GOVT. POLYTECHNIC, JAGATSINGHPUR

CIVIL ENGINEERING DEPARTMENT

LEARNING MATERIAL OF **STRUCTURAL MECHANICS**

3RD SEMESTER

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Simple Stress and Strain

Stress:-
The Internal resistance per unit area to deformation is called stress.

Example:-
When a body of uniform section 'A' subjected to a load 'p' transverese to the crosssection the Average stress 'σ' is given by P/A (N/m^2)

$\boxed{1\ N/m^2 = 1\ \text{pascal}}$ Stress $\sigma = \dfrac{\text{Force}}{\text{Cross-sectional Area}} = \dfrac{F}{A_0}$

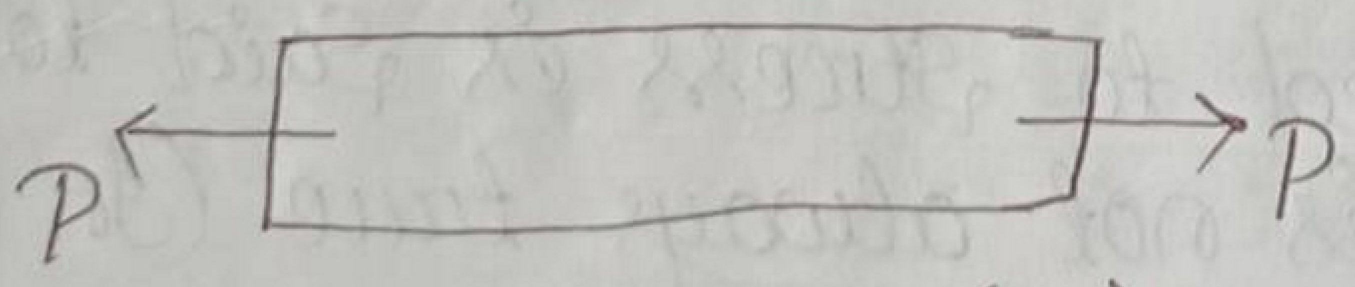
Imp $1\ \text{mega pascal} = 1\ N/mm^2$

$1\ \text{gega pascal} = 10^9$

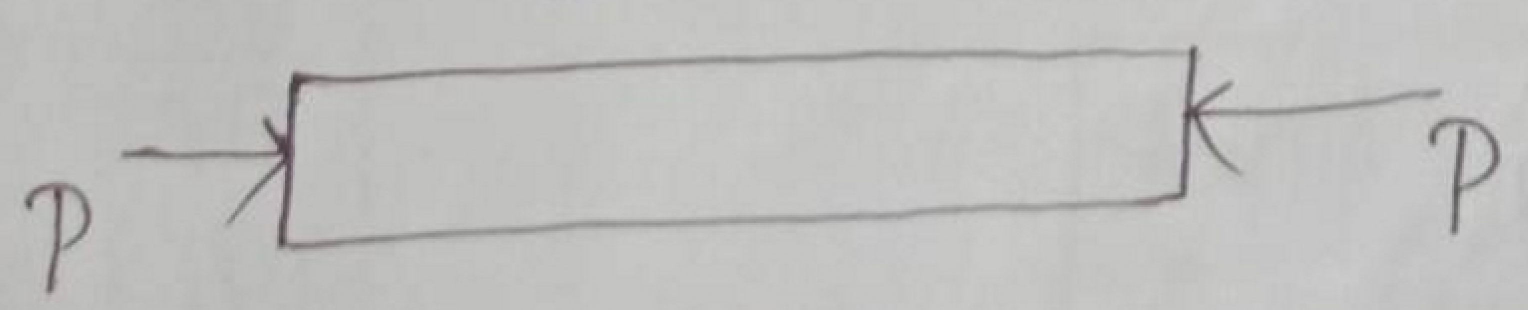
$1m = 1000\ mm$

Type of stresses
1. Tensile stress
2. compressive stress
3. Shear stress
4. Bending stress
5. Torsional stress

1. Tensile stress (σ_t)
→ If the applied force tends to increase the length of the solid body the stress induced is called Tensile stress.
→ It is denoted by σ_t



2. compressive stress: (σ_c)
→ If the applied force tends to decrease the length of the solid body, the stress induced is called compressive stress.
→ It is denoted by σ_c



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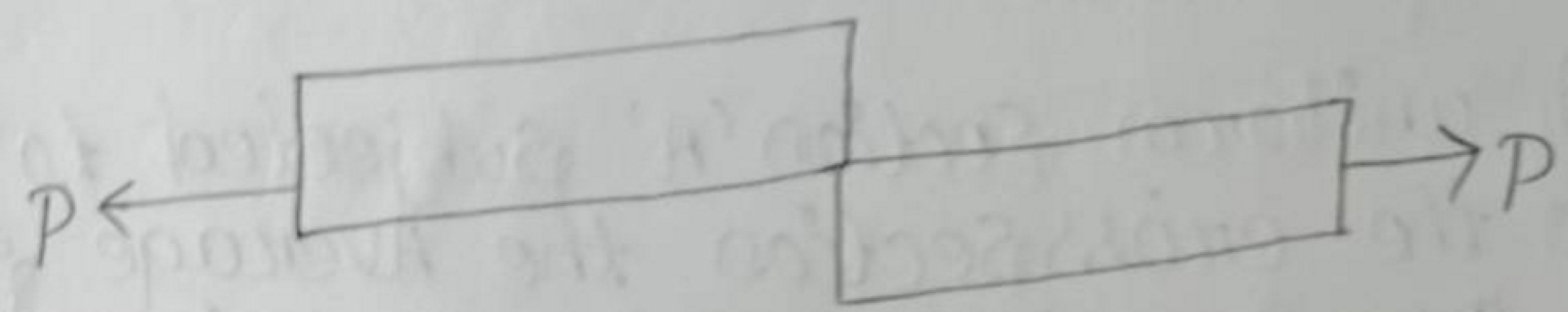
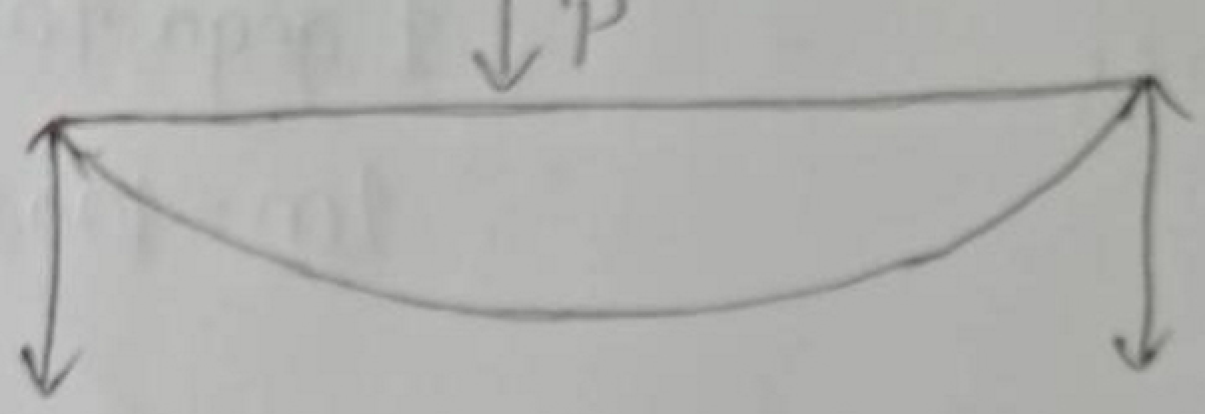
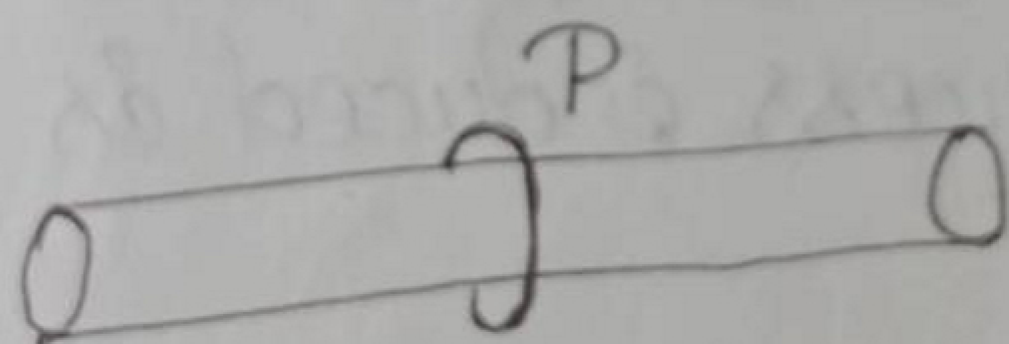
3. Shear Stress:- (τ)
→ It is the stress caused by force action along or parallel to the area resisting that force.
→ It is denoted by 'τ'



4. Bending Stress:- (σ_b)
The stress developed in a member due to Bending action of the transverse load is called Bending stress.



5. Torsional Stress:- (τ)
The stress developed against the deformation in a member due to torque is called Torsional stress.



Strain (ϵ)
When a body subjected to stress is said to be strained But reverse is not always true (In case of free thermal strain). $\boxed{\text{Strain} = \frac{\Delta l}{l}}$
Hence, the stress is the deformation produced by the strain. [Strain has no unit]
It is classified in 3 Parts:-
1. Tensile Strain
2. Compressive Strain
3. Shear Strain

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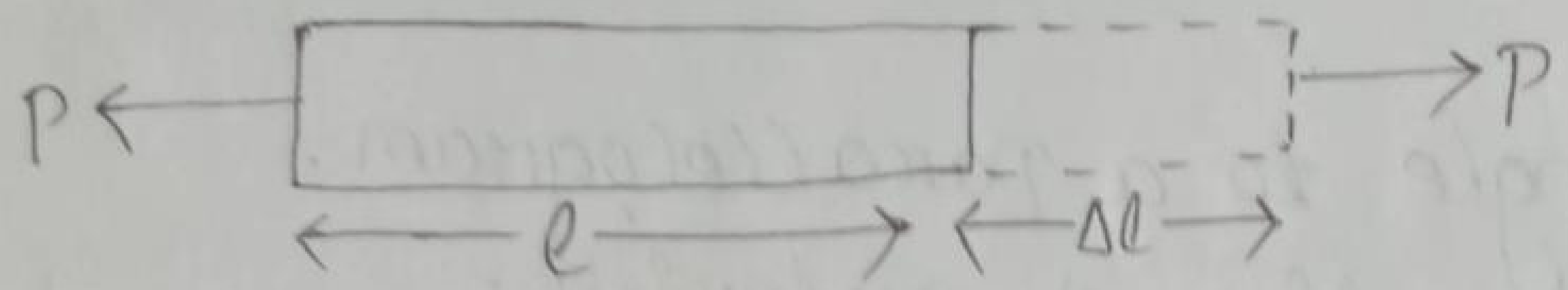
1. Tensile strain :-

→ The deformation due to the direct action of the tensile stress this is called Tensile strain.

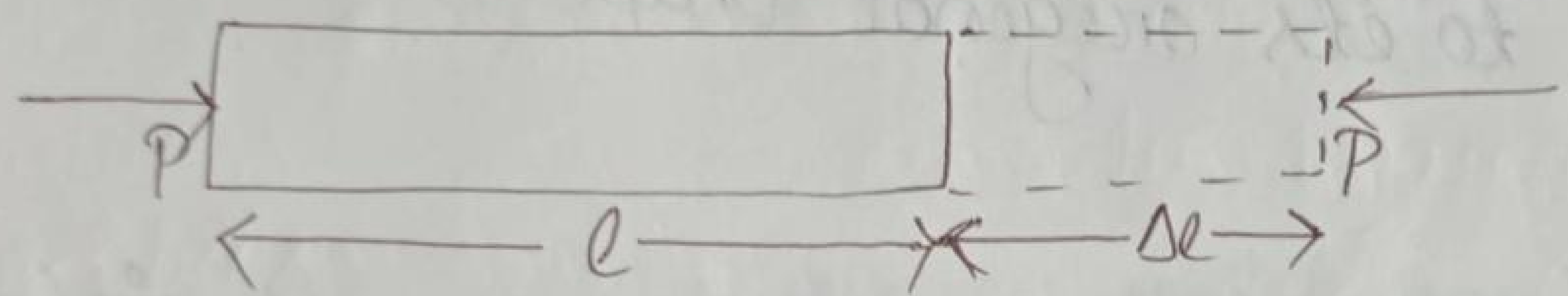


$$\boxed{E = \frac{\Delta l}{l}}$$

→ It has no unit or unitless.

Compressive strain :-

→ The deformation due to the direct action of the compressive stress this is called compressive strain.



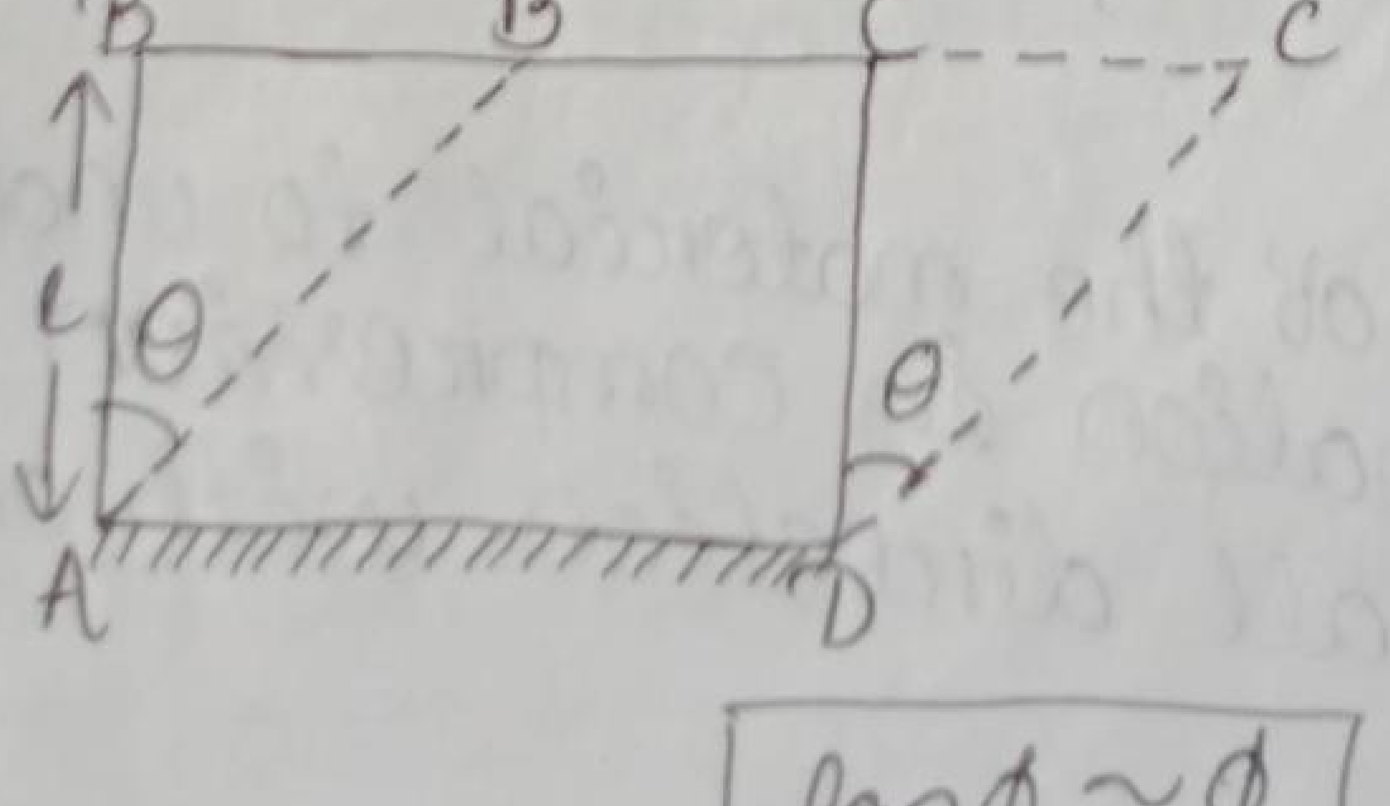
$$\boxed{E = \frac{\Delta l}{l}}$$

→ It has no unit or unitless.

(*) → Generally tensile strain is (+) strain and compressive strain is (−) strain.

Shear strain :-

→ It is defined as the tangent of the angle and is equal to the length of deformation at its maximum developed by the perpendicular length on the plane of force application.



$$\tan\phi = \frac{BB'}{AB}$$

$$\tan\theta = \frac{\Delta l}{l}$$

$$\tan\phi \simeq \phi$$

$$\phi = \frac{\Delta l}{l}$$

* When a body is subjected to shear does not change it's length, breath or thickness but, it undere goes a charge in shape.

Ex:-

From a rectangle to a parallelogram.

Mechanical properties of material:-

1. Rigidity:-

→ It referes to the property of a solid to resist change in shape.

2. Elasticity:-

→ It is defined as the ability of material to deform and return back to its original shape when the load is remove.

3. Plasticity:-

→ It is defined as the ability of a material to get permanent deformation under the action of load is called plasticity.

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4. Ductility:-

→ It is the ability of a material to under go large permanent deformation in tension.

For Ex:- The property which enables a materials to be drawn into wire.

Ex:- Mild steel.

5. Malleability:-

→ It is the ability of the material to under go large permanent deformation in compression and allows in to expand in all direction with out failure.

For Example:-

The property which enables a material to be beaten or rolled into thin sheets.

6. Compressibility :-

→ It is the property of a material due to which its volume decreases when pressure is applyed.

7. Hardness :-

→ The resistance of a material to indentation including, Scratching, or surface abrasion is called Hardness.

8. Toughness :-

→ It is the capacity of a structure to with stand and impact load.

Ex :- The capacity to absorb energy with out failure, It depends upon the ductility of a material and its ultimate strength.

9. Stiffness :-

→ It is the ability of material to resist any deformation under stress.

$$\text{Stiffness} = \frac{\text{Resisting force}(P)}{\text{long action}(\Delta)} = \frac{N}{mm}$$

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10. Brittleness :-

→ It is opposite to ductility.

For Example :- When a material can not be drawn out by tension to smaller section.

→ A brittle material fails instantly under the load with out showing any significant to deformation.

Ex :- Concrete or cast iron.

11. Fatigue :-

When loading are repeated thousands or millions of times, failure occures at a stress much below the static breaking strain this phenomenon is known as fatigue.

12. Creep:-

If the stress exceed the yeild point, the strain caused in the material by the application of load does not disappear totally of the removal of load. The plastic deformation caused to the material is known as creep.

13. Tenacity:-

→ Tenacity is the resistance of material to breaking.

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14. Durability:-

→ It is the property of a material in which it can with stand the wear or tear due to environmental effects such as:- wind, rain, hot, cold etc.

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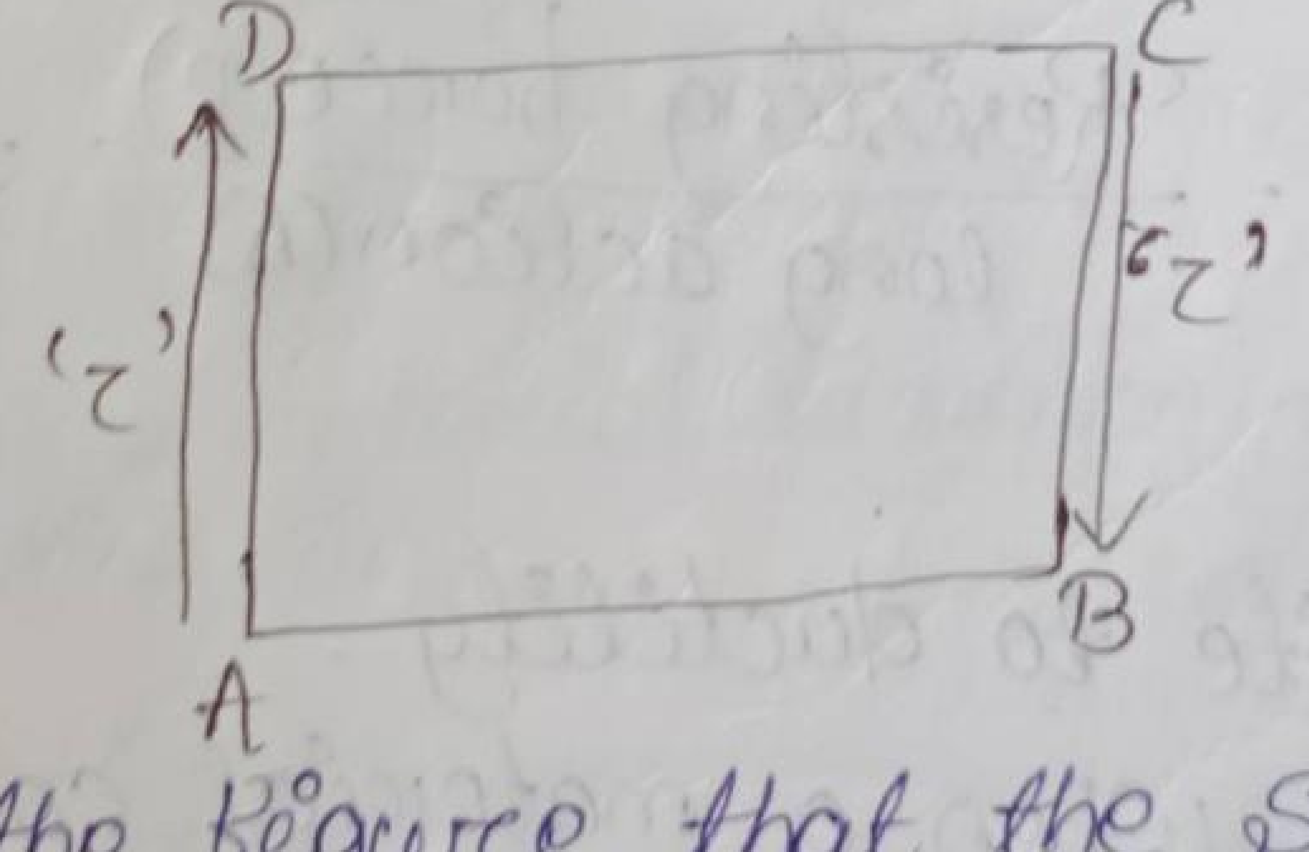
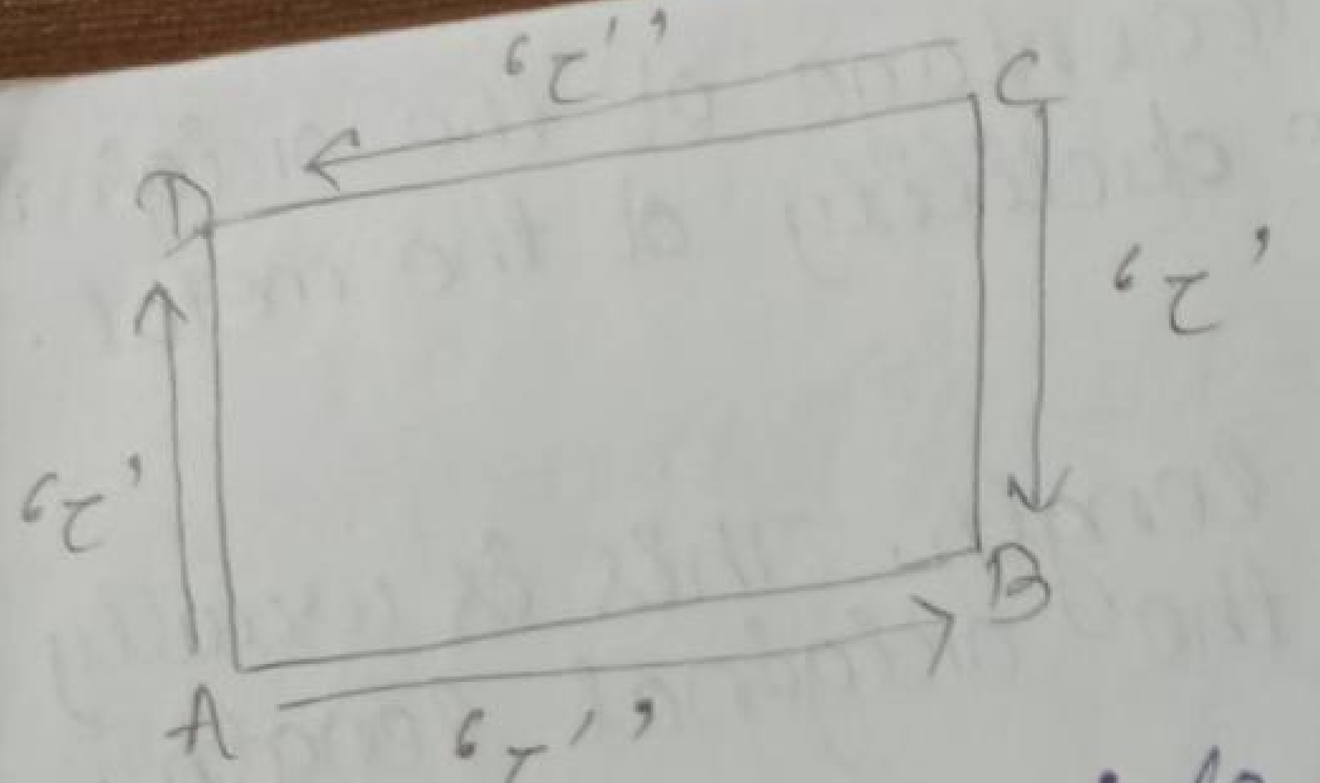
Complimentary Shear Stress:-

→ consider an infinitely small rectangular ABCD under shear stress of intensity 'τ' acting in plane AD and BC.



