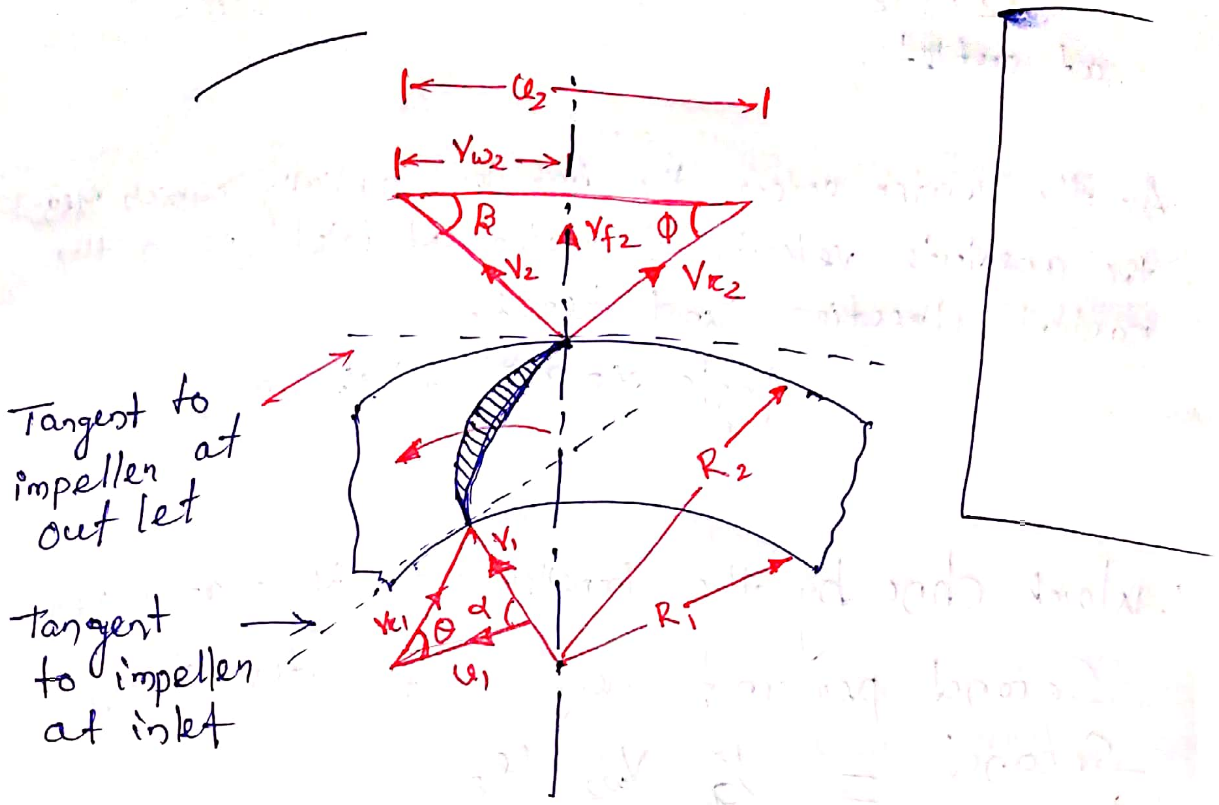
A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one way type of valve is fitted at the lower end of the pipe. The foot valve opens only in upward direct. A strainer is also fitted at the lower end of the suction pipe.

4. Delivery pipe:-

A pipe whose one end is connected to the outlet of the pump and other end delivers the water at a required height is known as delivery pipe.

Work Done By Centrifugal pump on water

In case of the centrifugal pump, work is done by the impeller on the water.



(Velocity triangle at inlet and out let.)

Let,

N = Speed of the impeller in r.p.m.

D_1 = Diameter of impeller at inlet

u_1 = Tangential velocity of impeller at inlet,

$$u_1 = \frac{\pi D_1 N}{60}$$

D_2 = Dia-meter of impeller at out let.

u_2 = Tangential velocity of impeller at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$

V_1 = Absolute velocity of water at inlet.

V_{r1} = Relative velocity of water at inlet.

α = Angle made by absolute velocity (V_1) at inlet with the direction of motion of vane.

θ = Angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane, ~~and~~

And V_2, V_{r_2}, β and ϕ are corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence,

$$\text{Angle } \alpha = 90^\circ \text{ and } V_{w_1} = 0$$

∴ Work done by the impeller on the water per second per unit weight of water striking per second $= \frac{1}{g} V_{w_2} \cdot u_2$

Work done by impeller on water per second

$$= \frac{W}{g} \cdot V_{w_2} \cdot u_2$$

where, W = Weight of water $= \rho \cdot g \cdot Q$

Q = Volume of water

and,

$$Q = \text{Area} \times \text{Velocity of flow}$$
$$= \pi D_1 B_1 \times V_{f_1}$$
$$= \pi D_2 B_2 \times V_{f_2}$$

B_1 and B_2 = width of impeller at inlet and outlet

V_{f_1} and V_{f_2} = Velocity of flow at inlet & outlet respectively.

So, $\boxed{\text{Discharge through pump} = \pi D_2 B_2 \times V_{f_2} \quad \text{OR} \quad \pi D_1 B_1 \times V_{f_1}}$

Definitions of Heads And Efficiencies of a Centrifugal pump.

1. **Suction Head:-** It is the vertical height of the center line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted.

   This height is also called suction lift and is denoted by 'h_s'.

2. **Delivery Head (H_d):-**
   The vertical distance between the centerline of the pump and the water surface in the tank to which water is delivered is known as delivery head. Denoted By 'h_d'.

3. **Static Head (H_s):-** The sum of suction head and delivery head is known as static head. Denoted as 'H_s'.

$$\boxed{H_s = H_s + H_d}$$

4. Manometric Head (H_m):- The Manometric head is defined as the head against which a centrifugal pump has to work.

Denoted By - H_m

(a) H_m = Head imparted by the impeller to the water − Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing}$$

$$= \frac{V_{w_2} u_2}{g} \quad \text{(if the loss of pump is Zero).}$$

(b) H_m = Total head at out let of the pump − Total head at the inlet of the pump.

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$$

where,

$\frac{P_o}{\rho g}$ = pressure head at outlet of the pump $= h_d$

$\frac{V_o^2}{2g}$ = Velocity head at out let of the pump

= velocity head in delivery pipe

$$= \frac{V_d^2}{2g}$$

Z_o = vertical height of the outlet of the pump from datum line and

$\frac{P_i}{\rho g}, \frac{V_i}{2g}, Z_i$ = Corresponding values of pressure head, velocity head and datum head of the inlet of the pump.

i.e.,

$h_s, \frac{V_s^2}{2g}$ and Z_s respectively.

(c) $H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$

where, h_s = Suction head

h_d = Delivery head

h_{fs} = frictional head loss in suction pipe,

h_{fd} = frictional head loss in delivery pipe

V_d = velocity of water in delivery pipe.

5. Efficiencies of Centrifugal pump:-

In case of a centrifugal pump, the power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller, from the impeller, the power is given to the water.

Thus power is decreasing from the shaft of the pump to the impeller and then to the water.

(a) Manometric efficiency (η_{man})

(b) Mechanical efficiency (η_m)

(c) Overall efficiency (η_o)

(a) Manometric efficiency (η_{man}):-

The ratio of the manometric head to the head imparted by the impeller to the water is known as manometric efficiency.

$$\boxed{\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}}$$

$$= \frac{H_m}{\left(\frac{V_{w2} u_2}{g}\right)} = \frac{g H_m}{V_{w2} u_2}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

$$\boxed{\eta_{man} = \frac{g \times H_m}{V_{w2} \times u_2}}$$

(b) Mechanical Efficiency (η_m):-

The power at the shaft of the centrifugal pump is more than the power available at the impeller of the pump. The ratio of the power availability at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency.

$$\boxed{\eta_m = \frac{\text{Power at the impeller}}{\text{power at the Shaft}}}$$

$$\boxed{\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} \cdot u_2}{1000}\right)}{S.P}}$$

S.P = Shaft power.