→ It is clear from the figure that the shear stress acting on the element will tend to rotate in clockwise direction.

→ As there is no other forces acting on the element, static equilibrium of the element can only be attended if another couple of the same magnitude is applied in the anti-clockwise direction. This can be achieved by having shear stress of intensity 'τ'' on the axis AB and CD.



→ Assume 'x' and 'y' to be the length of side AB and BC and a unit thickness perpendicular to the figure.

Force of the given couple $= \tau \times A$
$= \tau \times (y \times 1)$

moment of the given couple $=$ F x perpendicular distance
$= \tau \times y \times 1 \times x$.

Force of the balancing couple $= \tau' \times$ Area
$= \tau' \times (x \times 1)$

moment of the balancing couple $= F \times \perp d$
$= \tau' \times x \times 1 \times y$

For equilibrium:-

moment of the given couple = moment of the balancing couple

$$\tau \times y \times 1 \times x = \tau' \times x \times 1 \times y$$

$$\boxed{\tau = \tau'}$$

This shows that the magnitude of balancing shear stress is same as the applied shear stress. The shear stress on the transverse pair of faces are known as complimentary shear stress. Thus every shear stress is always accompained by an equal complimentary shear stresses.

Elongation:-

→ Elongation means increase in length which occur before a metal is failed when subjected in stress.

→ This is usually expressed as percentage of the original length and is a measure of the ductility of the metal.

Contraction:-

Contraction mean decrease in length. This is usually expressed as a percentage of the original length.

Longitudinal strain:-

The strain of a body in the direction of force is called longitudinal strain or linear strain.

It is expressed as $\frac{\Delta L}{L}$

Lateral strain:-

The strain of a body opposite to that of force and act right angle to its is called lateral strain.

It is expressed as $\frac{\Delta \text{diameter}}{\text{diameter}}$

Poisson's ratio:-

→ The ratio of the lateral strain to the longitudinal strain of a material when it is subjected to a longitudinal stress is known as poisson's ratio.

→ within the elastic limit lateral strain is directly proportional to longitudinal strain.

lateral strain = $\mu \times$ (longitudinal strain)

$$\frac{\text{lateral strain}}{\text{longitudinal strain}} = \mu$$

$$\mu = \frac{\Delta d/d}{\Delta l/l}$$

* Poisson's ratio is unit less.

$\boxed{\text{* poisson's ratio range} \rightarrow -0.5 \text{ to } 0.5}$

Hooke's law:- (1 dimension)

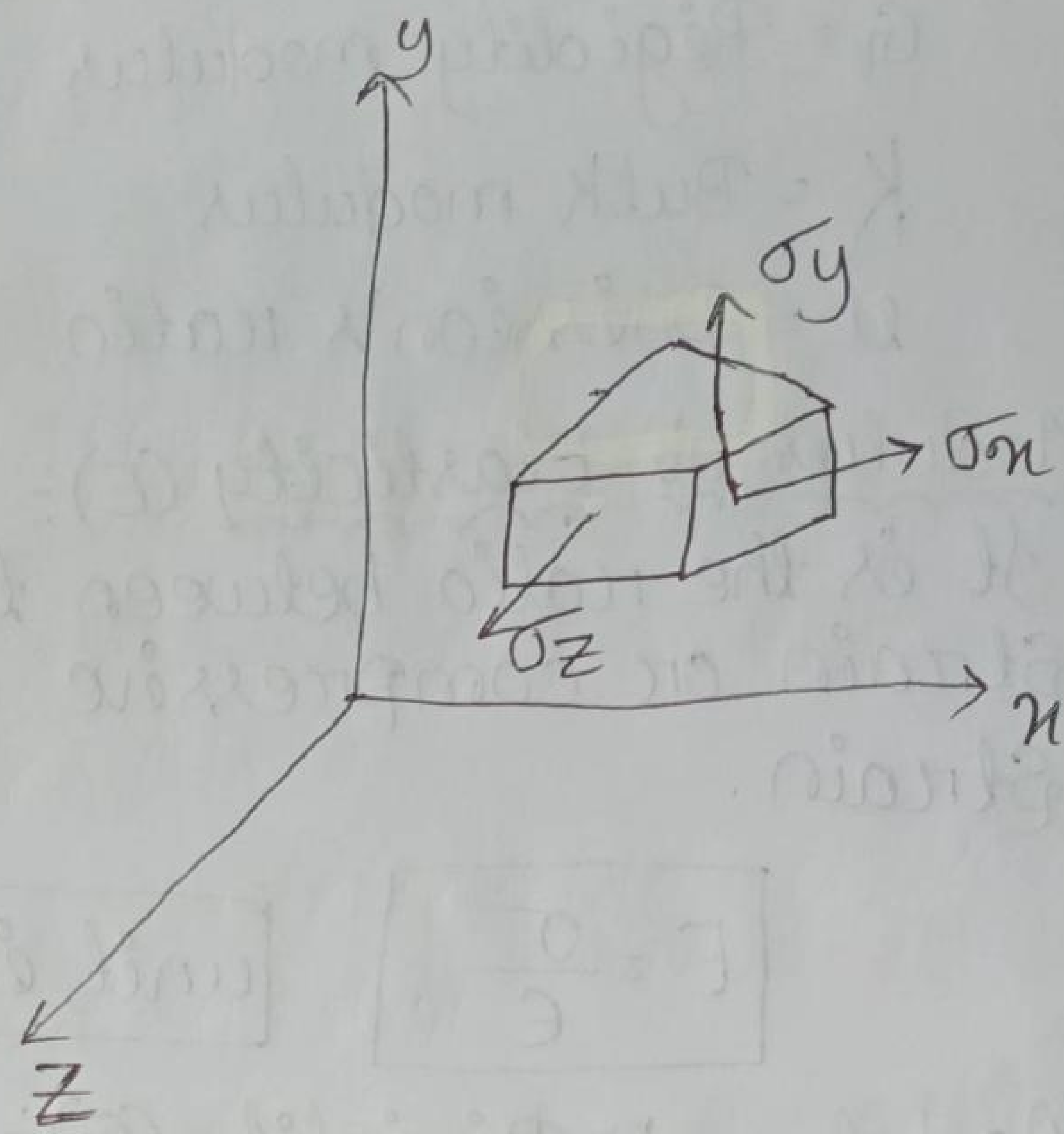
It states that when a body is loaded with in elastic (proportionality) limit, the stress is proportional to strain.

$\sigma \propto \epsilon$

$\sigma = E\epsilon$ $\quad$ E = N/mm²

E ↓ Young's modulus or modulus of elasticity.

Change in dimension and volume :-
(For uniaxial stress)



$$\epsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$$

$$\boxed{\sigma_y = \sigma_z = 0}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = -\mu\frac{\sigma_x}{E}$$

$$\epsilon_z = -\mu\frac{\sigma_x}{E}$$

Volumetric strain :-

It is defined as the change in volume by original when the body is subjected to stress.

$$\boxed{\epsilon_V = \frac{\Delta V}{V}} \quad \frac{\text{Change in volume } (\Delta V)}{\text{original vome } (V)}$$

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E}$$

$$\boxed{\epsilon_V = \frac{\sigma_x}{E}(1-2\mu)} \text{ for 1D}$$

For 3D

$$\epsilon_V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$\epsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

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Elastic constant :-

E = modulus of elasticity

G = Rigidity modulus

K = Bulk modulus

μ = Poisson's ratio

Modulus of Elasticity (E) :-

→ It is the ratio between tensile stress and tensile strain or compressive stress and compressive strain.

$$\boxed{E = \frac{\sigma}{\epsilon}}$$ unit is N/mm^2

Modulus of Rigidity (G) :-

→ It is the ratio between shear stress and shear strain, $\tau \propto \phi$

$$\boxed{\tau = G\phi}$$ unit is N/mm^2

Bulk modulus :- (K - (kapa))

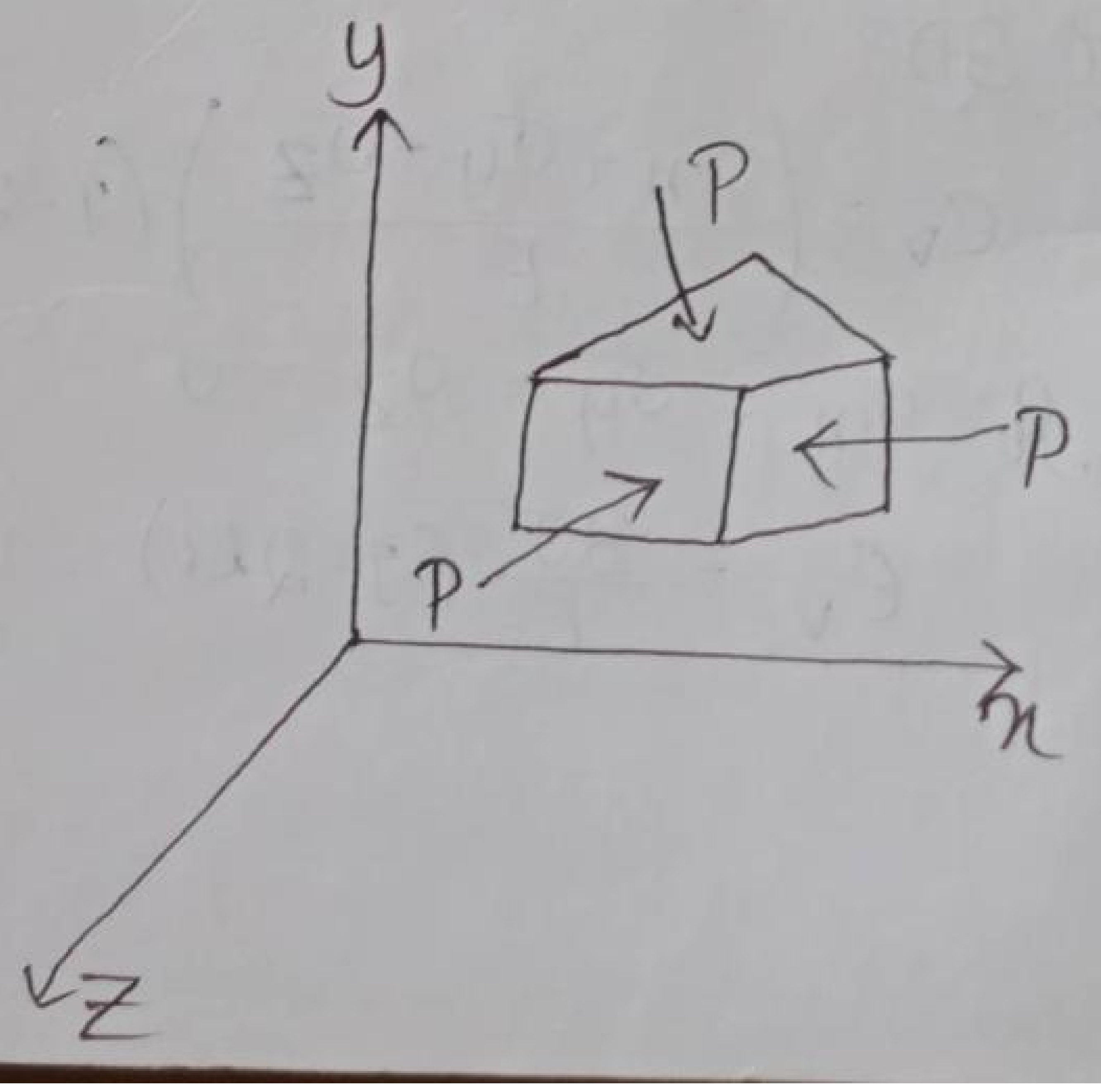
→ It is the ratio between normal stress and volumetric strain.

$$\boxed{K = \frac{\sigma}{\epsilon_v}} \quad \boxed{K = \frac{-P}{\epsilon_v}}$$ unit is N/mm^2

Relation between E and K :-

$$K = \frac{\sigma}{\epsilon_v}$$

$$= \frac{-P}{\epsilon_v}$$



$$\epsilon_v = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)$$

$$= \frac{-P-P-P}{E}(1-2\mu)$$

$$= \frac{-3P}{E}(1-2\mu)$$

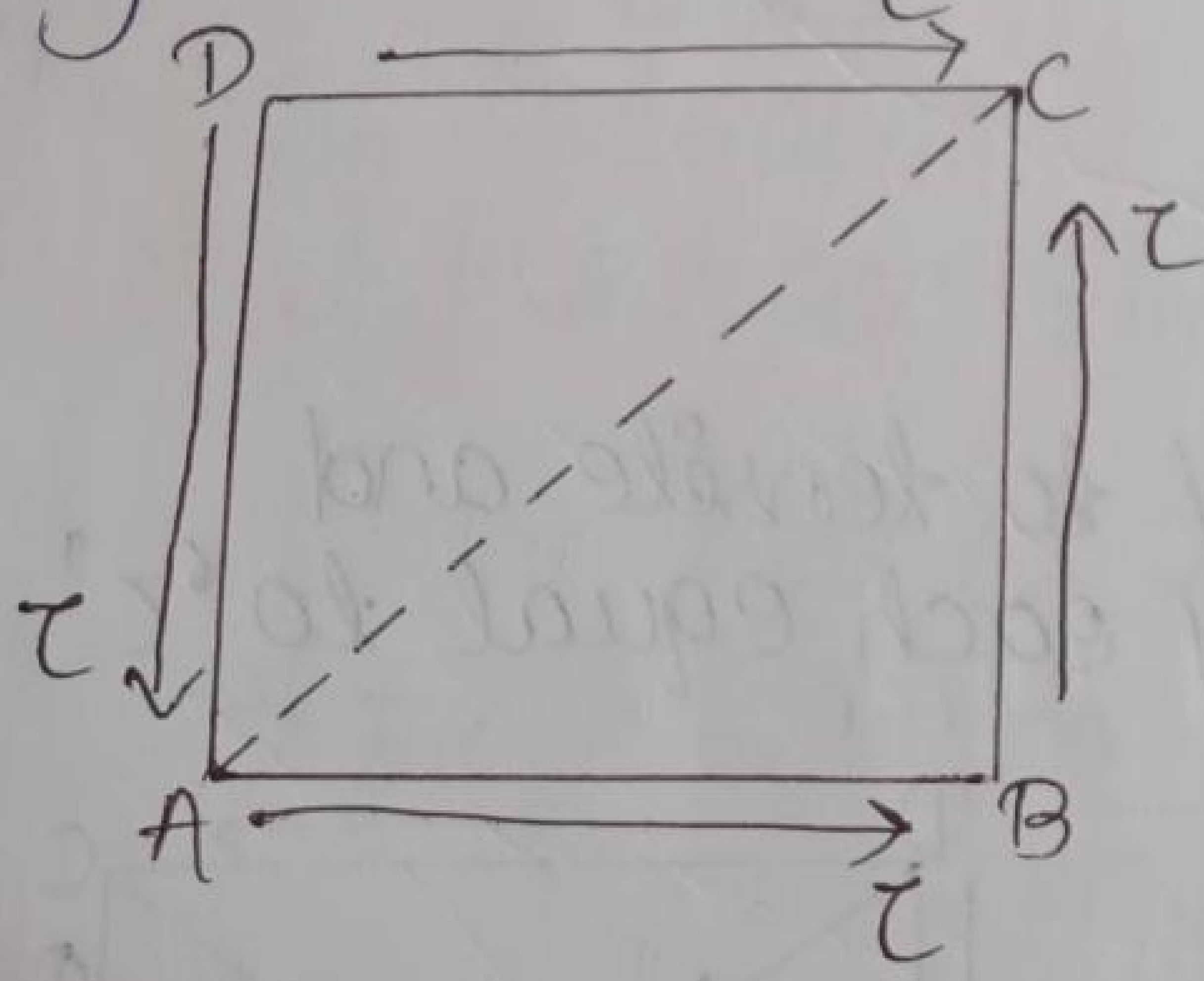
$$\epsilon_v = \frac{-3P}{E}(1-2\mu)$$

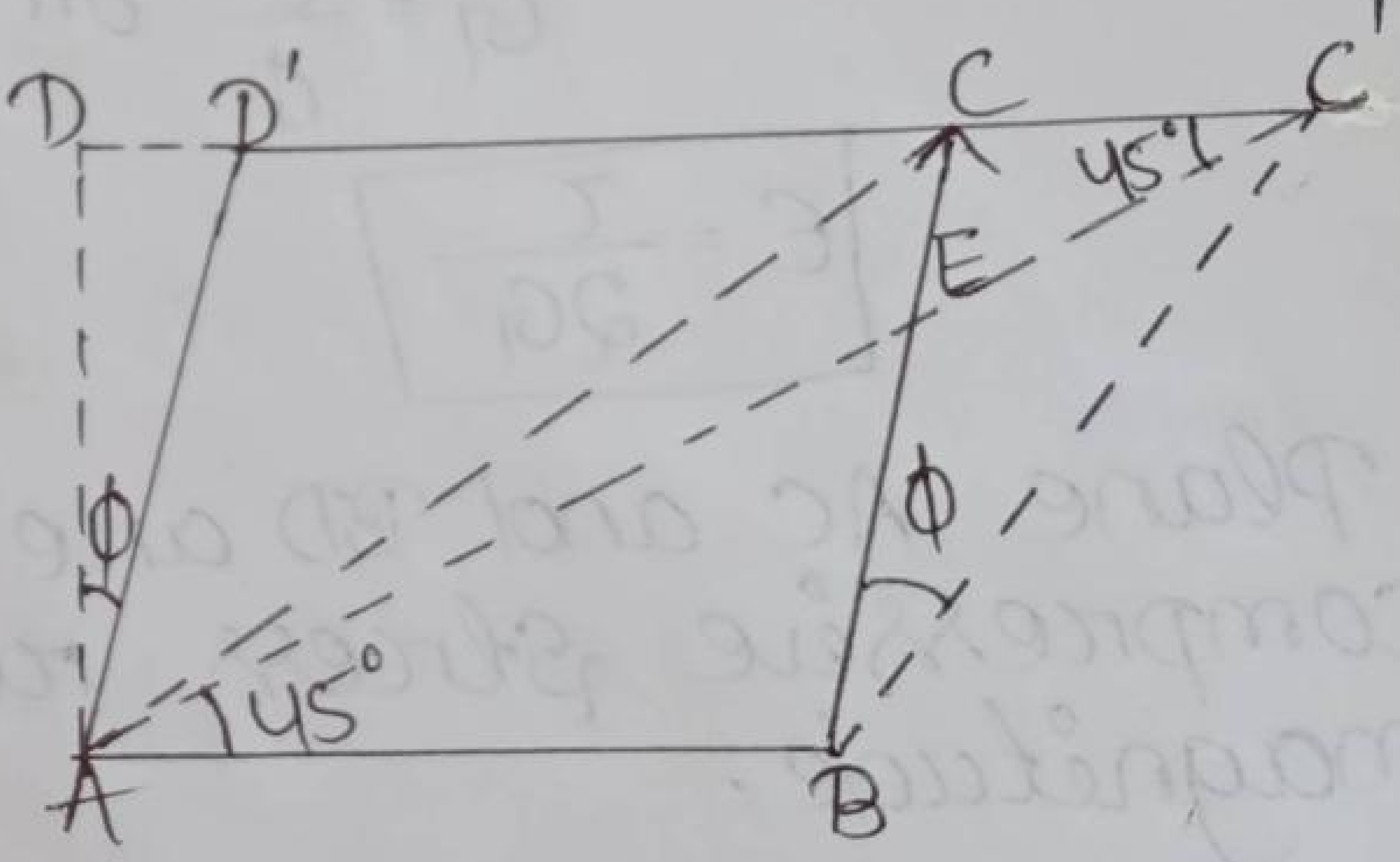
$$E = \frac{-3P}{\epsilon_v}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

Relation between E and G :-

→ Consider a square element ABCD, under the action of a simple shear stress (τ). The total change in angle is $\pm\phi$.





Linear strain of diagonal AC $= \frac{\Delta L}{L}$

$$= \frac{AC' - AC}{AC}$$

$$\epsilon = \frac{EC'}{AC} \quad \text{— (i)}$$

$$\cos 45^\circ = \frac{EC'}{CC'}$$

$$EC' = CC' \cos 45^\circ \quad \text{— (ii)}$$

$$\frac{AB}{AC} = \cos 45^\circ$$

$$AC = \frac{AB}{\cos 45^\circ} \quad \text{——— (iii)}$$

Put eq^n (ii) and (iii) in eq^n (i)

$$\epsilon = \frac{CC' \cos 45^\circ}{AB / \cos 45^\circ} \quad \text{——— (iv)}$$

$$\tan\phi = \frac{CC'}{BC} \qquad \tan\phi \simeq \phi \text{ (For Small angle)}$$

$$\phi BC = CC' \quad \text{——— (v)}$$

$$AB = BC$$

$$\epsilon = \frac{\phi \cancel{BC} \cos^2 45^\circ}{\cancel{BC}}$$

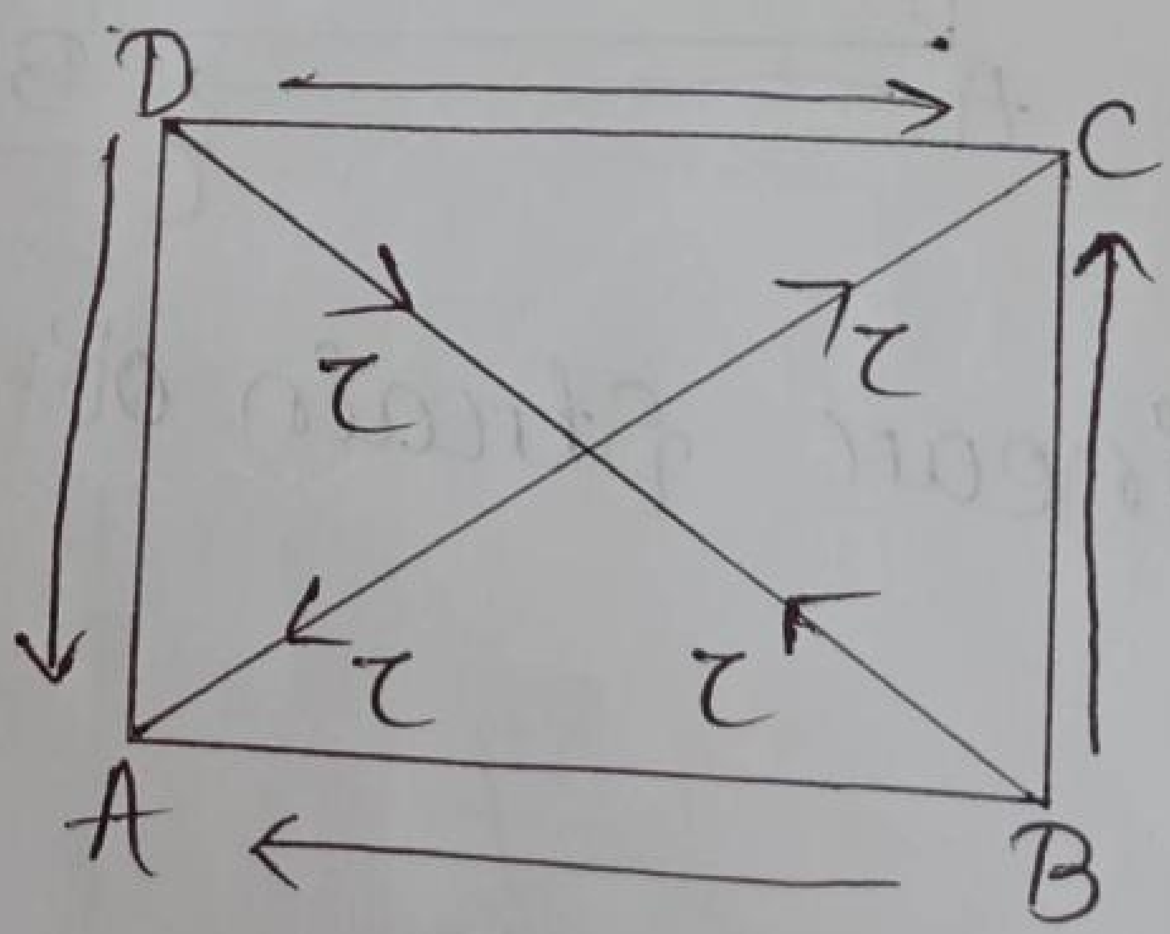
$$\boxed{\epsilon = \phi/2}$$

But modulus rigidity

$$G = \frac{\tau}{\phi} \text{ or } \phi = \frac{\tau}{G} \quad \text{as } \boxed{\epsilon = \frac{\phi}{2}}$$

$$\boxed{\epsilon = \frac{\tau}{2G}}$$

Plane AC and BD are subjected to tensile and compressive stress respectively each equal to 'τ' magnitude.



$$\epsilon = \frac{\tau}{E} - \left(-\mu \frac{\tau}{E}\right)$$

$$\epsilon = \frac{\tau}{E}(1+\mu)$$

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

Elastic constant formula :-

$$\boxed{\begin{aligned} E &= 2G(1+\mu) \\ E &= 3K(1-2\mu) \\ E &= \frac{9KG}{3K+G} \end{aligned}}$$

Q. A matercial as E of $2\times10^5\ N/mm^2$ and a Poission's ratio of 0.25. Calculate modulus of rigidity and bulk modulus.