(c) Overall Efficiency (η_o):- It is defined as the ratio of power out put of the pump to the power in put to the pump. The power out put of the pump in KW.

$$= \frac{\text{Weight of water lifted} \times H_m}{1000}$$

$$= \frac{W H_m}{1000}$$

power in put to the pump = power supplied by electric motor

= S.P. of the motor

$$\boxed{\therefore \eta_o = \frac{\frac{W H_m}{1000}}{S.P}}$$

Also $\boxed{\eta_o = \eta_{man} \times \eta_m}$

Problem 19.1 *The internal and external diameters of the impeller of a centrifugal pump are 200 mm and 400 mm respectively. The pump is running at 1200 r.p.m. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Solution. Given :

Internal diameter of impeller, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

External diameter of impeller, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

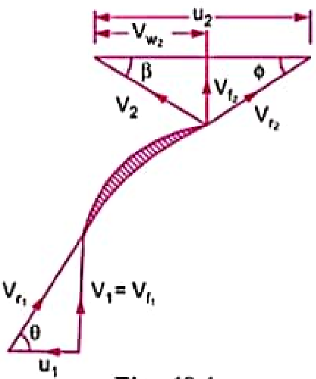


Fig. 19.4

From inlet velocity triangle, $\tan\theta = \dfrac{V_{f_1}}{u_1} = \dfrac{V_{f_1}}{12.56}$

$\therefore \quad V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore \quad V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \dfrac{V_{f_2}}{u_2 - V_{w_2}} = \dfrac{4.57}{25.13 - V_{w_2}}$

or

$$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30^\circ} = 7.915$$

$\therefore \quad V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (19.1) as

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \textbf{44.1 Nm/N. Ans.}$$

Problem 19.3 *A centrifugal pump delivers water against a net head of 14.5 metres and a design speed of 1000 r.p.m. The vanes are curved back to an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. Determine the discharge of the pump if manometric efficiency is 95%.*

Solution. Given :

Net head, $H_m = 14.5$ m

Speed, $N = 1000$ r.p.m.

Vane angle at outlet, $\phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

∴ Diameter, $D_2 = 300$ mm $= 0.30$ m

Outlet width, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$

Now using equation (19.8), $\eta_{man} = \dfrac{gH_m}{V_{w_2} \times u_2}$

$$\therefore \quad 0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$\therefore \quad V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

Refer to Fig. 19.5. From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

$$\therefore \quad V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

∴ Discharge, $Q = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3\text{/s} = \mathbf{0.1675 \text{ m}^3\text{/s}}$. **Ans.**

Problem 19.4 *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 r.p.m. works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine :*

(i) Vane angle at inlet, *(ii) Work done by impeller on water per second, and*

(iii) Manometric efficiency.

Solution. Given :

Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm $= 0.50$ m

Inner dia. of impeller, $D_1 = \dfrac{D_2}{2} = \dfrac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

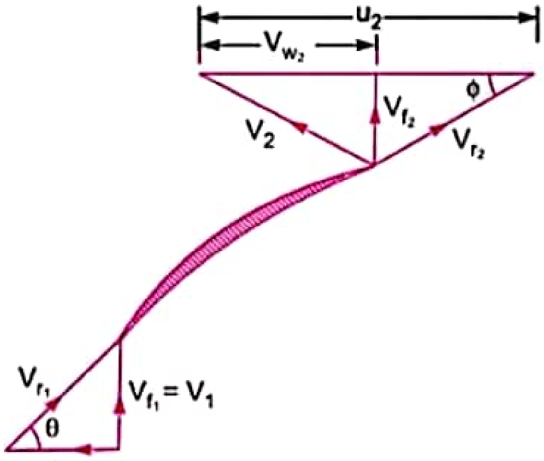


Fig. 19.6

Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}$.

(*i*) Vane angle at inlet (θ).

From inlet velocity triangle $\tan \theta = \dfrac{V_{f_1}}{u_1} = \dfrac{2.5}{13.09} = 0.191$

$\therefore$ $\theta = \tan^{-1} .191 = 10.81°$ or **10° 48′. Ans.**

(*ii*) Work done by impeller on water per second is given by equation (19.2) as

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18 \qquad ...(i)$$

But from outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$\therefore$ $$26.18 - V_{w_2} = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 40°} = 2.979$$

$\therefore$ $V_{w_2} = 26.18 - 2.979 = 23.2$ m/s.