Ans:-

$E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

$E = 2G(1+\mu)$

$2\times10^5 = 2G(1+0.25)$

$2G = \frac{2\times10^5}{1.25}$

$2G = 160000$

$G = 80,000\ N/mm^2 = 80\ GPa$

$E = 3K(1-2\mu)$

$\Rightarrow 3K = \frac{2\times10^5}{1-2\times0.25}$

$3K = 400000$

$K = \frac{400000}{3}$

$K = 133333.33\ N/mm^2$

$$\boxed{K = 133.33\ GPa}$$

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Q. A bar 24mm diameter and 400 mm length is acted upon by an uniaxial load of 38 kN, the elongation of the bar and change in diameter are measured as 0.165 mm and 0.0031 respectively determined Poisson's ratio and value of 3 elastic modulus.

Ans Given :- $d = 24 mm$

$l = 400 mm$

$\Delta l = 0.165 mm$

$\Delta d = 0.0031$

$P = 38 kN$

$$\text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\mu = \frac{\Delta d / d}{\Delta l / l}$$

$$\mu = \frac{\frac{0.0031}{24}}{\frac{0.165}{40}}$$

$$\mu = 0.313$$

$$\sigma = P/A$$

$$\sigma = \frac{38 \times 10^3 N}{\frac{\pi}{4} \times (24)^2 mm^2} \qquad \boxed{d = 24 mm}$$

$$\sigma = 84 N/mm^2 \text{ or } 84 MPa.$$

So. we know =

$$\sigma = E\epsilon$$

$$E = \frac{\sigma}{\epsilon}$$

$$E = \frac{84}{\Delta l / l}$$

$$E = \frac{84}{0.165/400} = 203636 MPa$$

or 203.636 GPa

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$E = 2G(1+\mu)$

$\Rightarrow 203.636 = 2G(1+0.313)$

$\Rightarrow 2G = \dfrac{203.636}{1+0.313}$

$\Rightarrow 2G = 155.09$

$\Rightarrow G = \dfrac{155.09}{2}$

$\Rightarrow G = 77.545$ GPa or 77546 MPa

$E = 3K(1-2\mu)$

$\Rightarrow 203636 = 3K(1-2\times 0.313)$

$\Rightarrow 3K = \dfrac{203636}{1-2\times 0.313}$

$\Rightarrow 3K = 544481.283$

$\Rightarrow K = \dfrac{54481.283}{3}$

$\Rightarrow K = 181493.76$ MPa

Q. A bar 12 mm in diameter is acted upon by a axial load 20KN change in diameter is measured 0.003 Determine the poisson's ratio, the modulus of elasticity and the bulk modulus. The value of modulus of rigidity is 80GPa.

Ans

$$\text{Area} = \frac{\pi}{4} \times (12)^2 = 36\pi \, mm^2$$

$$\sigma = \frac{20,000}{36\pi} = 176.84 MPa$$

Poisson's ratio = ?

$$\mu = \frac{\text{lateral strain}}{\text{linear strain}}$$

$\Rightarrow$ Lateral strain = $\mu \times \epsilon$(linear strain)

$$\Rightarrow \frac{\Delta d}{d} = \mu \times \epsilon$$

$$\Rightarrow \frac{0.003}{12} = \mu \times \epsilon$$

$$\Rightarrow \epsilon = \frac{0.00025}{\mu}$$

$$E = 2G(1+\mu)$$

$$= 2 \times 80,000 (1+\mu)$$

$$E = 160000 + 160000\mu \quad \text{——— (i)}$$

$$E = \frac{\sigma}{\epsilon}$$

$$= \frac{176.84}{\frac{0.00025}{\mu}}$$

$$E = 707360\mu \quad \text{——— (ii)}$$

equat
70
$\Rightarrow$ 5
$\Rightarrow$

equate (i) & (ii)

$707360\mu = 160000 + 160000\mu$

$\Rightarrow 547360\mu = 160000$

$\Rightarrow \mu = \frac{160000}{547360}$

$= 0.2923$

$E = 707360\mu$

$= 707360 \times 0.2923$

$= 206761$ MPa

or 206.761 GPa

$K = \frac{E}{3(1-2\mu)}$

$= \frac{206761}{3(1-2\times0.2923)}$

$= 165913.176$ MPa

Q: What will be percentage change in the volume of a steel bar of 20 mm diameter and 600 mm length when a tensile stress of 180 MPa is applied to it along its longitudinal axis?

$$E_S = 205 \text{ GPa}, \quad \mu = 0.3$$

Ans

Given,

$d = 20\text{mm}$ $\quad$ $L = 600\text{mm}$

$E_S = 205 \text{ GPa}$ $\quad$ $\mu = 0.3$

$\sigma = 180 \text{ MPa}$

Volume of the bar $= \frac{\pi}{4} \times d^2 \times h$

$= \frac{\pi}{4} \times (20)^2 \times 600$

$= 60,000\pi \text{ mm}^3$

Change in volume $= V \times \frac{\sigma(1-2\mu)}{E}$

$= 60,000\pi \times \frac{180(1-2\times0.3)}{205,000}$

$= 66.2 \text{ mm}^3$

Percentage change in volume $= \frac{66.2}{60,000\pi} \times 10$

$= 0.035$

$\boxed{\text{volume of cylinder} = \pi r^2 h}$

Q: A bar of 240 mm long and $40 \times 30\ mm^2$ crosssection is subjected to an axile tensile force of 100 kN. Find the change in length, bredth, thickness and volume. Take $E = 200\ GPa$, $\mu = 0.3$.

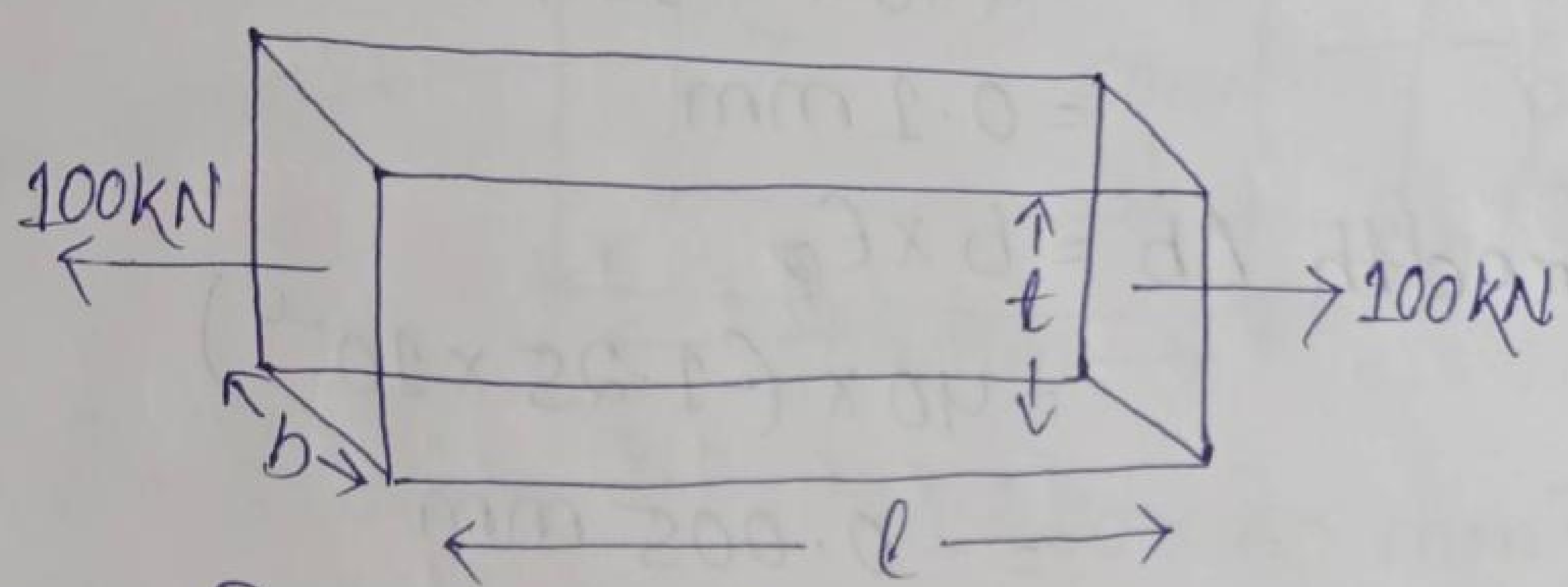
Ans

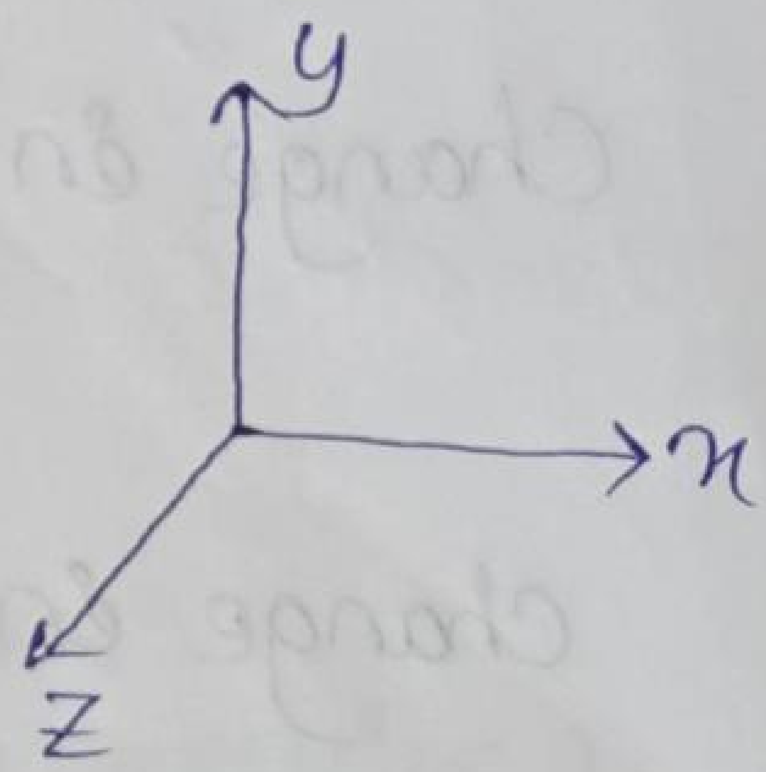
Given data:-

$l = 240\ mm$ $\mu = 0.3$

$b = 40\ mm$ $E = 200\ GPa$

$t = 30\ mm$ $P = 100\ kN$





$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{40 \times 30} = 83.34\ N/mm^2$$

$$\sigma_y = 0, \quad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \mu(\cancel{\sigma_y}^{0} + \cancel{\sigma_z}^{0})\right)$$

$$= \frac{1}{E}(\sigma_x)$$

$$= \frac{83.34}{200 \times 10^3}$$

$$= 4.167 \times 10^{-4}$$

$$\epsilon_y = \frac{1}{E}\left(\cancel{\sigma_y}^{0} - \mu(\sigma_x + \cancel{\sigma_z}^{0})\right)$$

$$= \frac{-\mu\sigma_x}{E}$$

$$= \frac{-0.3 \times 83.34}{200 \times 10^3}$$

$$= -1.25 \times 10^{-4}$$

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$$\epsilon_z = \frac{1}{E}\left(\cancel{\sigma_z}^0 - \mu(\sigma_x + \cancel{\sigma_y}^0)\right)$$

$$= \frac{-\mu \sigma_x}{E}$$

$$= \frac{-0.3 \times 83.34}{200 \times 10^3}$$

$$= -1.25 \times 10^{-4}$$

Change in length $\Delta l = l \times \epsilon_x$

$= 240 \times 4.167 \times 10^{-4}$

$= 0.1\ mm$

Change in bredth $\Delta b = b \times \epsilon_y$

$= 40 \times (-1.25 \times 10^{-4})$

$= -0.005\ mm$

change in thickness $\Delta t = t \times \epsilon_z$

$= 30 \times (-1.25 \times 10^{-4})$

$= -0.0037\ mm$

volumetric strain, $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$\epsilon_v = (4.167 - 1.25 - 1.25) \times 10^{-4}$

$= 1.667 \times 10^{-4}$

Change in volume $\Delta V = V \times \epsilon_v$

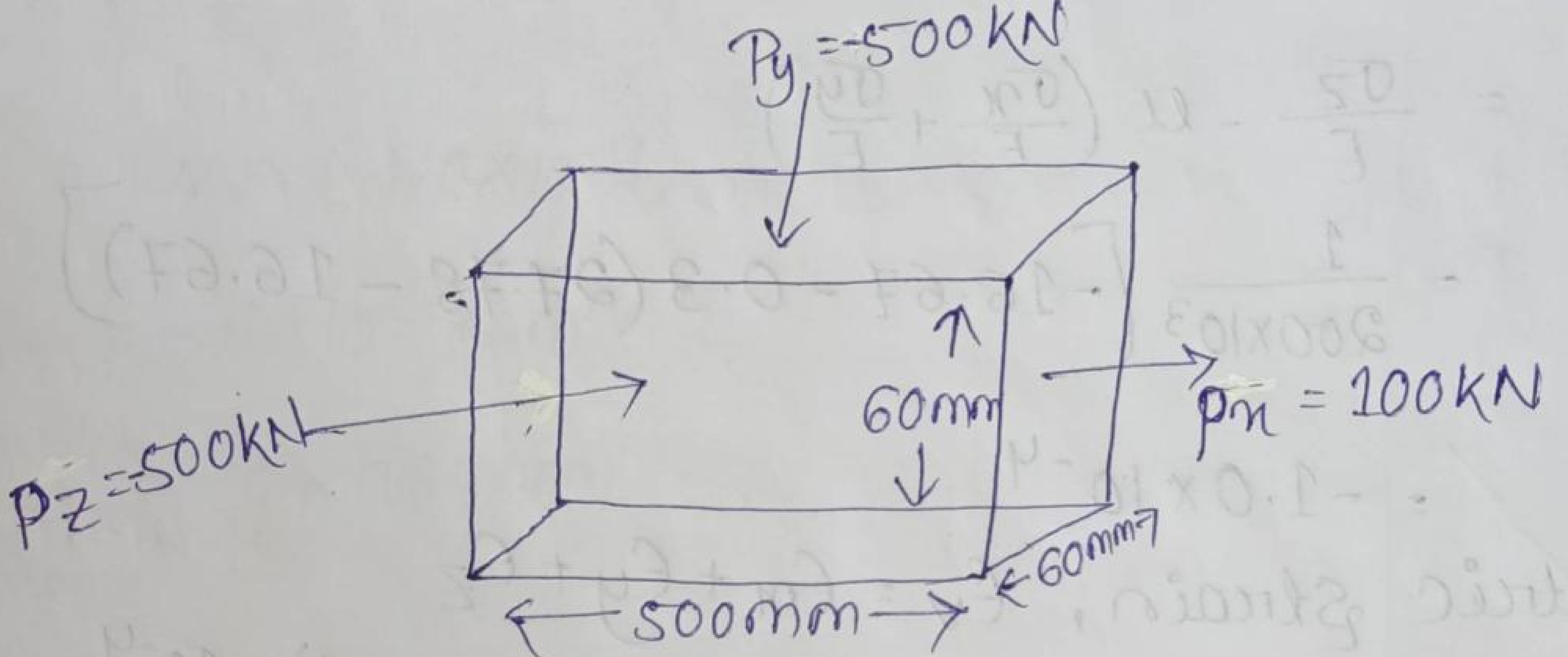
$= 240 \times 40 \times 30 \times 1.667 \times 10^{-4}$

$= 48\ mm^3$

$$\boxed{\Delta V = 48\ mm^3}$$

Q: A bar 500 mm long is having squral crosssection of size 60mm x 60mm. If the bar is subjected to an axial load of 100KN and a lateral compression of 500KN in face of size 60mm, 500mm. Find the change in size and volume. Take E = 200 GPa, $\mu = 0.3$

Ans



$l = 500\text{mm}$ $\quad A = 60 \times 60 \text{ mm}^2$

$P_x = 100 \text{ KN}$ (Tensile) $\quad P_y = P_z = -500 \text{ KN}$ (compressive)

$E = 200 \times 10^3 \text{ N/mm}^2, \ \mu = 0.3$

Now, $\sigma_x = \frac{P_x}{A} = \frac{100 \times 10^3}{60 \times 60} = 27.78 \text{ N/mm}^2$

$\sigma_y = \sigma_z = P/A = \frac{-500 \times 10^3}{60 \times 500} = -16.67 \text{ N/mm}^2$

Strain in the direction of 'x'

$$\epsilon_x = \frac{\sigma_x}{E} - \mu(\sigma_y + \sigma_z)$$

$$= \frac{1}{E}\left(27.78 - 0.3(-16.67 - 16.67)\right)$$

$$= \frac{1}{200 \times 10^3}\left(27.78 + 0.3(2 \times 16.67)\right)$$

$$= 1.889 \times 10^{-4}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$$

$$= \frac{1}{200\times10^3}\left[-16.67 - 0.3(-16.67 + 27.78)\right]$$

$$= -1.0\times10^{-4}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$$

$$= \frac{1}{200\times10^3}\left[-16.67 - 0.3(27.78 - 16.67)\right]$$

$$= -1.0\times10^{-4}$$

volumetric strain, $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$$\epsilon_v = (1.889 - 1.0 - 1.0)\times10^{-4}$$

$$= -0.111\times10^{-4}$$

Change in volume $= \frac{\Delta V}{V}$ $\quad (\because \Delta V = V\times\epsilon_v)$

$$\frac{\Delta V}{V} = \epsilon_v$$

$$\Rightarrow \Delta V = -0.111\times10^{-4}\times V$$

$$\Rightarrow \Delta V = -0.111\times10^{-4}\times500\times60\times60$$

$$\Rightarrow \boxed{\Delta V = -19.98\ mm^3}$$

change in size ?

$\Delta L = L\times\epsilon_x \qquad \Delta t = t\times\epsilon_t$

$\Delta b = b\times\epsilon_y$

$\Delta_L = 500\times1.889\times10^{-4}$

$= 0.0945\ mm$

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$$\Delta_b = 60 \times (-1.0 \times 10^{-4})$$

$$= -6 \times 10^{-3} \text{ mm}$$

$$= -0.006 \text{ mm}$$

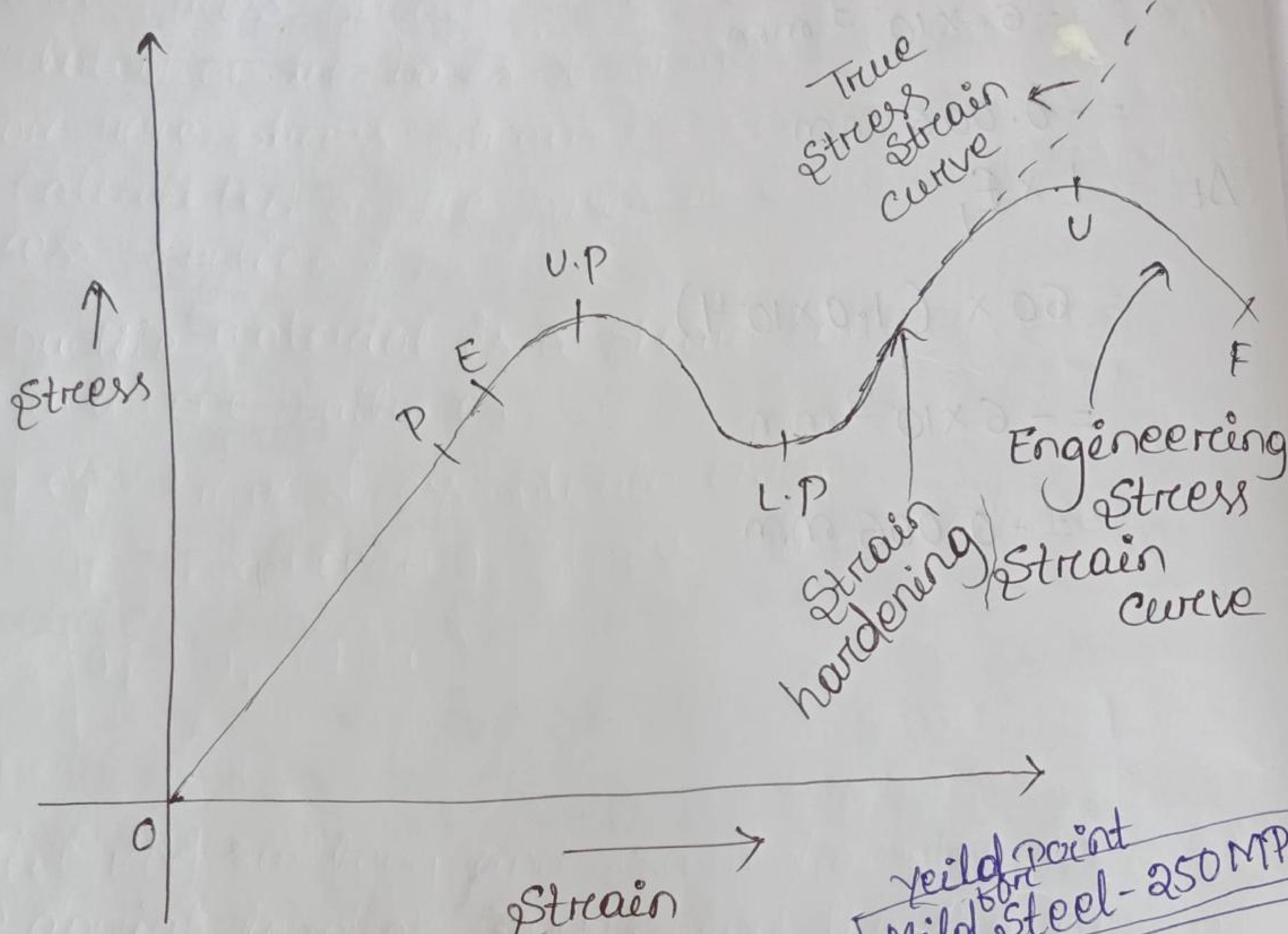
$$\Delta_t = t \times \epsilon_t$$

$$= 60 \times (-1.0 \times 10^{-4})$$

$$= -6 \times 10^{-3} \text{ mm}$$

$$= -0.006 \text{ mm}$$

Stress Strain curve of Ductile material



Yeild point for Mild Steel - 250 MPa

Mild Steel Strain - 22% - 25%

P = proportionality limit
E = Elastic limit
U.P = upper yeild point
L.P = Lower yeild Point
U = Ultimate Point
F = Breaking Point or Fracture point

Salient feature of diagram:-

→ The stress strain curve is obtained by tensile test using ductile material as the standard specimen. The load is hydraulically applied and measurement is done. The elongation of the specimen is noted simultaneously. The measured data, are plotted in x - y plane as shown.

Portion O-P :-
It is straight i.e. stress, is proportional to strain. The point 'p' is known as limit of proportionality. In other word this is the limit of linear elasticity.

Portion P-E :-
In this portion the curve departs from linearity, but the material is still elastic i.e. in this portion the stress is no longer proportional to strain. The point 'E' is known as elastic limit. It is the point of greatest stress that the material can withstand without giving a permanent deformation when the load is removed.

Portion E-UP-LP :-
After Elastic limit, yeilding start (Plastic flow of materials). The yeild point is the point at which there is an appreciable change in length without any corresponding increase of load, even it decreases. The structural members have uppear yeild point and lower yeild point.

Portion L.P-U :-
After the lower yeild point, the curve becomes smooth and much flatter. It rises till a point U, known as ultimate point. It is the point of maximum stress that the specimen is capable of sustaining its original area of cross-section.

Portion U-F :-
After the point U, the area of cross-section is reduced appreciably and this phenomenon is called a necking simultaneously the apparent stress decreases and the material quickly breaks at F known as breaking point.

Nominal stress strain curve ⇒ all the stresses are calculated on the bases of original cross-section. But for True stress strain curve ⇒ all the stresses are calculated on the bases of instantaneous area of cross-section.

Imp
* A Ductile material is a shear failure or cup and cone failure.

* σ_t = True stress strain curve
  σ_E = Engineering stress strain curve

$$\boxed{\sigma_t = \sigma_E (1+\epsilon)}$$

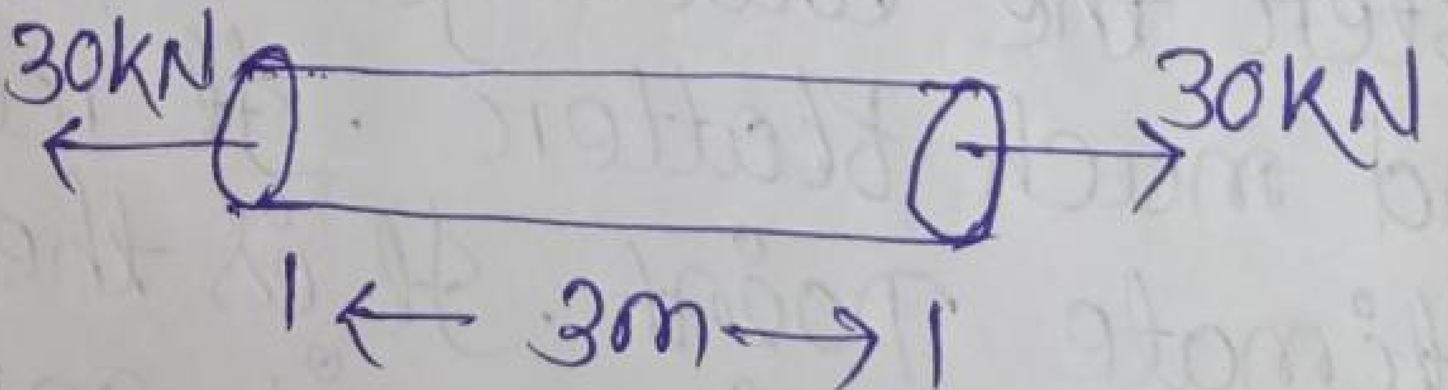
H.W.
Q. A steel rod 3m long and 33mm diameter is subjected to an axial load of 30 kN. Find the change in length, diameter and volume of the rod. Take E as 200 GPa and Poission's ratio as 0.32.

Ans ~~Gt~~ Given,

$l = 3m = 3000mm$

$d = 30mm$, $P = 30kN$

$\mu = 0.32$, $E = 200 GPa$



To find :- Δl, Δd, ΔV

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\left(\cancel{\frac{\sigma_y}{E}}^{0} + \cancel{\frac{\sigma_z}{E}}^{0}\right) \quad \boxed{\sigma_y = \sigma_z = 0}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\sigma_x = P/A = \frac{30\times10^3}{\frac{\pi}{4}\times(30)^2} = 42.46 \text{ N/mm}^2 \text{ or } 42.46 \text{ MPa}.$$

$$\epsilon_x = \frac{42.46}{200\times10^3} = 0.2123\times10^{-3}$$

$\frac{\Delta l}{l} = 0.2123 \times 10^{-3}$

$\Delta l = 0.2123 \times \cancel{10^{-3}} \times 3 \times \cancel{10^{3}}$

$\Delta l = 0.6369\ mm$

$\epsilon_y = -\mu \frac{\sigma_x}{E}$

$\epsilon_y = -0.32 \times \frac{42.46}{200 \times 10^3}$

$\epsilon_y = -0.0679 \times 10^{-3}\ mm$

$\frac{\Delta d}{d} = -0.0679 \times 10^{-3}$

$\Delta d = -0.0679 \times 10^{-3} \times 30$

$\Delta d = -2.037 \times 10^{-3}\ mm$

* negative is for direction

$\epsilon_z = \cancel{\frac{\sigma_z}{E}}^{0} - \mu \left(\frac{\sigma_x}{E} + \cancel{\frac{\sigma_y}{E}}^{0} \right)$

$= -\mu \frac{\sigma_x}{E}$

$\boxed{\epsilon_z = \epsilon_y = -0.0679 \times 10^{-3}\ mm}$

$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$= 0.2123 \times 10^{-3} - 0.0679 \times 10^{-3} - 0.0679 \times 10^{-3}$

$\epsilon_v = 0.0765 \times 10^{-3}$

$\frac{\Delta V}{V} = 0.0765 \times 10^{-3}$

$V = \pi \frac{d^2}{4} \times h$

$= \pi \times \frac{(30)^2}{4} \times 3 \times 10^3$

$V = 2120.18 \times 10^3$

$\Delta V = 2120.18 \times \cancel{10^3} \times 0.0765 \times \cancel{10^{-3}}$

$\boxed{\Delta V = 162.194}\ mm^3$

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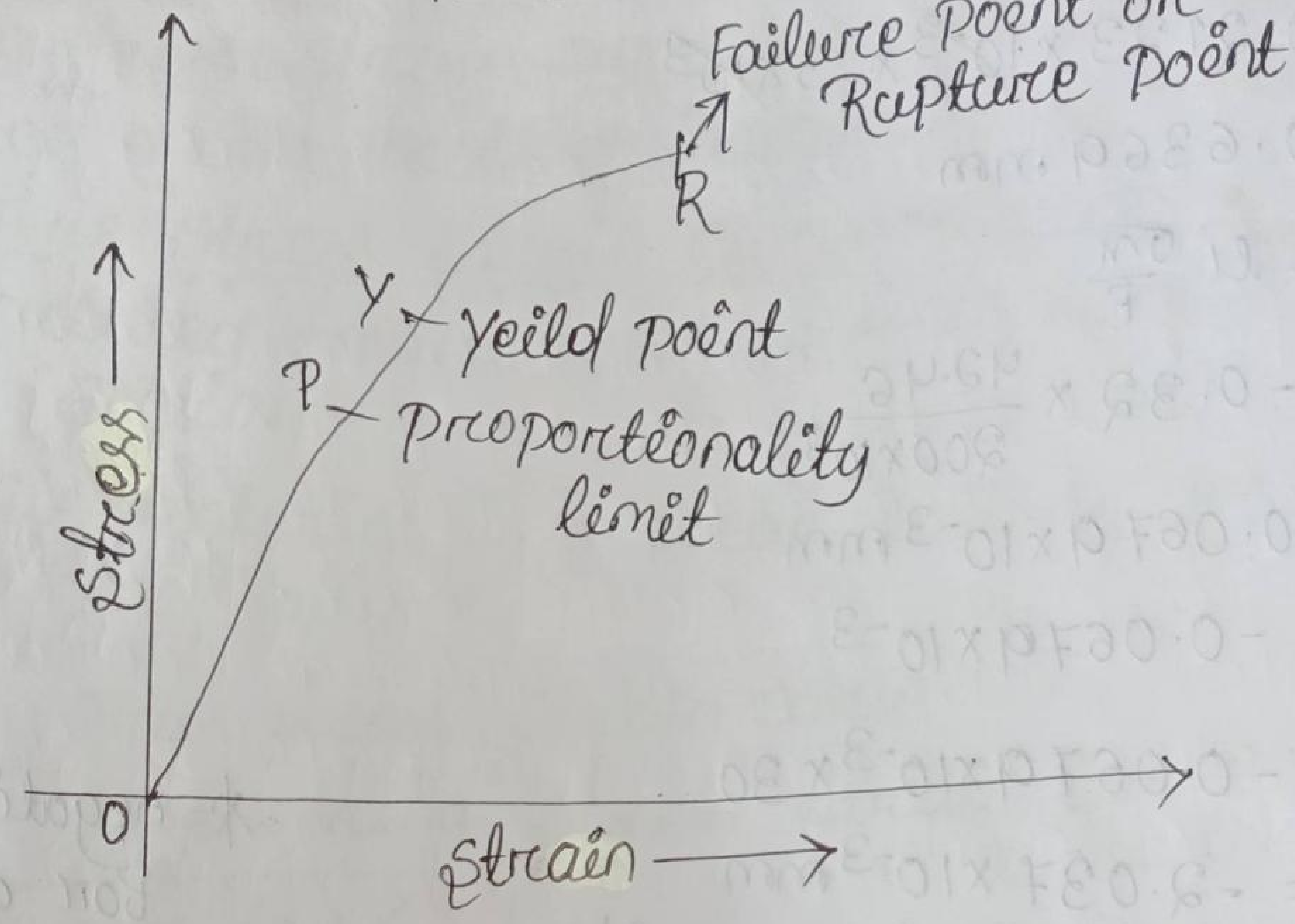
Stress Strain curve of Brittle material:



Example of Brittle material:- castIron and concrete.

* Brittle material have a very low strain and fails immediately after yeilding.
* Ductile materials have a large significant permanent deformation and does not fail immediately.

Proportionality limit:-
This is called as limit of linear elasticity and to this point stress is directly proportional to strain.

Elastic limit:-
It is the point of greatest stress that the material can with stand without giving a permanent deformation when the load is removed.

Yeild stress:-
The stress which produces permanent deformation in the material is called as yeild stress and the point

corresponding to this stress is called yeild point.

Ultimate Stress :-

It is the point of maximum stress that the material capable of substaining in its original area of cross - section.

Rupture Stress :-

The stress at which material fails is called Breaking stress and the point corresponding to this stress is called Rupture point or Breaking point.

Percentage of elongation :-

It is ratio between change in length at rupture and the original length expressed in percentage.

if l' = Final change in length.

l = original length.

Imp % Elongation $= \frac{l' - l}{l} \times 100$

Percentage elongation is the measure of ductility i.e. the higher the value of percentage elongation, the more ductile is the material.

Percentage reduction in area :-

The reduction in area of the specimen or material at the neck divided by the original area of specimen or material expressed as percentage is known as percentage reduction in area.

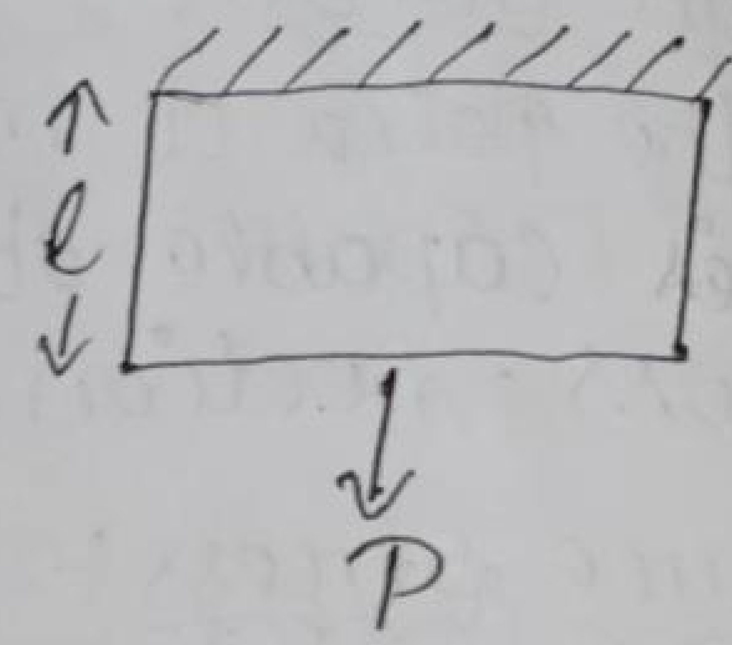
A' = Reduced area at the neck.

A_0 = Original area.

$$\boxed{\text{Percentage Reduction} = \frac{A_0 - A'}{A_0} \times 100}$$

Deformation of Prismatic bar in uniaxial loading:

$\sigma = P/A$



$\sigma = E\epsilon$

$P/A = E\epsilon$

$P/A = E \times \frac{\Delta l}{l}$

$$\boxed{\Delta l = \frac{Pl}{AE}}$$

Imp

Principle of Superposition:-

The net elongation of the body is equal to the algebraic sum of elongation of the individual section. This principle of finding elongation is known as principle of superposition.

$$\boxed{\Delta l_{net} = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3}}$$

v.v. Imp.

Elongation of body due to self weight:-

consider a bar AB hanging freely under the action of its own weight.

Let, A = cross-sectional Area

l = original length

E = young's modulus

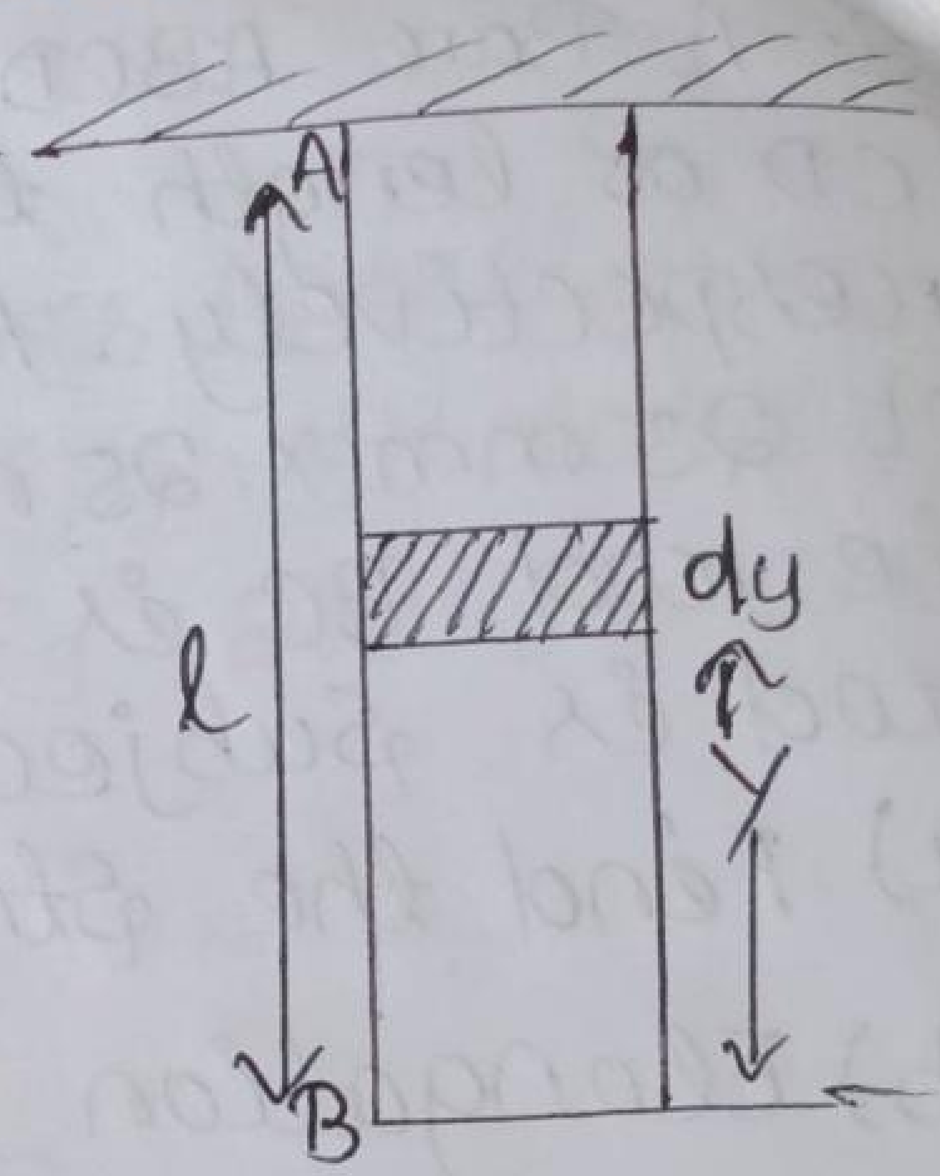
w = weight density of the bar material

Let us consider an elementary length dy at a distance of y from free end. The total pull acting on this length dy is the weight of the length y

Hence total pull on elemantary length $P = wAy$

Elongation of elematary length

$$\Delta y = \frac{P\,dy}{AE} = \frac{wAy\,dy}{AE} = \frac{wy\,dy}{E}$$



Therefore total elongation of body due to self weight

$$\Delta l = \int \Delta y = \int_0^l \frac{wy\,dy}{E}$$

or $$\boxed{\delta L = \frac{wl^2}{2E}}$$

Total weight $= wAl$

$W = wAl$.

$\boxed{\Delta = \delta}$

Elongation can be written as:-

$$\boxed{\delta L = \frac{Wl}{2AE}}$$

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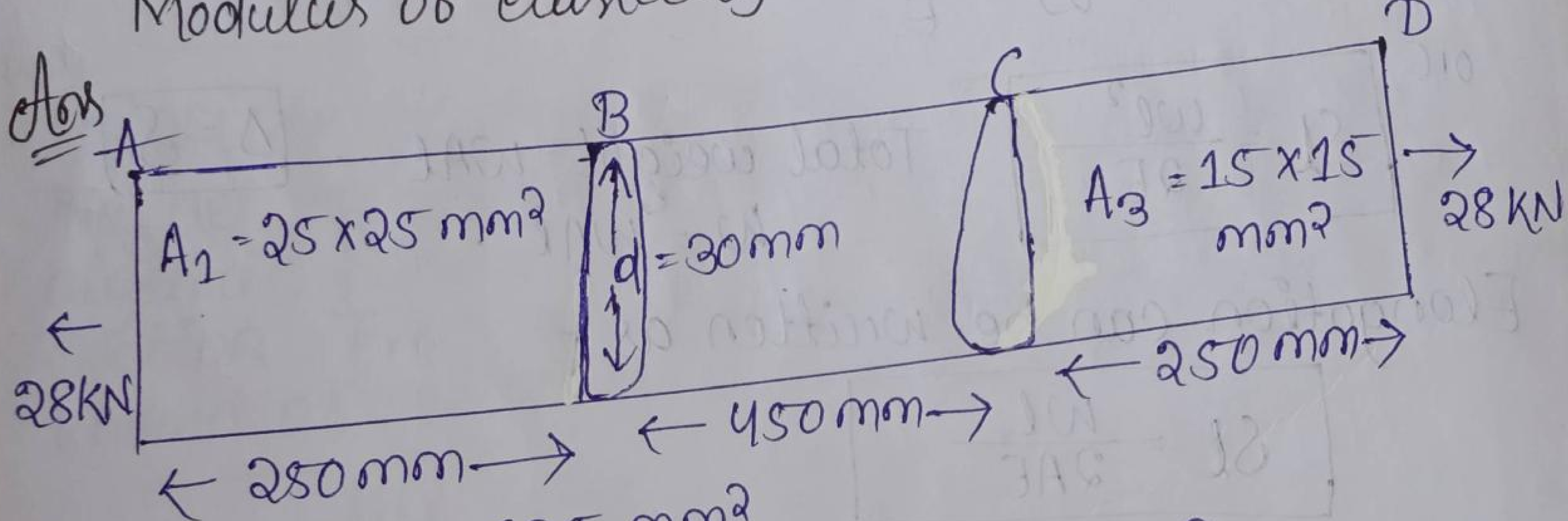
Q. A Bare ABCD 950 mm is made up 3 parts AB, BC & CD of length to 250 mm, 450 mm and 250 mm respectively. AB and CD are square cross-section of 25 mm x 25 mm and 15 mm x 15 mm respectively. The the rod BC is spherical of diameter 30 mm. The rod is subjected to a pull of 28 KN.

(a) Find the stress in 3 parts of the rod?

(b) Elongation of the rod?

Modulus of elasticity, $E = 2 \times 10^5 \text{ N/mm}^2$

Ans



$A_1 = 25 \times 25 = 625 \text{ mm}^2$

$A_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (30)^2 = 706.73 \text{ mm}^2$

$A_3 = 15 \times 15 = 225 \text{ mm}^2$

$\sigma_{AB} = \frac{28 \times 10^3 \text{ N}}{25 \times 25} = 44.8 \text{ N/mm}^2$

$\sigma_{BC} = \frac{28 \times 10^3}{\pi/4 \times d^2} = \frac{28 \times 10^3}{706.73} = 39.62 \text{ N/mm}^2$

$\sigma_{CD} = \frac{28 \times 10^3}{15 \times 15} = 124.44 \text{ N/mm}^2$

$\delta l = \delta l_1 + \delta l_2 + \delta l_3$

$\delta l = \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$

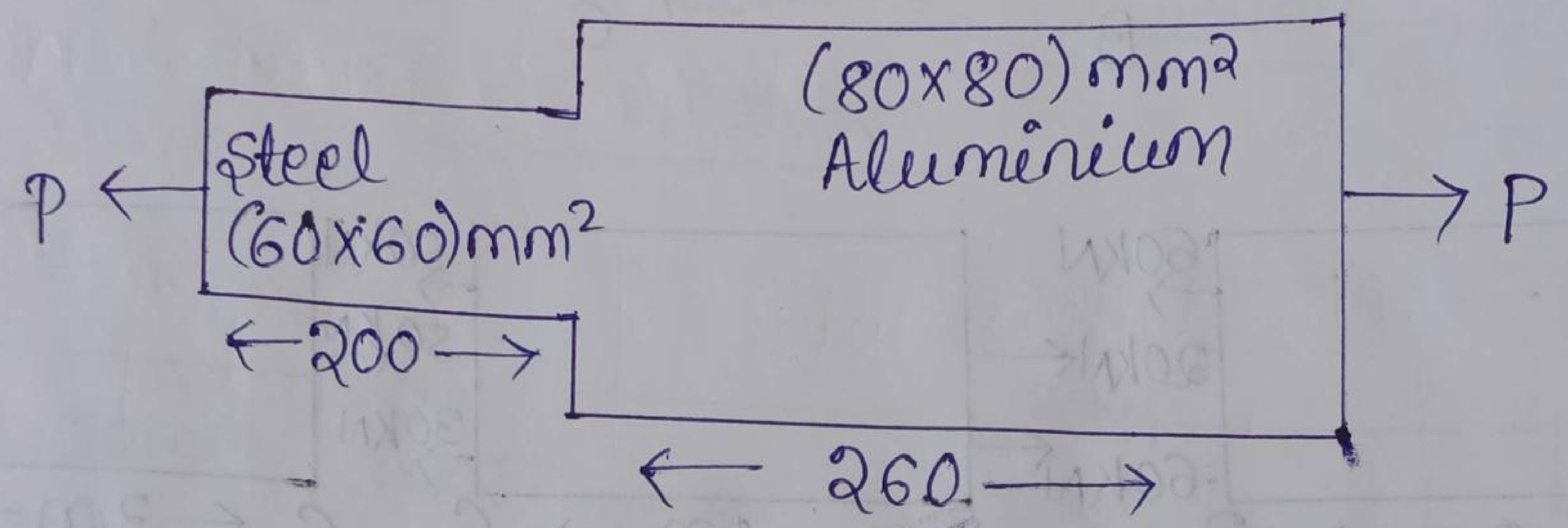
$= \frac{28 \times 10^3 \times 250}{25 \times 25 \times 2 \times 10^5} + \frac{28 \times 10^3 \times 450}{706.73 \times 2 \times 10^5} + \frac{28 \times 10^3 \times 250}{15 \times 15 \times 2 \times 10^5}$

$= 0.056 + 0.0891 + 0.155$

$= 0.3$ mm.

Q. A member formed by 2 different bars (steel & Aluminium) is subjected to a load of 'P'. If total extension is 0.3 mm. Find 'P'. take $E_S = 200$ GPa and $E_A = 70$ GPa.

Ans



$$\delta l = \delta l_1 + \delta l_2$$

$$\delta l = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E}$$

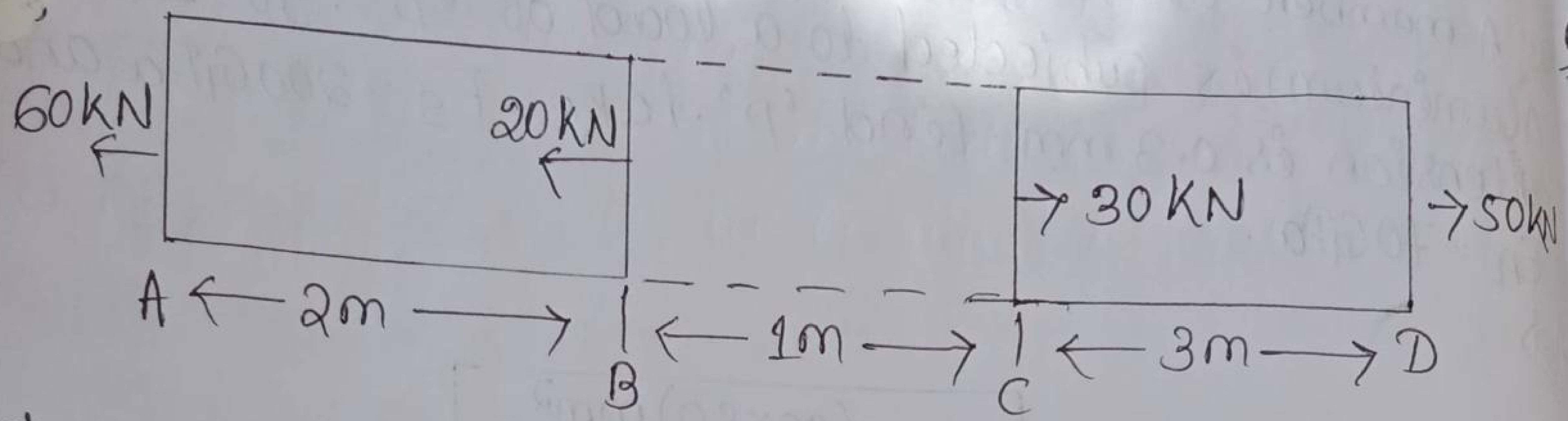
$$0.3 = \frac{P \times 200}{60 \times 60 \times 2 \times 10^5 \text{ N/mm}^2} + \frac{P \times 260}{80 \times 80 \times 70 \times 10^3}$$

$$0.3 = 0.0277 \times 10^{-5} P + 0.058 \times 10^{-5} P$$

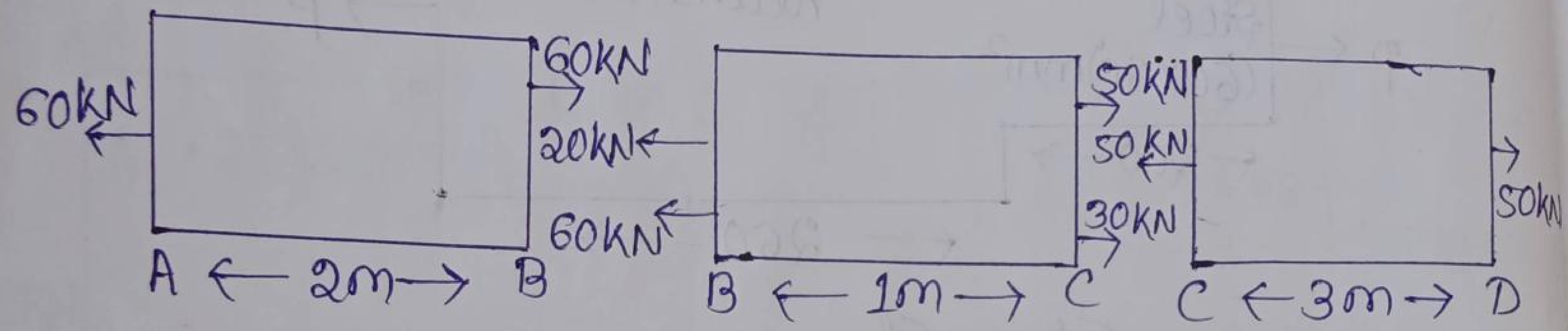
$$0.3 = 0.857 \times 10^{-5} P$$

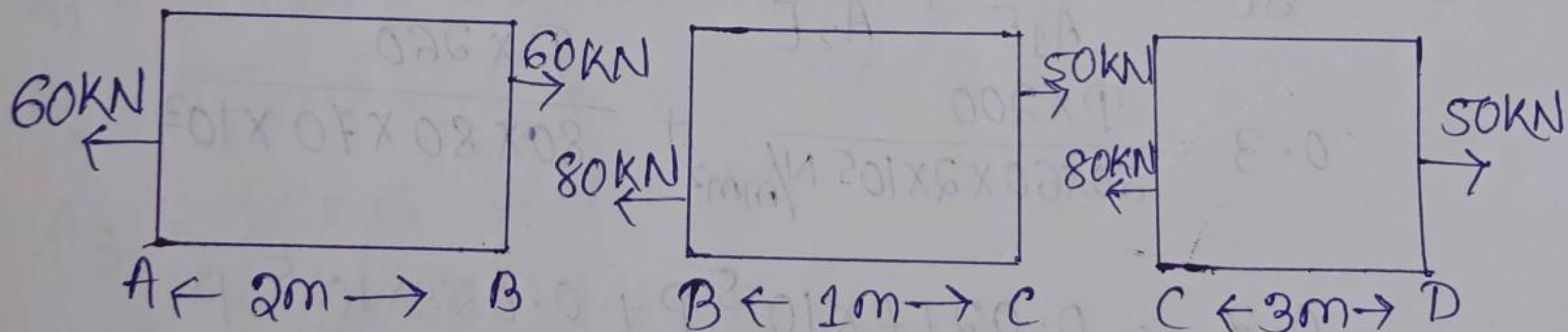
$$\boxed{P = 350.058 \text{ kN}}$$

Q Imp A steel bar of 25 mm diameter is acted upon by forces as soon in fig. What is the elongation of the bar, E = 190 GPa.