Substituting this value of V_{w_2} in equation (*i*), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$= $ **119227.9 Nm/s. Ans.**

(*iii*) Manometric efficiency (η_{man}). Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \textbf{64.4\%}. \textbf{ Ans.}$$

Problem 19.9 *Find the power required to drive a centrifugal pump which delivers 0.04 m^3/s of water to a height of 20 m through a 15 cm diameter pipe and 100 m long. The overall efficiency of the pump is 70% and co-efficient of friction 'f' = 0.15 in the formula* $h_f = \dfrac{4fLV^2}{d \times 2g}$.

Solution. Given :

Discharge, $Q = 0.04 \text{ m}^3/\text{s}$

Height, $H_s = h_s + h_d = 20 \text{ m}$

Dia. of pipe, $D_s = D_d = 15 \text{ cm} = 0.15 \text{ m}$

Length, $L_s + L_d = L = 100 \text{ m}$

Overall efficiency, $\eta_o = 70\% = 0.70$

Co-efficient of friction, $f = .015$

Velocity of water in pipe, $V_s = V_d = V = \dfrac{\text{Discharge}}{\text{Area of pipe}} = \dfrac{0.04}{\frac{\pi}{4}(.15)^2} = 2.26 \text{ m/s}.$

Frictional head loss in pipe,

$$(h_{f_s} + h_{f_d}) = \frac{4fLV^2}{d \times 2g} = \frac{4 \times .015 \times 100 \times 2.26^2}{.15 \times 2 \times 9.81} = 10.41 \text{ m}.$$

Using equation (19.7), we get manometric head as

$$H_m = (h_s + h_d) + (h_{f_s} + h_{f_d}) + \frac{V_d^2}{2g}$$

$$= 20 + 10.41 + \frac{2.26^2}{2 \times 9.81} \qquad (\because \;\; h_s + h_d = H_s = 20 \text{ m})$$

$$= 30.41 + 0.26 = 30.67 \text{ m}.$$

Overall efficiency is given by equation (19.10) as

$$\eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}} = \frac{\rho g \times Q \times H_m}{1000 \times \text{S.P.}}$$

$$\therefore \qquad \text{S.P.} = \frac{\rho g \times Q \times H_m}{1000 \times \eta_o} = \frac{1000 \times 9.81 \times .04 \times 30.67}{1000 \times 0.70} = \mathbf{17.19 \text{ kW. Ans.}}$$

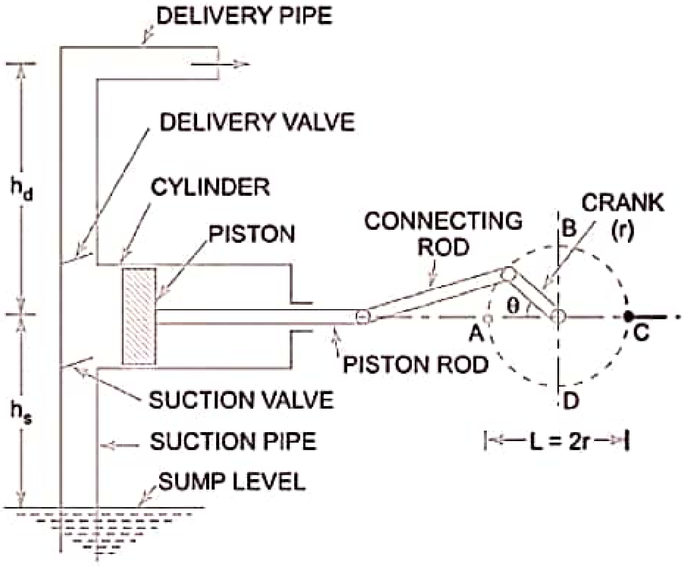
S.P. is the power required to drive the centrifugal pump.

RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy, by means of centrifugal force acting on the liquid, the pump is known as centrifugal pump. But if the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump.

MAIN PARTS OF A RECIPROCATING PUMP

The following are the main parts of a reciprocating pump as shown in Fig.



Main parts of a reciprocating pump.

1. A cylinder with a piston, piston rod, connecting rod and a crank,
2. Suction pipe,
3. Delivery pipe,
4. Suction valve, and
5. Delivery valve.