Ans





$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$= \frac{P l_{AB}}{A_{AB} \times E} + \frac{P l_{BC}}{A_{BC} \times E} + \frac{P l_{CD}}{A_{CD} \times E}$$

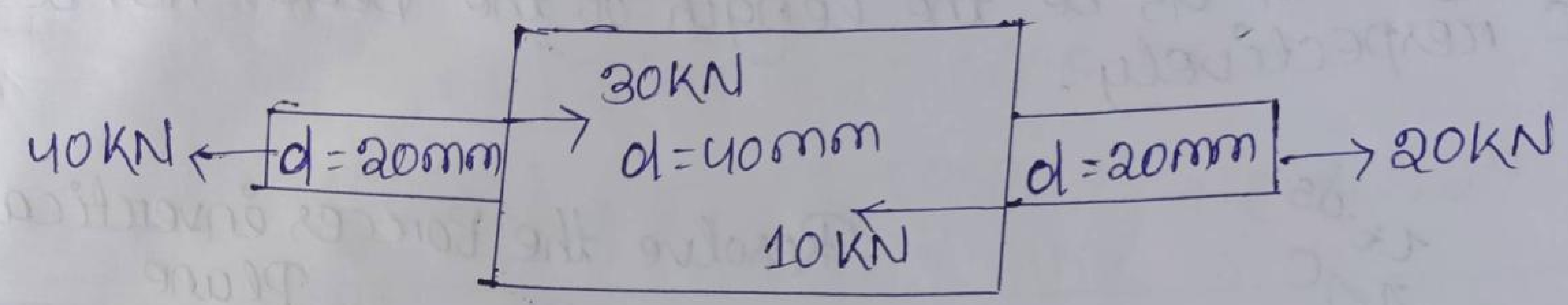
$$= \frac{60 \times 10^3 \times 2000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3} + \frac{80 \times 10^3 \times 1000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3} + \frac{50 \times 10^3 \times 3000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3}$$

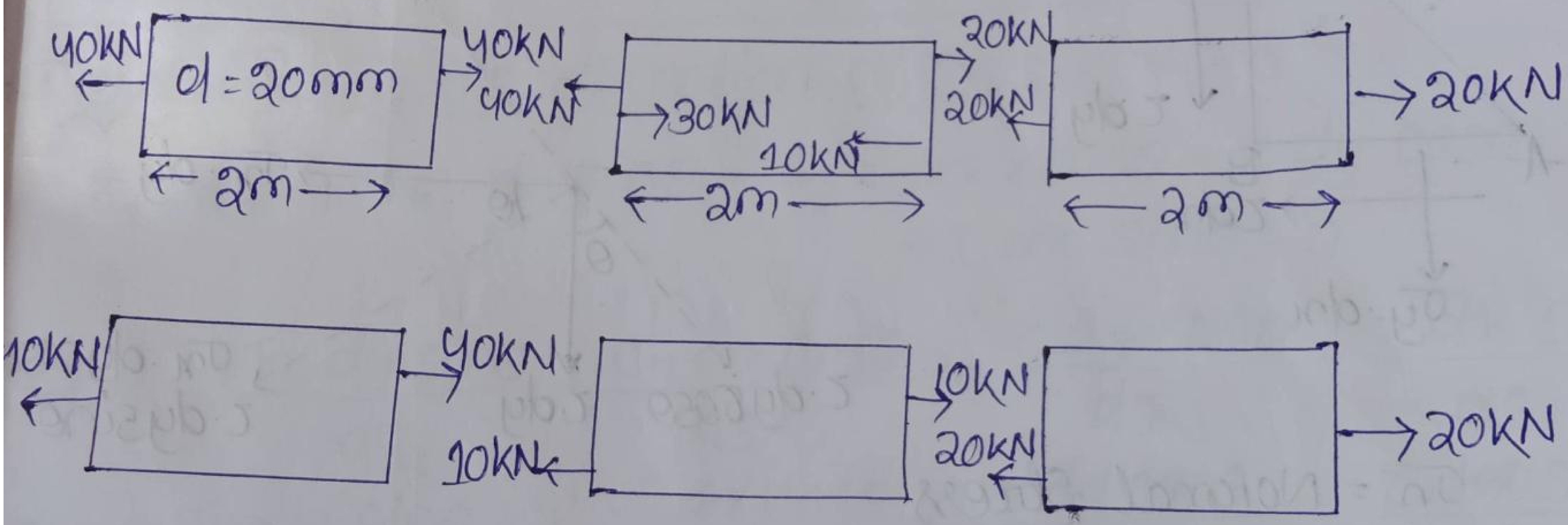
$$= 1.287 + 0.858 + 1.609$$

$= 3.75 \text{ mm}$

Q: A steel bar ABCD 6m. long consisting of different section is subjected to force as soon as figure. Find the elongation of the bar. Take $E = 200$ GPa.

Ans





$$\delta l = \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

$$= \frac{40 \times 10^3 \times 2000}{\frac{\pi}{4} \times (20)^2 \times 200 \times 10^3} + \frac{10 \times 10^3 \times 2000}{\frac{\pi}{4} \times (40)^2 \times 200 \times 10^3} + \frac{20 \times 10^3 \times 2000}{\frac{\pi}{4} \times 20^2 \times 200 \times 10^3}$$

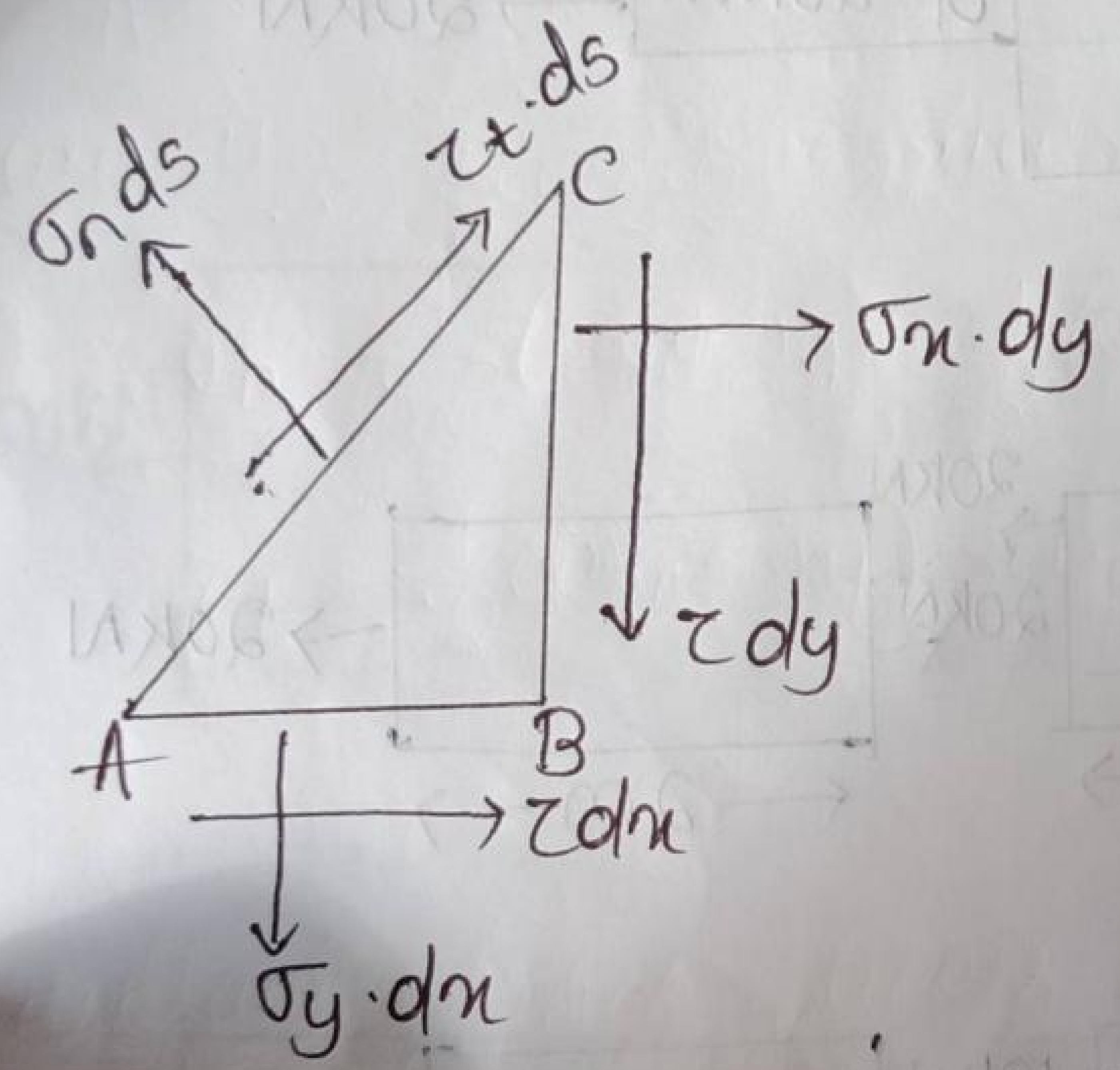
$$= 1.273 + 0.079 + 0.636$$

$$= 1.988 \text{ mm}$$

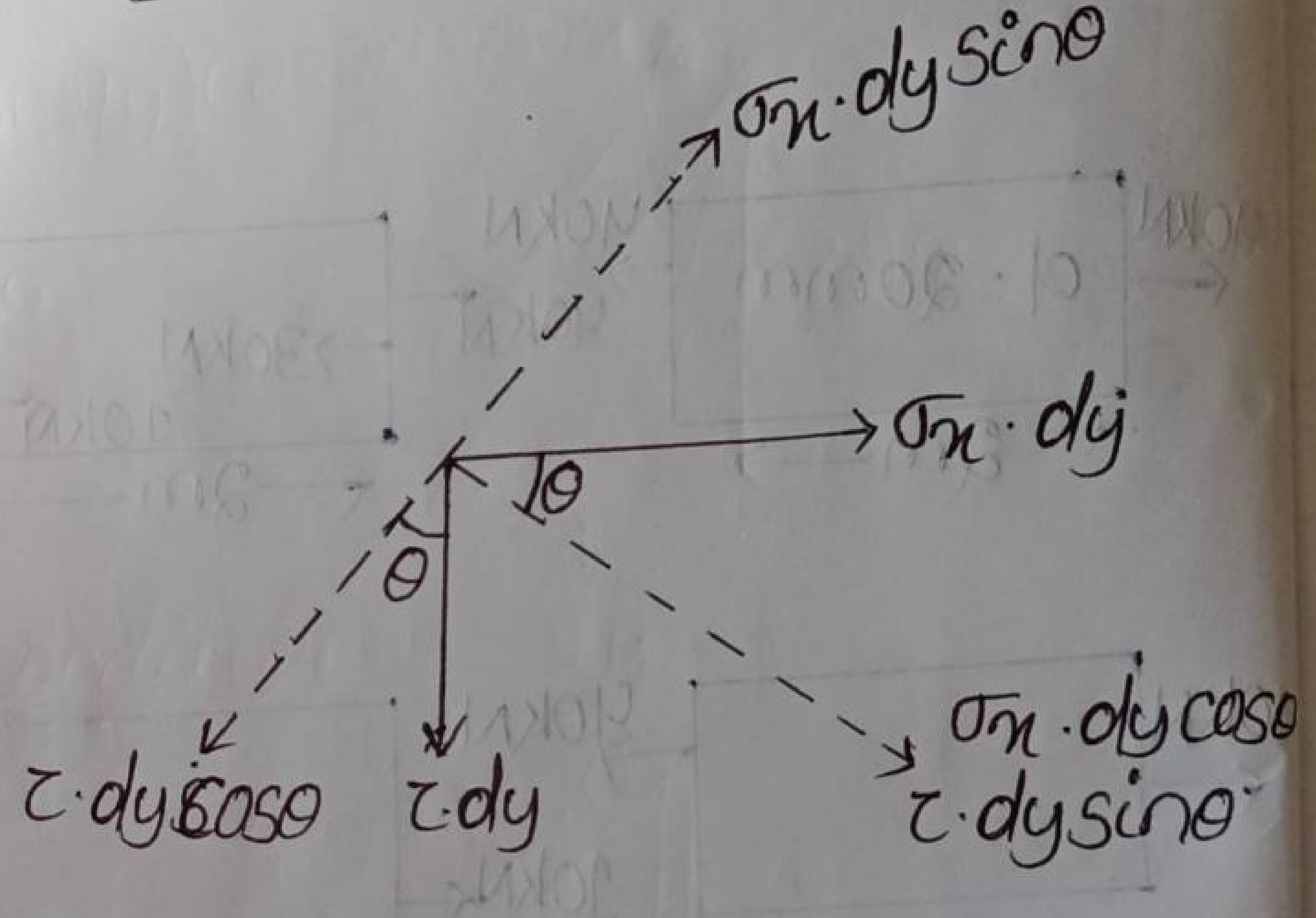
Complex stress and strain

Biaxial and shear stress condition

Let an element of a body acted upon by two tensile stresses along with shear stresses acting on two perpendicular planes of the body as shown in figure. Let dx, dy and ds be the length of the sides AB, BC and AC respectively.



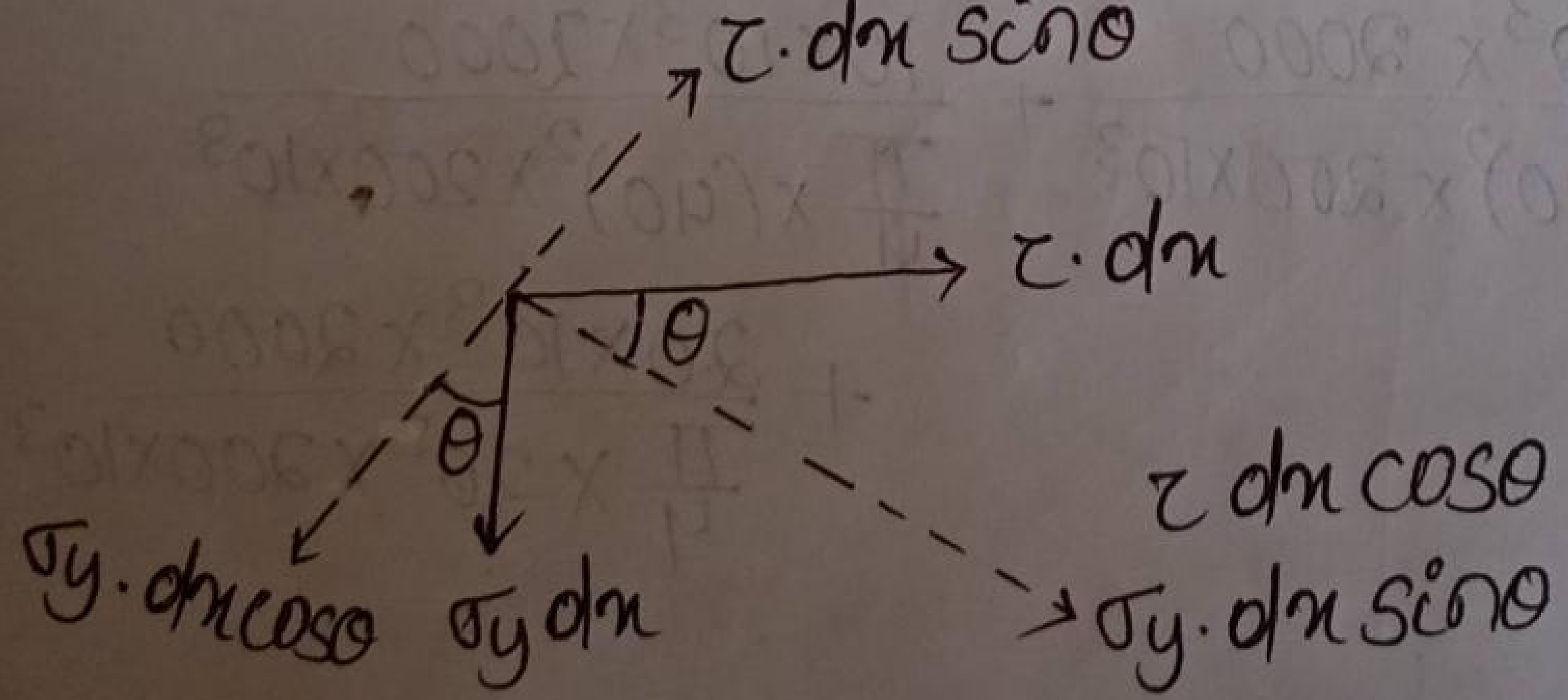
Resolve the forces in vertical plane



σ_n = Normal stress

τ_t = Tangential stress

Resolve the force in horizontal direction :



considering unit thickness of the body and resolving the forces in the direction of σ_n .

$$\sigma_n \cdot d_s = \sigma_x \cdot dy \cos\theta + \sigma_y \cdot dx \sin\theta + \tau dy \sin\theta + \tau \cdot dx \cos\theta$$

$$\sigma_n = \frac{\sigma_x \cdot dy\cos\theta}{ds} + \frac{\sigma_y\,dx\sin\theta}{ds} + \frac{\tau\,dy\sin\theta}{ds} + \frac{\tau \cdot dx \cdot \cos\theta}{ds}$$

$$\sigma_n = \frac{\sigma_x \cdot \cancel{dy}\cos\theta}{\cancel{dy}/\cos\theta} + \frac{\sigma_y\,dx\sin\theta}{dx/\sin\theta} + \frac{\tau\,\cancel{dy}\sin\theta}{\cancel{dy}/\cos\theta} + \frac{\tau\,\cancel{dx}\cos\theta}{\cancel{dx}/\sin\theta}$$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau\sin\theta\cos\theta + \tau\sin\theta\cos\theta$$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau\sin\theta\cdot\cos\theta$$

$$\sigma_n = \sigma_x\left(\frac{1+\cos^2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos2\theta}{2}\right) + \tau\sin2\theta$$

$$\boxed{\sigma_n = \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos2\theta + \tau\sin2\theta}$$

Resolving in the direction of τ_t :

$$\tau_t \cdot ds + \sigma_x \cdot dy\sin\theta - \sigma_y \times dx \times \cos\theta - \tau\,dy\cos\theta + \tau\,dx\sin\theta = 0$$

$$\tau_t = -\frac{\sigma_x\,dy\sin\theta}{ds} + \frac{\sigma_y\,dx\cos\theta}{ds} + \frac{\tau\,dy\cos\theta}{ds} - \frac{\tau\,dx\sin\theta}{ds}$$

$$\tau_t = \frac{-\sigma_x\,\cancel{dy}\sin\theta}{\cancel{dy}/\cos\theta} + \frac{\sigma_y\,\cancel{dx}\cos\theta}{\cancel{dx}/\sin\theta} + \frac{\tau\,\cancel{dy}\cos\theta}{\cancel{dy}/\cos\theta} - \frac{\tau\,\cancel{dx}\sin\theta}{\cancel{dx}/\sin\theta}$$

$$\tau_t = -\sigma_x\sin\theta\cos\theta + \sigma_y\sin\theta\cos\theta + \tau\cos^2\theta - \tau\sin^2\theta$$

$$= -\frac{1}{2}(\sigma_x-\sigma_y)\sin2\theta + \tau\left[\left(\frac{1+\cos2\theta}{2}\right) - \left(\frac{1-\cos^2\theta}{2}\right)\right]$$

$$\boxed{\tau_t = -\frac{1}{2}(\sigma_x-\sigma_y)\sin2\theta + \tau\cos2\theta}$$

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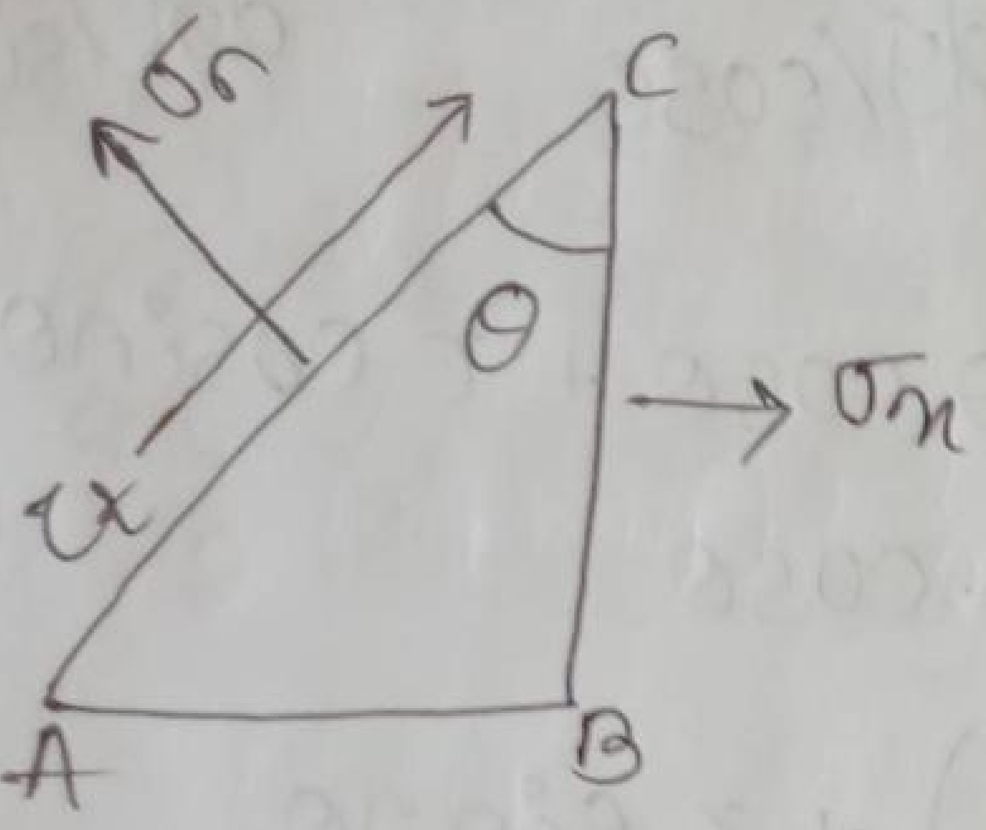
Various cases in Plane Stress Condition:

Case-I :- Direct stress condition



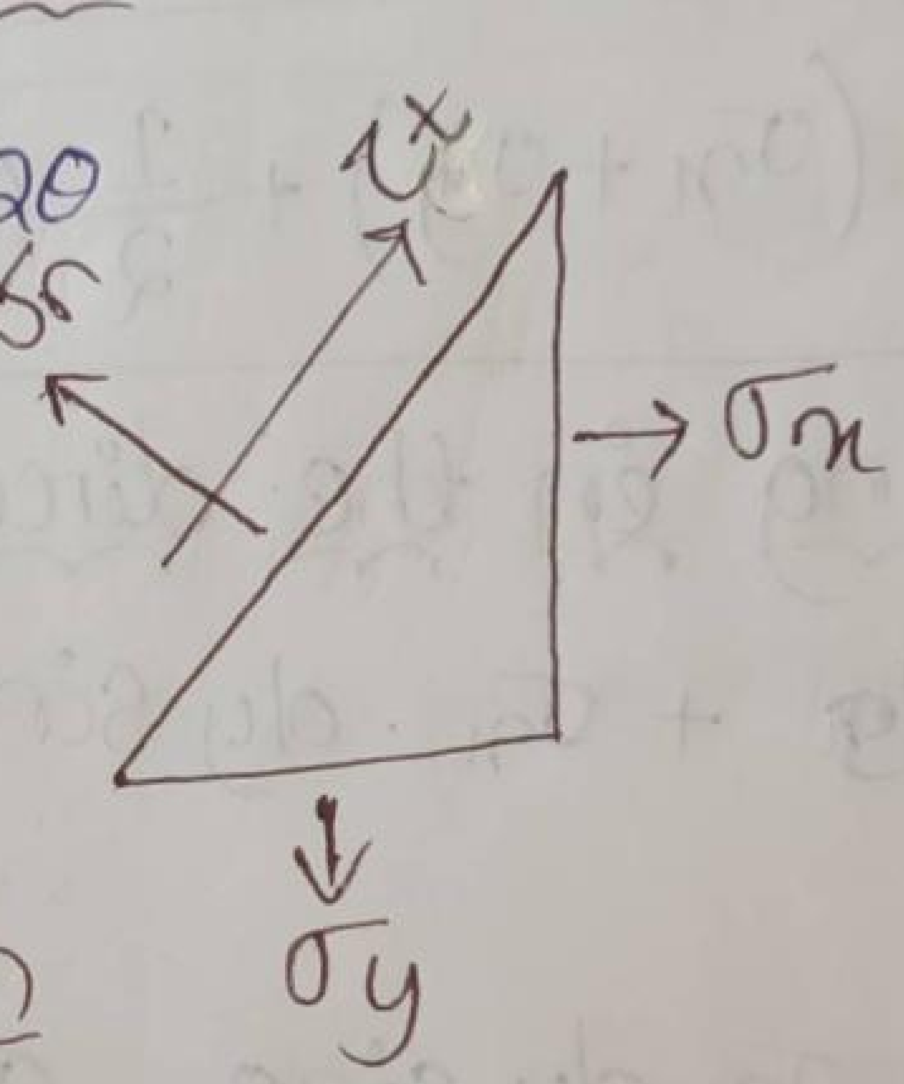
$$\sigma_n = \sigma_x \cos^2\theta$$

$$\tau_t = -\frac{1}{2}\sigma_x \sin 2\theta$$

Case-II :- Biaxial stress condition

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$$

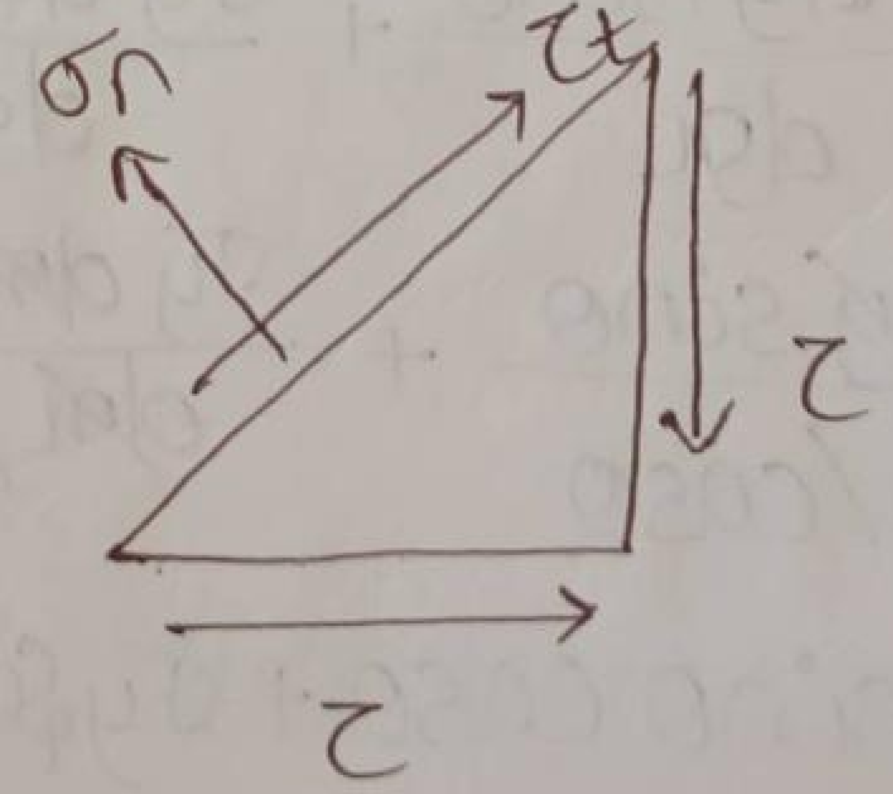
$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$



Case-III :- Pure stress condition

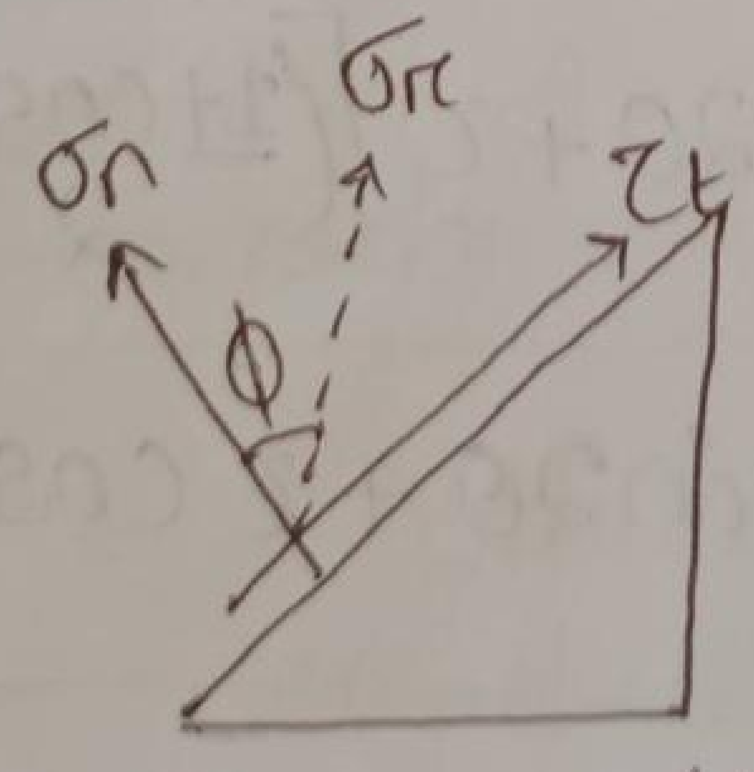
$$\sigma_n = \tau \sin 2\theta$$

$$\tau_t = \tau \cos 2\theta$$



Resultant Stress :-

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$



Angle of inclination of the resultant with σ_n :

$$\boxed{\tan\phi = \frac{\tau_t}{\sigma_n}}$$

Principal stress and principal plane :

Principal planes are those plane on which shear stress (τ_t) is zero. These planes are mutually perpendicular.

Principal stresses are the maximum and minimum normal stresses.

The maximum normal stress is called major principal stress.

The minimum normal stress is called minor principal stress.

The planes on which the maximum normal stress acts, called major principal plane.

The plane on which the minimum normal stress acts, called minor principal plane.

As shear stress is zero in principal plane.

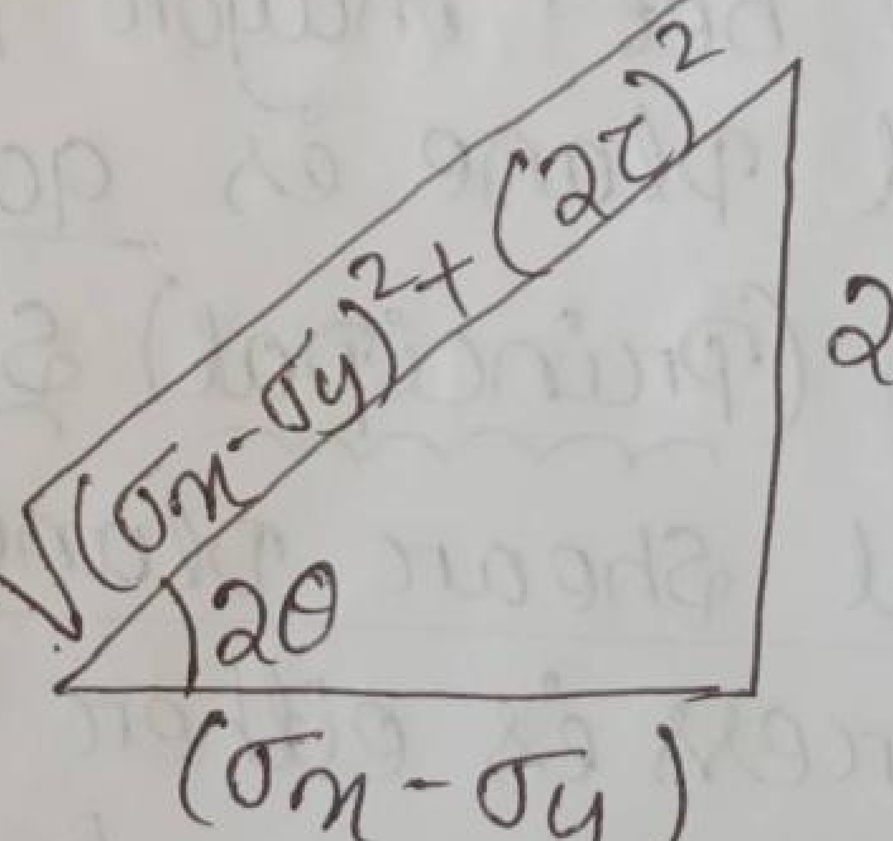
$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau\cos 2\theta$$

$$0 = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \cos 2\theta$$

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$\sin 2\theta = \pm\frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\cos 2\theta = \pm\frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$



Put the value, of $\sin 2\theta$ and $\cos 2\theta$ in 'σ_n'

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau\sin 2\theta$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\frac{(\sigma_x - \sigma_y)^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}} \pm \tau\frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2} \frac{(\sigma_x - \sigma_y)^2 + 4\tau^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\boxed{\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

→ major principal stress

$$\boxed{\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

→ minor principal stress

Direction of principal plane :-

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

Minor principal plane $(\theta_2) = (\theta_1) + 90^\circ$

θ_1 = Major principal plane.

The angle betn major principal plane and minor principal plane is 90°.

Maximum (Principal) Shear Stress :

Principal shear planes are those planes on which shear stress is either maximum or minimum. Those planes are mutually perpendicular to each other.

The normal stress is not zero in this plane

$$\tau_{1,2} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

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$$\tau_1 = +\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

↳ maximum shear stress

$$\tau_2 = -\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

↳ minimum shear stress.

→ The angle between maximum shear stress plane and minimum shear stress plane is 90°.

→ The angle between principal maxm shear plane and principal plane is 45°.

$$\boxed{\theta_S = \theta_P + 45^\circ}$$

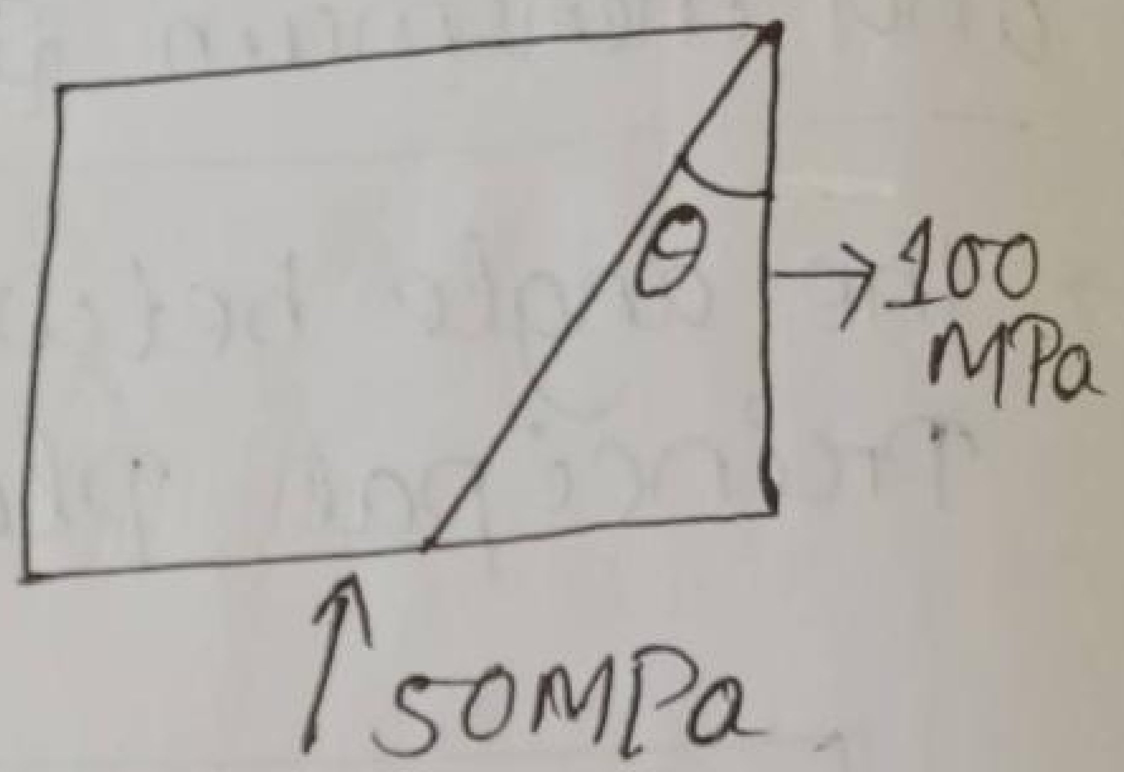
$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$$

↓

maximum shear stress.

Q. The stress at a point in a components are 100 MPa tensile and 50 MPa compressive. Determine the magnitude of the normal stress and shear stress on a plane inclined at an angle of 25° to its tensile, Also determine the direction of resultant stress and the magnitude of maximum shear stress.

Ans



$\sigma_x = 100\,MPa$

$\sigma_y = -50\,MPa$

$\theta = 25°$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{100-50}{2}\right) + \left(\frac{100+50}{2}\right)\cos 2\times 25°$$

$$= 50 + 75\cos 50°$$

$$= 73.209\ MPa$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(100+50)\sin 2\times 25°$$

$$= -\frac{1}{2}\times 150 \times \sin 50°$$

$$= -57.4533\,MPa$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{(73.209)^2 + (-57.4533)^2}$$

$$= 93.06\ MPa$$

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$$\tan\phi = \frac{\tau_t}{\sigma_n}$$

$$\tan\phi = \frac{-57.45}{73.209} = -0.7847$$

$$\phi = \tan^{-1}(-0.7847) = -38^\circ 12'$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \qquad \boxed{\tau = 0}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$= \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{100 + 50}{2}$$

$$\boxed{\tau_{max} = \pm 75 \text{ MPa}}$$

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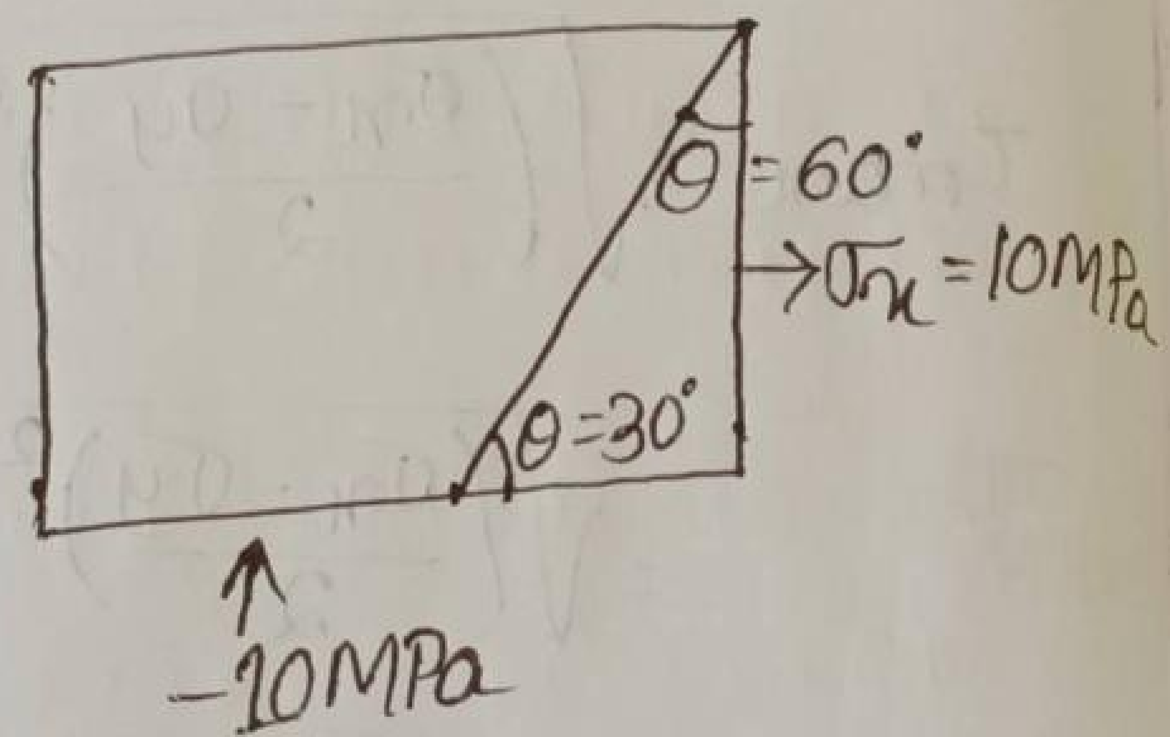
Q. A rectangular block is subjected to two perpendicular stress of 10 MPa (tensile) and 10 MPa (compressive). Determine stresses on plane inclined at 30°, 45° and 60° with the plane of compressive stress.

Ans

$\sigma_x = 10\,MPa$

$\sigma_y = -10\,MPa$



When $\theta = 30°$

Let's calculate 'θ' with σ_x

$$\theta = 90° - 30° = 60°$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta$$

$$= \cancel{\left(\frac{10-10}{2}\right)}^{0} + \left(\frac{10+10}{2}\right) \cos 2 \times 60°$$

$$= \frac{20}{2} \times \cos 120°$$

$$= -5\,MPa$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$$

$$= -\frac{1}{2}(10+10) \sin 2 \times 60°$$

$$= -\frac{1}{2} \times 20 \times \sin 120°$$

$$= -5\sqrt{3}\,MPa$$

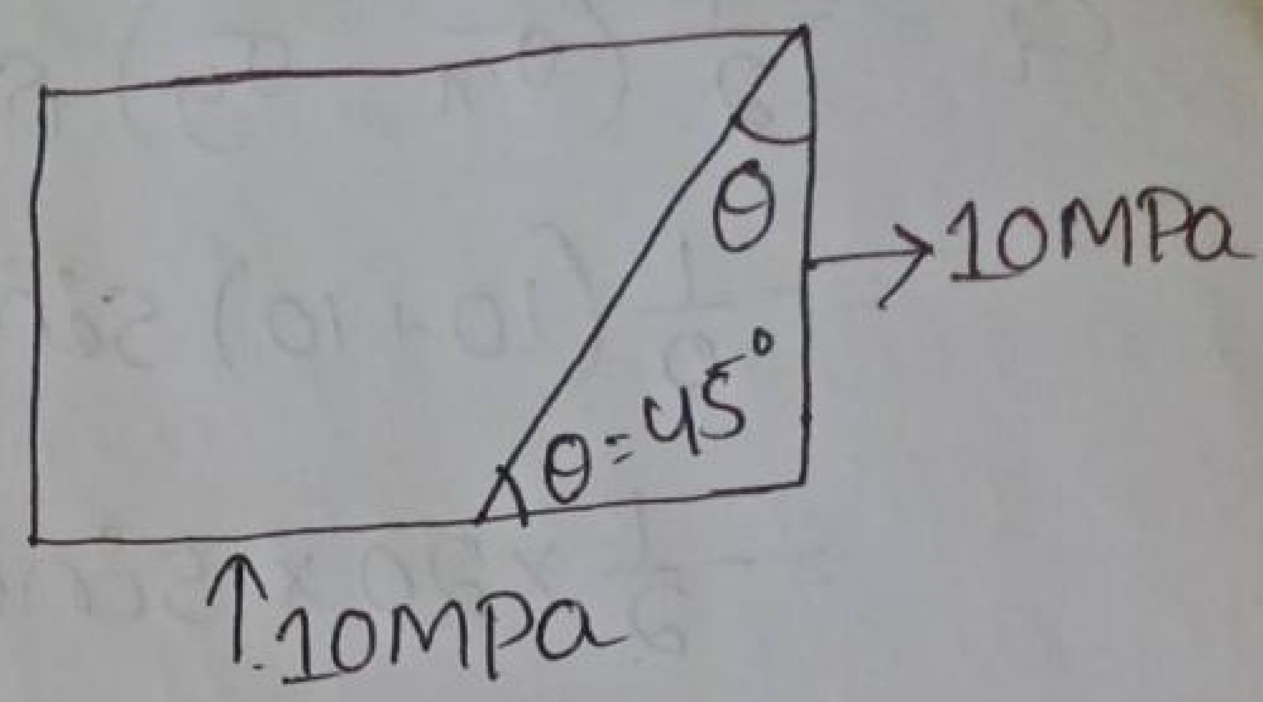
$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{(-5)^2 + (-5\sqrt{3})^2}$$

$$= 10\,MPa$$

when $\theta = 45°$

calculate 'θ' with 'σ_x' axis



$$\theta = 90° - 45° = 45°$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{10-10}{2}\right) + \left(\frac{10+10}{2}\right)\cos 2\times45°$$

$$= 0 + 0$$

$$= 0 \text{ MPa}.$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(10+10)\sin 2\times45°$$

$$= -\frac{1}{2}\times 20\times \sin 90°$$

$$= -10 \text{ MPa}$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{0^2 + (-10)^2}$$

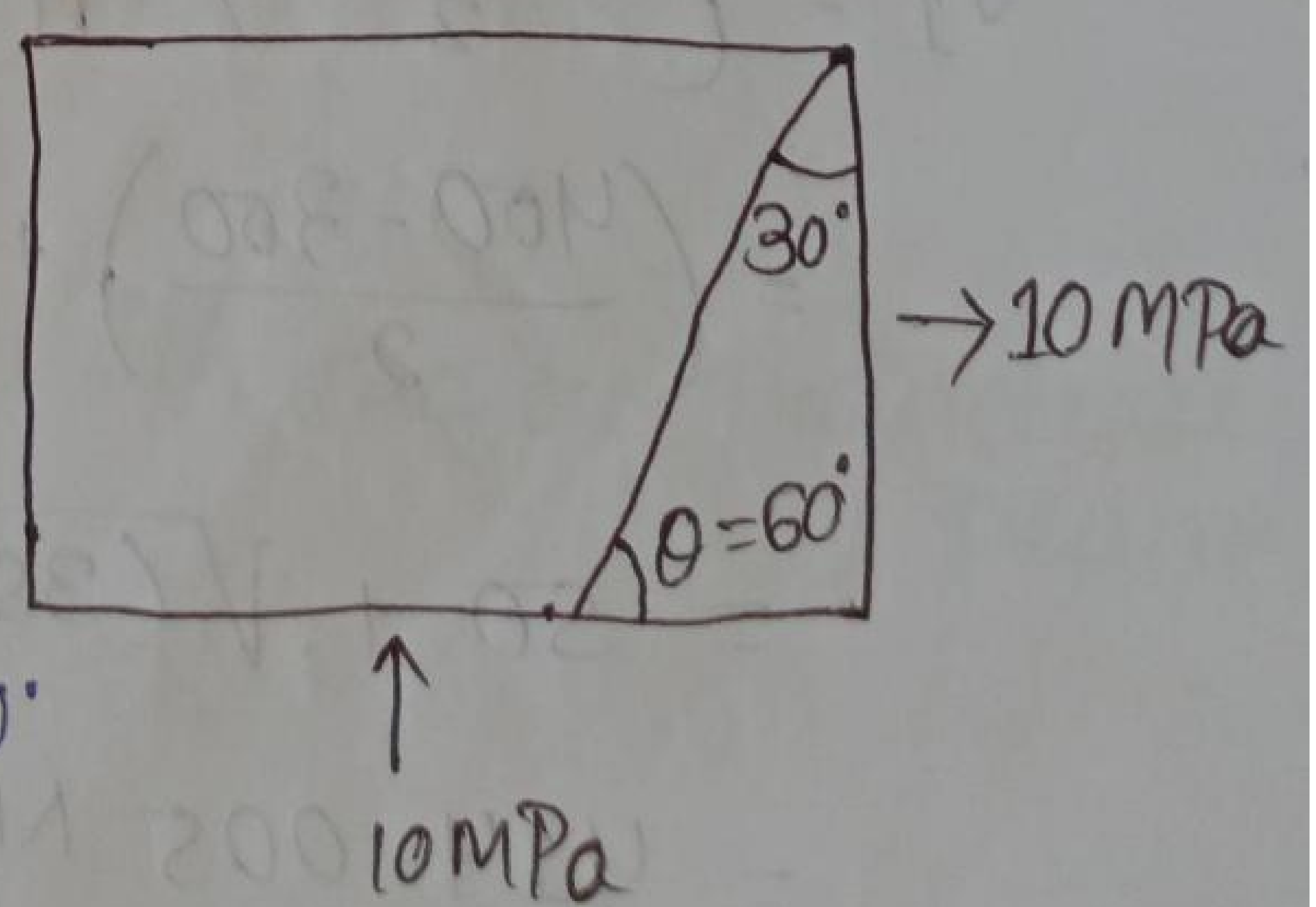
$$= 10 \text{ MPa}$$

When $\theta = 60°$

calculate 'θ' with 'σ_x' axis

$$\theta = 90° - 60°$$

$$= 30°$$



$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{10-10}{2}\right) + \left(\frac{10+10}{2}\right)\cos 2\times30°$$

$$= 10\cos 60° = 5 \text{ MPa}.$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(10+10)\sin 2\times 30^\circ$$

$$= -\frac{1}{2}\times 20 \times \sin 60^\circ$$

$$= -5\sqrt{3}\ \text{MPa}$$

$$\sigma_R = \sqrt{(\sigma_n)^2 + (\tau_t)^2}$$

$$= \sqrt{(5)^2 + (-5\sqrt{3})^2}$$

$$= 10\ \text{MPa}$$

Q:

In an experiment it was found that the tensile stress of 400 N/mm² and a compressive stress of 300 N/mm² acting on mutually plane and equal shear stress of 100 N/mm² on this plane. Find the principal stress and the position of principal plane. Find also maximum shear stress.