WORKING OF A RECIPROCATING PUMP

Fig. shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

When crank starts rotating, the piston moves to and fro in the cylinder. When crank is at A., the piston is at the extreme left position in the cylinder. As the crank is rotating from A to C, (*i.e.*, from $\theta = 0^\circ$ to $\theta = 180^\circ$), the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface of the liquid in the sump atmospheric pressure is acting, which is more than the pressure inside the cylinder. Thus, the liquid is forced in the suction pipe from the sump. This liquid opens the suction valve and enters the cylinder.

When crank is rotating from C to A (*i.e.*, from $\theta = 180^\circ$ to $\theta = 360^\circ$), the piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

Discharge Through a Reciprocating Pump. Consider a single* acting reciprocating pump as shown in Fig.

Let D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$= \frac{\pi}{4} D^2$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke $= 2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$$= \text{Area} \times \text{Length of stroke} = A \times L$$

Number of revolution per second, $= \dfrac{N}{60}$

$\therefore$ Discharge of the pump per second,

$$Q = \text{Discharge in one revolution} \times \text{No. of revolution per second}$$

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60}$$

Weight of water delivered per second,

$$W = \rho \times g \times Q = \frac{\rho g A L N}{60}.$$

Work done by Reciprocating Pump. Work done by the reciprocating pump per second is given by the reaction as

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d) \quad \text{...}(i)$$

where $(h_s + h_d)$ = Total height through which water is lifted.

From equation (20.2), Weight, W, is given by

$$W = \frac{\rho g \times ALN}{60}.$$

Substituting the value of W in equation (i), we get

$$\text{Work done per second} = \frac{\rho g \times ALN}{60} \times (h_s + h_d)$$

∴ Power required to drive the pump, in kW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} \text{ kW}$$

Discharge, Work done and Power Required to Drive a Double-acting Pump. In case of double-acting pump, the water is acting on both sides of the piston as shown in Fig. 20.2. Thus, we require two suction pipes and two delivery pipes for double-acting pump. When there is a suction stroke on one side of the piston, there is at the same time a delivery stroke on the other side of the piston. Thus for one complete revolution of the crank there are two delivery strokes and water is delivered to the pipes by the pump during these two delivery strokes.

Let D = Diameter of the piston,

d = Diameter of the piston rod

∴ Area on one side of the piston,

$$A = \frac{\pi}{4} D^2$$

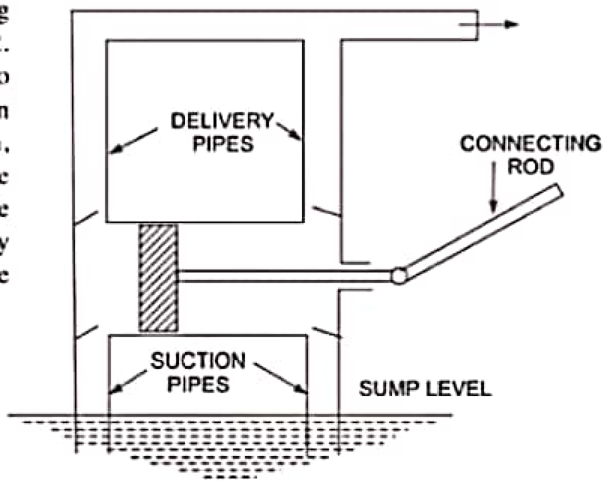


Fig.

Area on the other side of the piston, where piston rod is connected to the piston,

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \frac{\pi}{4} (D^2 - d^2).$$

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$\therefore$ Volume of water delivered in one revolution of crank

$$= A \times \text{Length of stroke} + A_1 \times \text{Length of stroke}$$

$$= AL + A_1L = (A + A_1)L = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L$$

$\therefore$ Discharge of pump per second

= Volume of water delivered in one revolution × No. of revolution per second

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

If 'd' the diameter of the piston rod is very small as compared to the diameter of the piston, then it can be neglected and discharge of pump per second,

$$Q = \left(\frac{\pi}{4}D^2 + \frac{\pi}{4}D^2\right) \times \frac{L \times N}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{L \times N}{60} = \frac{2ALN}{60}$$

Equation (gives the discharge of a double-acting reciprocating pump. This discharge is two times the discharge of a single-acting pump.