Ans

$\sigma_x = 400\ \text{N/mm}^2$ (Tensile)

$\sigma_y = -300\ \text{N/mm}^2$ (Compressive)

$\tau = 100\ \text{N/mm}^2$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \left(\frac{400-300}{2}\right) + \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 50 + \sqrt{(350)^2 + (100)^2}$$

$$= 414.005\ \text{N/mm}^2$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{400-300}{2} - \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 50 - \sqrt{(350)^2 + (100)^2}$$

$$= 50 - 364.005$$

$$= -314.005 \text{ N/mm}^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 364.005 \text{ N/mm}^2$$

The position of principal plane :-

$$\tan 2\theta_1 = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$= \frac{2 \times 100}{400+300}$$

$$= 0.285$$

$$2\theta_1 = \tan^{-1}(0.285)$$

$$= 15.907^\circ$$

$$\theta_1 = \frac{15.907^\circ}{2}$$

$= 7.953^\circ \rightarrow$ Angle with major principal plane.

$$\theta_2 = 90^\circ + \theta_1$$

$$= 90^\circ + 7.953^\circ$$

$= 97.953^\circ \rightarrow$ Angle with minor principal plane.

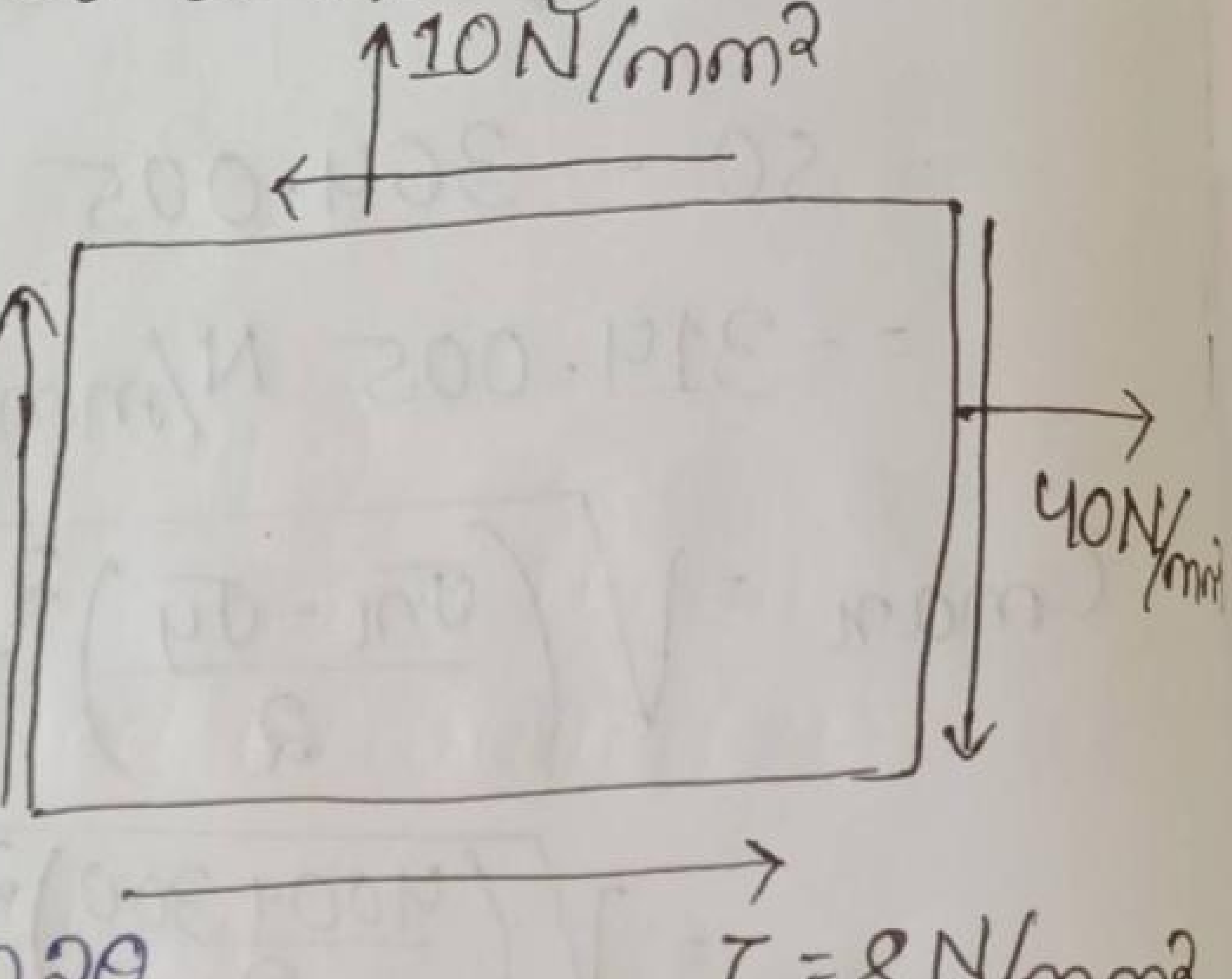
Q. At a point in a rectangular block the stress on mutually perpendicular plane are 40 N/mm² (tensile) and 10 N/mm² (tensile). The shear stress across the plane is 8 N/mm². Find the magnitude and direction of resultant stress on a plane making an angle 30° with the plane of first stress.

Ans

$\sigma_x = 40\ N/mm^2$

$\sigma_y = 10\ N/mm^2$

$\tau = 8\ N/mm^2$



$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{40+10}{2}\right) + \left(\frac{40-10}{2}\right)\cos 2\times 30^\circ + 8 \sin 2\times 30^\circ$$

$$= 25 + 15 \times \cos 60^\circ + 8 \sin 60^\circ$$

$$= 39.428\ N/mm^2$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau \cos 2\theta$$

$$= -\frac{1}{2}(40-10)\sin 2\times 30 + 8 \cos 2\times 30^\circ$$

$$= -\frac{1}{2} \times 30 \times \sin 60^\circ + 8 \cos 60^\circ$$

$$= -8.990\ N/mm^2$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{(39.428)^2 + (-8.990)^2}$$

$$= 40.439\ N/mm^2$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(40-10)^2 + 4\times 8^2}$$

$$= 17 \text{ MPa}$$

$$\tan\phi = \frac{\tau_t}{\sigma_n}$$

$$= \frac{-8.990}{39.428}$$

$$= -0.228$$

$$\phi = \tan^{-1}(-0.228)$$

$$= -12^\circ.84'$$

Note:

clockwise $\Rightarrow$ +ve

Anti-clockwise direction $\Rightarrow$ -ve

Tensile $\Rightarrow$ +ve

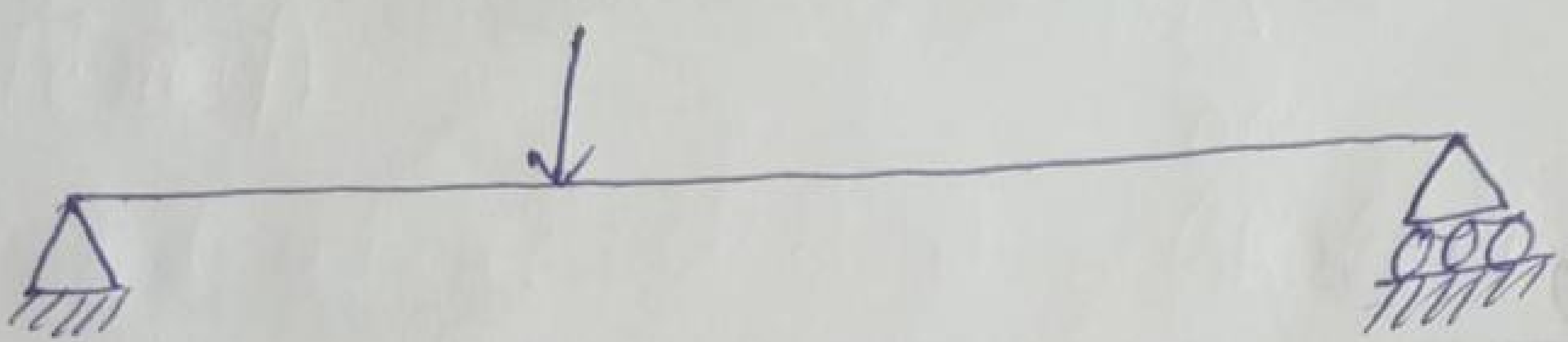
compressive $\Rightarrow$ -ve

Chapter-5

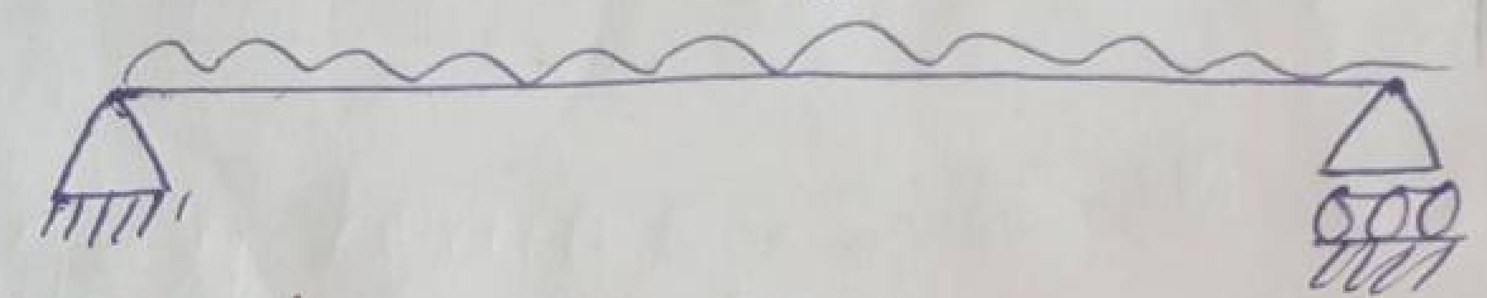
Types of loads and beam:

Types of load on a beam are :-

1. Point load or concentrated load →
It is assumed to act at a point



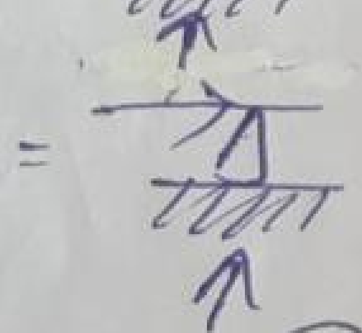
2. uniformly distributed load (U.D.L.) :-
→ It is distributed uniformly over some length. The intensity of loads is constant and measured as load per unit length.

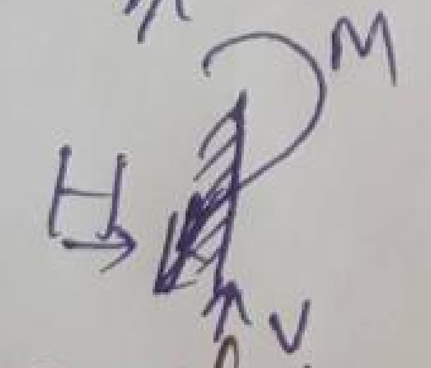


V.V.I.

Types of Support :

1. Roller  ⟶ Reaction → (1)

2. Hinge =  ⟶ Reaction ⟶ (2)

3. Fixed → 

⟶ Reaction ⟶ (3)

1. Roller Support :-
→ When beam rests on a sliding surface such as roller or any flat surface like masonary wall, the support is known as roller support.
→ This support offers normal reaction or vertical reaction.

2. Hinge Support :-
In a Hinge support no translational displacement of the beam possible however it is force to rotate as it has two reaction (Horizontal and vertical reaction).

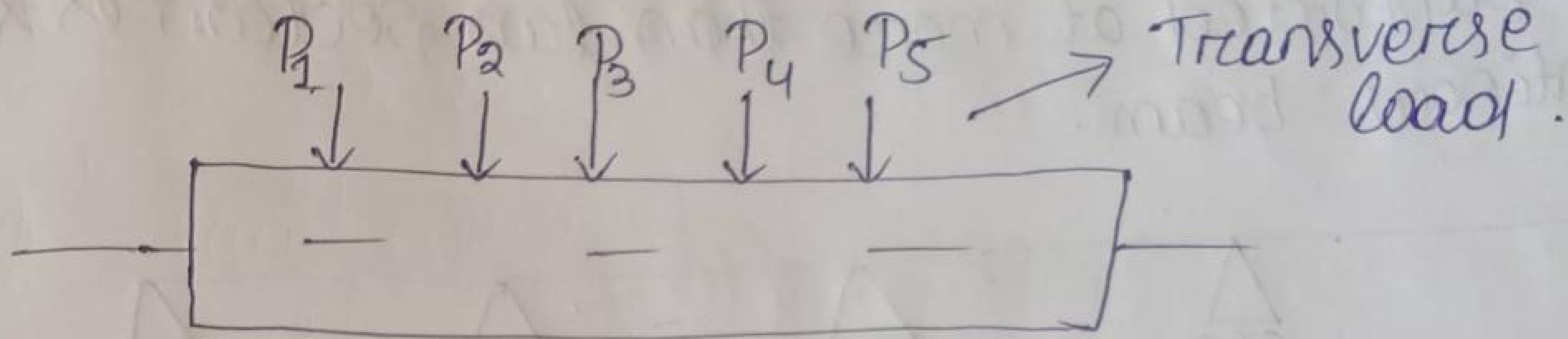
3. Fixed Support:

This is also called Built in or clamped support which does not allow any type of movement or rotation. It offers 3 reaction (Horizontal, vertical and moment reaction)

Types of Beam:

Defination of Beam :-

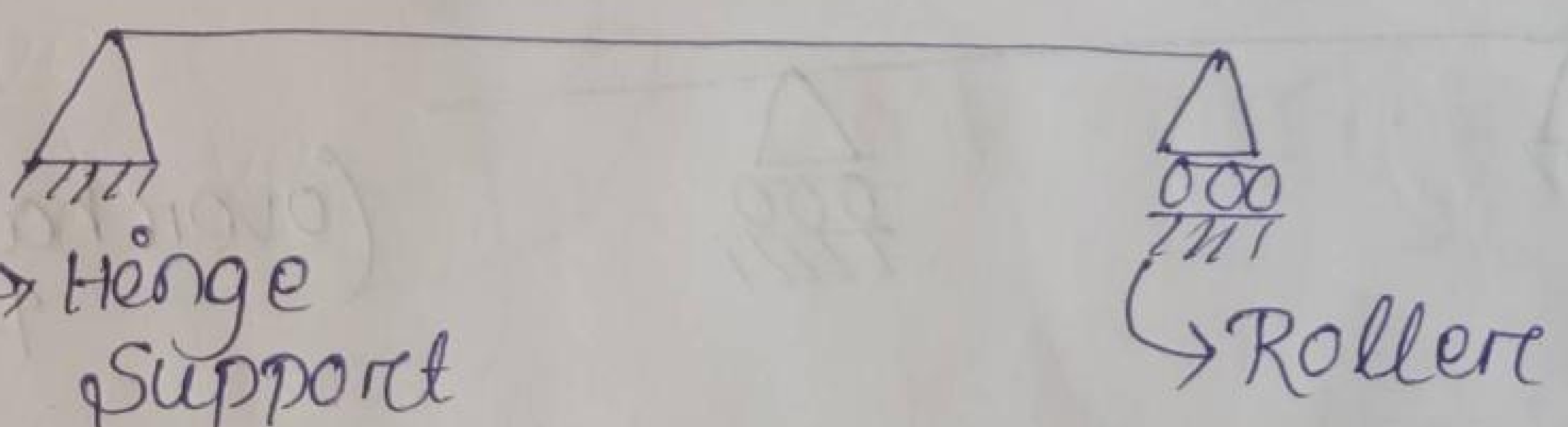
It is a structural element which is subjected to load transverse to its axis is known as beam.



Types of beam:

1. Simply Supported beam:-(S.S.B)

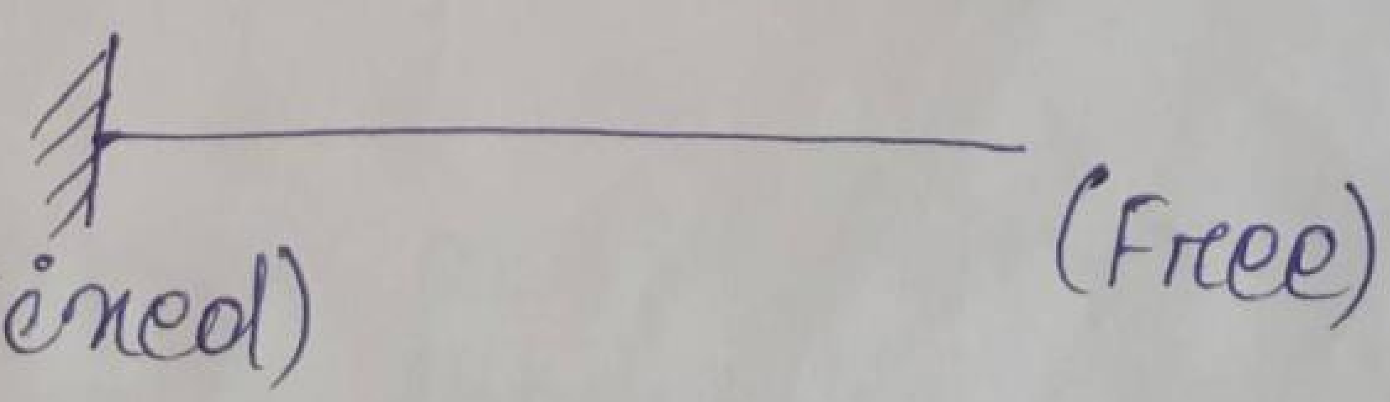
A beam having its both ends freely resting on supports is known as simply supported beam.



2. cantilever beam:-

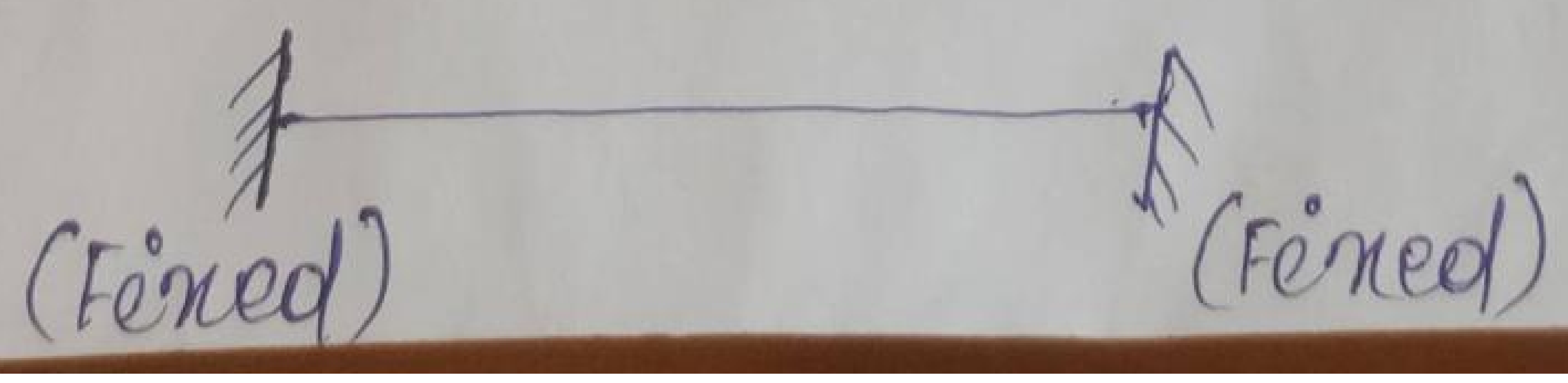
A beam with one end fixed and other free is called cantilever beam.

* Free end has no reaction



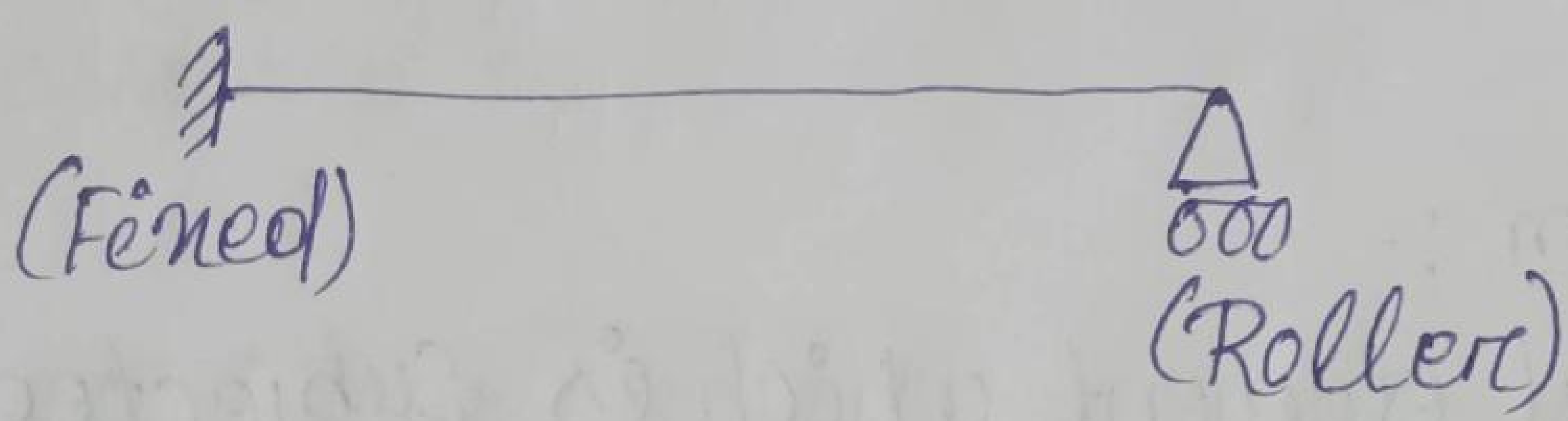
3. Fixed beam:

When both ends are fixed is called fixed beam.



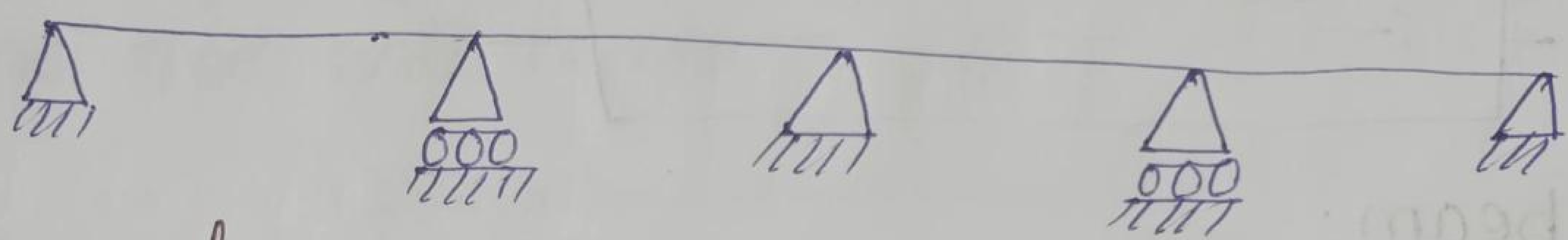
4. Propped cantilever:-

Beams with one end fixed and the other simply supported (roller, hinge) are known as propped cantilever.



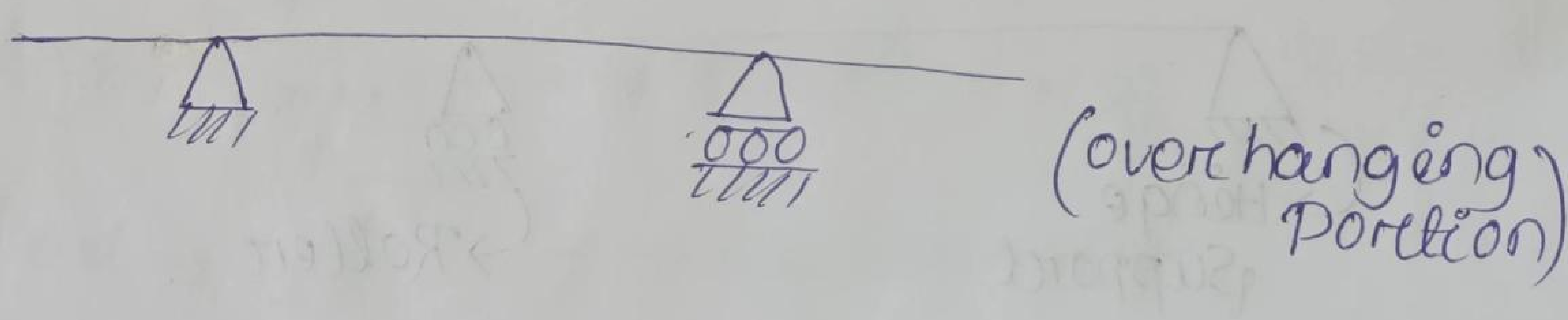
5. continous Beam:-

Beams supported at more than two section is known as continous beam.