Work done by double-acting reciprocating pump

Work done per second = Weight of water delivered × Total height

$$= \rho g \times \text{Discharge per second} \times \text{Total height}$$

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d) = 2\rho g \times \frac{ALN}{60} \times (h_s + h_d)$$

$\therefore$ Power required to drive the double-acting pump in kW,

$$P = \frac{\text{Work done per second}}{1000} = 2\rho g \times \frac{ALN}{60} \times \frac{(h_s + h_d)}{1000}$$

$$= \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

. SLIP OF RECIPROCATING PUMP

Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump. The discharge of a single-acting pump given by equation (20.1) and of a double-acting pump given by equation (20.5) are theoretical discharge. The actual discharge of a pump is less than the theoretical discharge due to leakage. The difference of the theoretical discharge and actual discharge is known as slip of the pump. Hence, mathematically.

$$\text{Slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$$

where C_d = Co-efficient of discharge.

Negative Slip of the Reciprocating Pump. Slip is equal to the difference of theoretical discharge and actual discharge. If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve. In that case, the slip of the pump is known as negative slip.

Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

CLASSIFICATION OF RECIPROCATING PUMPS

The reciprocating pumps may be classified as :

1. According to the water being in contact with one side or both sides of the piston, and
2. According to the number of cylinders provided.

If the water is in contact with one side of the piston, the pump is known as single-acting. On the other hand, if the water is in contact with both sides of the piston, the pump is called double-acting. Hence, classification according to the contact of water is :

(*i*) Single-acting pump, and (*ii*) Double-acting pump.

According to the number of cylinder provided, the pumps are classified as :

(*i*) Single cylinder pump, (*ii*) Double cylinder pump, and

(*iii*) Triple cylinder pump.

Problem *A single-acting reciprocating pump, running at 50 r.p.m., delivers 0.01 m³/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine :*

(i) The theoretical discharge of the pump, (ii) Co-efficient of discharge, and (iii) Slip and the percentage slip of the pump.

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01 \text{ m}^3/\text{s}$

Dia. of piston, $D = 200 \text{ mm} = .20 \text{ m}$

∴ Area, $A = \frac{\pi}{4} (.2)^2 = .031416 \text{ m}^2$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$.

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047 \text{ m}^3/\text{s. Ans.}}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955. \text{ Ans.}}$$

(*iii*) Using equation , we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047 \text{ m}^3/\text{s. Ans.}}$$

And percentage slip

$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%. \text{ Ans.}}$$

Problem *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m^3 of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Solution. Given:

Speed of pump, $N = 40$ r.p.m.

Actual discharge, $Q_{act} = 1.0 \text{ m}^3/\text{min} = \frac{1.0}{60} \text{ m}^3/\text{s} = 0.01666 \text{ m}^3/\text{s}$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$

Diameter of piston, $D = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.031416 \text{ m}^2$

Suction head, $h_s = 5$ m

Delivery head, $h_d = 20$ m.

Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times .031416 \times 0.4 \times 40}{60} = .01675 \text{ m}^3/\text{s}.$$

Using equation Slip $= Q_{th} - Q_{act} = .01675 - .01666 =$ **.00009 m^3/s. Ans.**

Power required to drive the double-acting pump is given by equation (20.7) as,

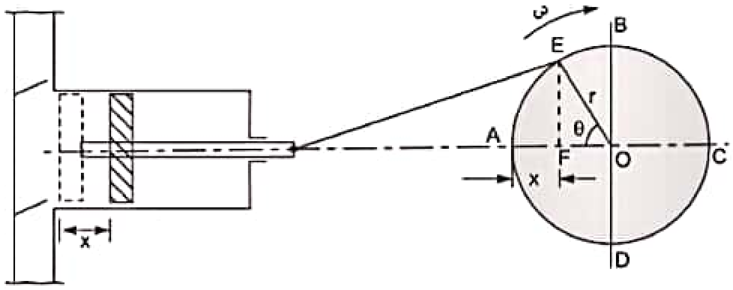
$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000} = \frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times (5 + 20)}{60{,}000}$$

= **4.109 kW. Ans.**

VARIATION OF VELOCITY AND ACCELERATION IN THE SUCTION AND DELIVERY PIPES DUE TO ACCELERATION OF THE PISTON

when crank starts rotating, the piston moves forwards and backwards in the cylinder. At the extreme left position and right position of the piston in the cylinder, the velocity of the piston is zero. The velocity of the piston is maximum at the centre of the cylinder. This means that at the start of a stroke (may be suction or delivery stroke), the velocity of the piston is zero and this velocity becomes maximum at the centre of each stroke and again becomes zero at the end of each stroke. Thus at the beginning of each stroke, the piston will be having an acceleration and at the end of each stroke, the piston will be having a retardation. The water in the cylinder is in contact with the piston and hence the water, flowing from the suction pipe or to the delivery pipe will have an acceleration at the beginning of each stroke and a retardation at the end of each stroke. This means the velocity of flow of water in the suction and delivery pipe will not be uniform. Hence, an accelerative or retarding head will be acting on the water flowing through the suction or delivery pipe. This accelerative or retarding head will change the pressure inside the cylinder.

If the ratio of length of connecting rod to the radius of crank (*i.e.*, L/r) is very large, then the motion of the piston can be assumed as simple harmonic in nature. Fig. shows the cylinder of a reciprocating single-acting pump, fitted with a piston which is connected to the crank. Let the crank is rotating at a constant angular speed.



Velocity and acceleration of piston.

Let ω = Angular speed of the crank in rad./s,

A = Area of the cylinder,

a = Area of the pipe (suction or delivery),

l = Length of the pipe (suction or delivery), and

r = Radius of the crank.

In the beginning, the crank is at A (which is called inner dead centre) and the piston in the cylinder is at a position shown by dotted lines. The crank is rotating with an angular velocity ω and let in time 't' seconds, the crank turns through an angle θ (in radians) from A (*i.e.*, inner dead centre). The displacement of the piston in time 't' is 'x' as shown in Fig. 20.3.

Now θ = Angle turned by crank in radians in time 't'

$$= \omega t \quad \text{...(i)}$$

The distance x travelled by the piston is given as

$$x = \text{Distance } AF = AO - FO$$

$$= r - r\cos\theta \qquad (\because AO = r,\ FO = r\cos\theta)$$

$$= r - r\cos(\omega t) \qquad (\because \text{From } (i),\ \theta = \omega t) \text{ ...(ii)}$$

The velocity of the piston is obtained by differentiating equation (*ii*) with respect to 't'.

$\therefore$ Velocity of piston,
$$V = \frac{dx}{dt} = \frac{d}{dt}[r - r\cos(\omega t)]$$

$$= 0 - r[-\sin\omega t]\times\omega \qquad (\because r \text{ is constant})$$

$$= \omega r \sin \omega t.$$

Now from continuity equation, the volume of water flowing into cylinder per second is equal to the volume of water flowing from the pipe per second.

$\therefore$ Velocity of water in cylinder $\times$ Area of cylinder

= Velocity of water in pipe $\times$ Area of pipe

or $V \times A = v \times a$ ($\because$ Velocity of water in cylinder = Velocity of piston = V)

where v = Velocity of water in pipe

$$\therefore \quad v = \frac{V\times A}{a} = \frac{A}{a}\times V$$

$$= \frac{A}{a}\,\omega r \sin \omega t \qquad [\because \quad V = \omega r \sin \omega t]$$

The acceleration of water in pipe is obtained by differentiating equation (20.11) with respect to 't'.

∴ Acceleration of water in pipe

$$= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a}\omega r \sin \omega t\right) = \frac{A}{a}\omega^2 r \cos \omega t$$

Mass of water in pipe = ρ × Volume of water in pipe

$$= \rho \times [\text{Area of pipe} \times \text{Length of pipe}] = \rho \times [a \times l] = \rho a l$$

∴ Force required to accelerate the water in the pipe

= Mass of water in pipe × Acceleration of water in pipe

$$= \rho a l \times \frac{A}{a}\omega^2 r \cos \omega t$$

∴ Intensity of pressure due to acceleration

$$= \frac{\text{Force required to accelerate the water}}{\text{Area of pipe}}$$

$$= \frac{\rho a l \times \frac{A}{a}\omega^2 r \cos \omega t}{a} = \rho l \times \frac{A}{a}\omega^2 r \cos \omega t$$

$$= \rho l \times \frac{A}{a}\omega^2 r \cos \theta \qquad (\because \omega t = \theta)$$

∴ Pressure head (h_a) due to acceleration

$$h_a = \frac{\text{Intensity of pressure due to acceleration}}{\text{Weight density of liquid}}$$

$$= \frac{\rho l \times \frac{A}{a}\omega^2 r \cos \theta}{\rho g} = \frac{l}{g} \times \frac{A}{a}\omega^2 r \cos \theta.$$

The pressure head due to acceleration in the suction and delivery pipes is obtained from equation by using subscripts 's' and 'd' as

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s}\omega^2 r \cos \theta$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d}\omega^2 r \cos \theta.$$

The pressure head (h_a) due to acceleration, given by equation varies with θ. The values of 'h_a' for different values of θ are :

1. When $\theta = 0°$, $\quad h_a = \frac{l}{g} \times \frac{A}{a}\omega^2 r \quad$ as $\cos 0° = 1$

2. When $\theta = 90°$, $\quad h_a = 0 \quad$ as $\cos 90° = 0$

3. When $\theta = 180°$, $\quad h_a = -\frac{l}{g} \times \frac{A}{a}\omega^2 r \quad$ as $\cos 180° = -1$

$\therefore$ Maximum pressure head due to acceleration

$$(h_a)_{max} = \frac{l}{g} \times \frac{A}{a} \omega^2 r$$

EFFECT OF VARIATION OF VELOCITY ON FRICTION IN THE SUCTION AND DELIVERY PIPES

The velocity of water in suction or delivery pipe is given by equation (20.11) as

$$v = \frac{A}{a} \omega r \sin \omega t = \frac{A}{a} \omega r \sin \theta \qquad ...(i)$$

Loss of head due to friction in pipes is given by

$$h_f = \frac{4flv^2}{d \times 2g} \qquad ...(ii)$$

where f = Co-efficient of friction, l = Length of pipe,
d = Diameter of pipe, and v = Velocity of water in pipe.

Substituting equation (*i*) into equation (*ii*), we get

$$h_f = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r \sin \theta\right]^2$$

The variation of h_f with θ is parabolic. The loss of head due to friction in suction and delivery pipes is obtained from equation (20.17) by using subscripts '*s*' for suction pipe and '*d*' for delivery pipe as

$$h_{fs} = \frac{4fl_s}{d_s \times 2g} \times \left[\frac{A}{a_s} \omega r \sin \theta\right]^2$$

$$h_{fd} = \frac{4fl_d}{d_d \times 2g} \times \left[\frac{A}{a_d} \omega r \sin \theta\right]^2$$

The loss of head due to friction in pipes given by equation (20.17) varies with θ as :

1. When $\theta = 0°$, $\sin \theta = 0$ $\therefore$ $h_f = \frac{4fl}{d \times 2g} \times 0 = 0$

2. When $\theta = 90°$, $\sin 90° = 1$ $\therefore$ $h_f = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2$

3. When $\theta = 180°$, $\sin 180° = 0$ $\therefore$ $h_f = 0$

$\therefore$ Maximum value of loss of head due to friction ;

$$(h_f)_{max} = \frac{4fl}{d \times 2g} \times \left[\frac{A}{a} \omega r\right]^2$$

COMPARISON BETWEEN CENTRIFUGAL PUMPS AND RECIPROCATING PUMPS

Centrifugal pumps	*Reciprocating pumps*
1. The discharge is continuous and smooth.	1. The discharge is fluctuating and pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liquids.
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge and high heads.
5. Cost of centrifugal pump is less as compared to reciprocating pump.	5. Cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. Centrifugal pump runs at high speed. They can be coupled to electric motor.	6. Reciprocating pump runs at low speed. Speed is limited due to consideration of separation and cavitation.
7. The operation of centrifugal pump is smooth and without much noise. The maintenance cost is low.	7. The operation of reciprocating pump is complicated and with much noise. The maintenance cost is high.
8. Centrifugal pump needs smaller floor area and installation cost is low.	8. Reciprocating pump requires large floor area and installation cost is high.
9. Efficiency is high.	9. Efficiency is low.