6. over hanging beam:-

A beam having its end portion extended beyond the support is known as over hanging beam.



Sheare Force and Bending Moment

Sheare Force:-

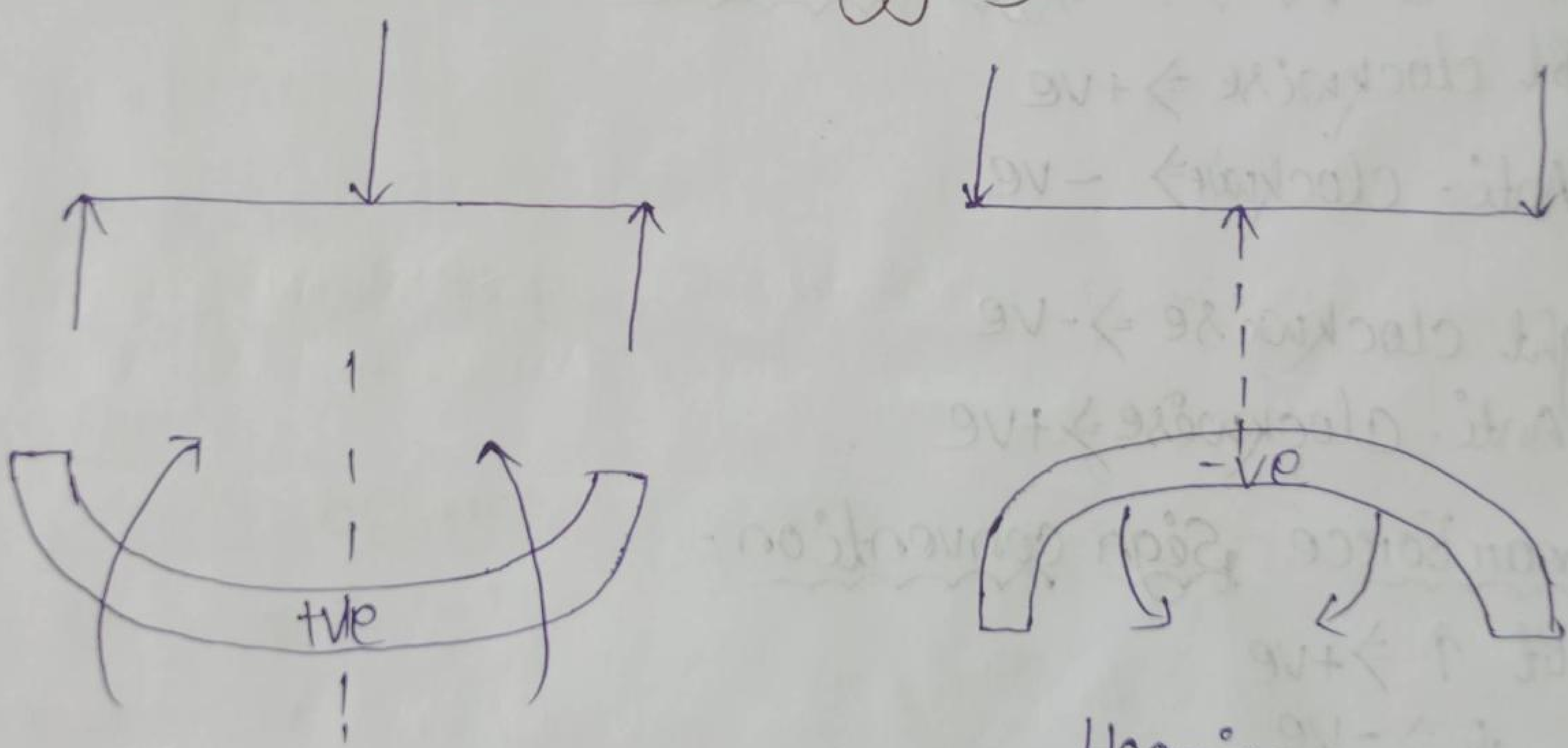
The sheare force at the cross-section of a beam may be defined as the algebraic sum of unbalanced vertical forces to right or left of the section.

Sign convention
* Left $\uparrow \Rightarrow +ve$, $\downarrow \Rightarrow -ve$
* Right $\uparrow \Rightarrow -ve$, $\downarrow \Rightarrow +ve$

Bending moment:-

The bending moment at the cross-section of a beam may be defined as the algebraic sum of the moment of the vertical force to the left or right of the section.

Concept of Hogging and Sagging



Sagging

Hogging

Relation between the load, Sheareforce and Bending moment.

(i) The ratio of change of S.F (Sheare Force) at any section represents the ratio of loading at the section

$$W = \frac{d(S.F)}{dx}$$

(ii) The rate of change of Bending moment at any section represents the Shear force at the section.

$$S.F = \frac{d(BM)}{dx}$$

So, $$\boxed{w = -\frac{d(S.F)}{dx} = -\frac{d^2(B.M)}{dx^2}}$$

→ At a point where $\frac{d(B.M)}{dx} = 0$ or shear force is zero, the Bending moment will have maximum value. That point also the position where shear force changes sign.

Imp

Bending moment sign convention:

Left clockwise ⇒ +ve

Anti-clockwise ⇒ -ve

Right clockwise ⇒ -ve

Anti-clockwise ⇒ +ve

Shear force sign convention:-

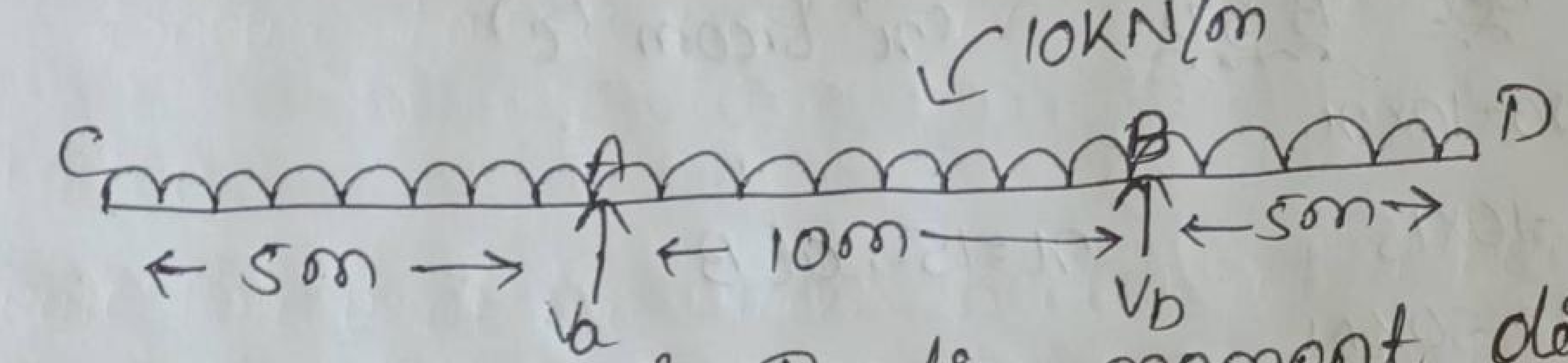
Left ↑ ⇒ +ve

↓ ⇒ -ve

Right ↑ ⇒ -ve

↓ ⇒ +ve

Q:



Draw Shear Force & Bending moment diagram?

Ans $\Sigma V = 0$

$\Rightarrow V_a + V_b = 10\times10 + 10\times(5+5)$

$= 200 KN$

$\Sigma M_A = 0$

$\Rightarrow -V_b \times 10 + \cancel{10}^{5} \times 15 \times \frac{15}{\cancel{2}} - \cancel{10}^{5} \times 5 \times \frac{5}{\cancel{2}} = 0$

$\Rightarrow V_b = \frac{1000}{10}$

$\Rightarrow V_b = 100 KN$

$V_a = 200 - V_b$

$= 200 - 100 = 100 KN$

Shear Force

Take section between C and A

'x' from 'C'

$(S.F)_{xx} = -10x$

$(S.F)_C = -10\times0 = 0KN$, $x=0$ at C

$(S.F)_A = -10\times5 = -50KN$ $x=5$ at A

Take section betn A & B 'x' from 'C'

$(S.F)_{xx} = 100 - 10x$

$(S.F)_A = 100 - 10\times5$ $x = 5$, at A

$= 100 - 50 = 50KN$

$(S.F)_B = 100 - 10\times15$ $x = 15$, at B

$= 100 - 150$

$= -50 KN$

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Shear Force betn B & D ('x' from 'c')

$(S.F)_{xx} = 100+100-10\times x$

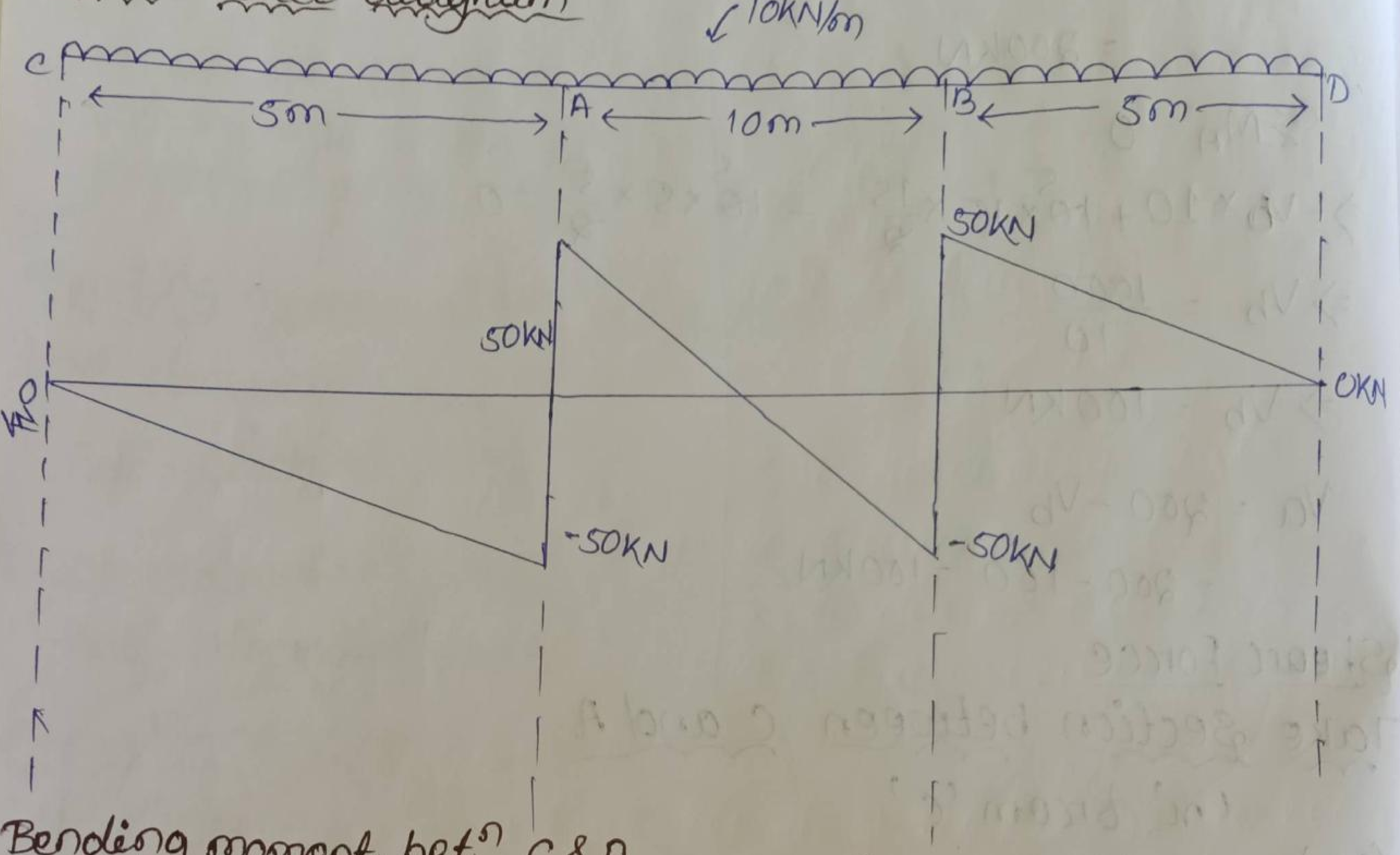
$(S.F)_B = 100+100-10\times 15$ $x=15$, at B

$= 200-150 = 50KN$

$(S.F)_D = 200-10\times 20$

$= 200-200 = 0KN$ $x = 20$ at D

Shear Force diagram



Bending moment betn C & A ('x' from 'c')

$(B.M)_{xx} = -10\times x\times \frac{x}{2}$

$(B.M)_C = -10\times 0\times 0/2 = 0KNm$ $x=0$, at C

$(B.M)_A = -\cancel{10}^{5}\times 5\times 5/\cancel{2} = -125KNm$ $x=5$, at A

Bending moment betn A & B ('x' from 'c')

$(B.M)_{xx} = -10\times \frac{x^2}{2} + 100(x-5)$

$(B.M)_{xx} = -5x^2+100(x-5)$

$(B.M)_A = -5\times(5)^2+100(5-5)$ $x=5$, at 'A'

$= -125$ KN-m

- As the shear force changes sign betn 'A' and 'B' at point 'M' so, we have to calculate Bending moment at 'm'.

$(B.M)_m = -5 \times (10)^2 + 100(10-5)$

$= -500 + 500 = 0$ KN-m $x = 10$, at 'M'

$(B.M)_B = -5 \times (15)^2 + 100(15-5)$

$= -125$ KN-m $x = 15$, at B

Take section betⁿ B & D: 'x' from 'C'

$(B.M)_{xx} = -10 \times \frac{x^2}{2} + 100(x-5) + 100(x-15)$

$= -5x^2 + 100(x-5) + 100(x-15)$

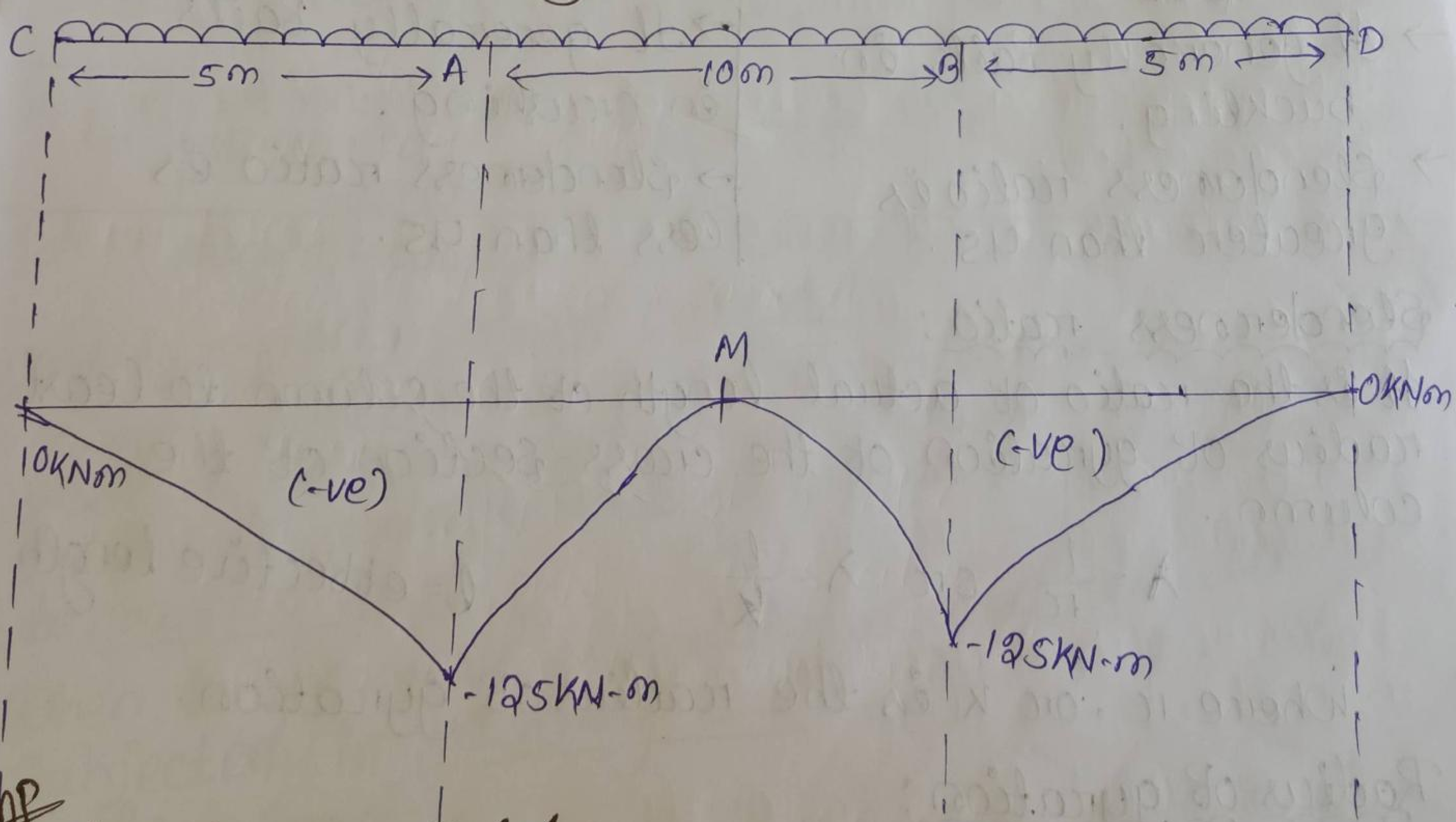
$(B.M)_B = -5 \times (15)^2 + 100(15-5) + 100(15-15)$ $x = 15$, at B

$= -125$ KN-m

$(B.M)_D = -5 \times (20)^2 + 100(20-5) + 100(20-15)$ $x = 20$, at D

$= 0$ KN-m

Bending moment diagram



Imp

Contraflexure Point:

The point at which the Bending moment is zero is called contraflexure point.

Column and Struts

→ Column is a vertical member, it sustain compressive load.

→ Strut is a vertical, horizontal and inclined member, it also sustain compressive load.

Long column	Short column
→ The column whose lateral dimension is small when compared to its length.	→ The column whose lateral dimension is very large when compared to its length.
→ Ratio of effective length to least lateral dimension is greater than 12. $\frac{l}{D(\text{or})b} \geq 12$	→ Ratio of effective length to least lateral dimension is less than 12. $\frac{l}{D(\text{or})b} < 12$
→ It generally fails in buckling.	→ It generally fails in crushing.
→ Slenderness ratio is greater than 45.	→ Slenderness ratio is less than 45.

Slenderness ratio:
It is the ratio of actual length of the column to least radius of gyration of the cross-section of the column.

$$\lambda = \frac{l}{r} \quad \text{or} \quad \lambda = \frac{l}{k}$$

l = effective length

Where r, or k is the radius of gyration.

Radius of gyration:
It is defined as the distance between the reference axis to the centre of gravity. $r = \sqrt{\frac{I}{A}}$

Equivalent length or effective length:-
→ The equivalent length of a given column with given end condition is the length of an equivalent column of

the same material and cross-section in which both end hinged and having the value of crippling load equal to that of the given column.

Sl. NO.	Endcondition	Relation of effective and actual length	crippling load
1	Both Hinged	$L_e = l$	$P_E = \frac{\pi^2 EI}{le^2}$ $P_E = \frac{\pi^2 EI}{l^2}$
2.	Both end Fixed	$L_e = l/2$	$P_E = \frac{\pi^2 EI}{(l/2)^2}$ $= \frac{4\pi^2 EI}{l^2}$
3.	one end fixed other end free	$L_e = 2l$	$P_E = \frac{\pi^2 EI}{(2l)^2}$ $P_E = \frac{\pi^2 EI}{4l^2}$
4.	one end fixed other end hinged.	$L_e = l/\sqrt{2}$	$P_E = \frac{\pi^2 EI}{(l/\sqrt{2})^2}$ $P_E = \frac{2\pi^2 EI}{l^2}$

E = Young's modulus , I = moment of Inertia

What is axially loaded column?

→ column subjected to load acting along the longitudinal axis or centroid of the column section. When short column is axially loaded, it will be subjected to crushing load.

When long column is axially loaded, it will be subjected to buckling load.

crushing load:

When the load is gradually increased, the short column will reach a stage at which it is subjected to ultimate crushing stress, beyond the stress the column will fail. The load corresponding to the crushing stress is called crushing load.

Buckling load :-

When a long column is subjected to a compressive stress, if the load is gradually increased the column will reach a stage when it will start buckle. The load at which the column just buckle is called buckling load.

Assumption in Euler's theory :-

→ Initially the column is perfectly straight and the load applied is truly axial.

→ The cross-section of the column is uniform throughout its length.

→ The column is perfectly elastic homogenous and isotropic and obey's hooke's law.

→ The length of the column is very large as compare to its cross-section.

→ The shortening of column due to direct compressive is neglected.

→ The failure of column occurs due to buckling alone.

$$P_E = \frac{\pi^2 EI}{(l_e)^2}$$

Isotropic - properties are equal in every direction.

Ortho-tropic - mutually perpendicular & diffn direction.

Anistropic - properties are different in all direction.

Homogenous - uniform mass

moment of Inertia

$$I = \frac{\pi d^4}{64} \rightarrow \text{circule}$$

$$I = \frac{\pi (D^4 - d^4)}{64} \rightarrow \text{Holo}$$

$$I = \frac{bd^3}{12} \rightarrow \text{rectangle}$$

Square in $b = a$ $\quad I = \frac{a^4}{12}$

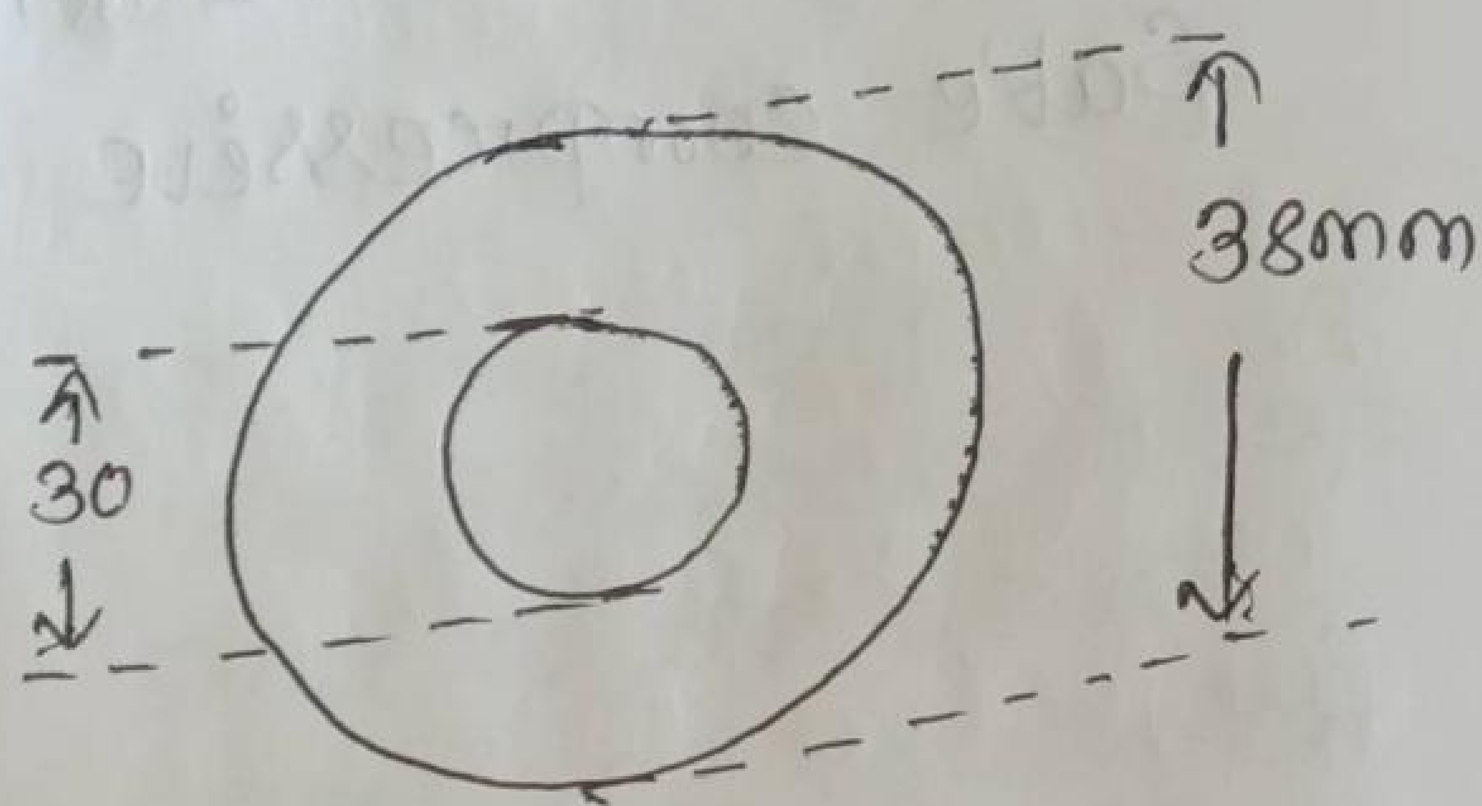
Q.1 A mild steel tube 4 meters long 30mm internal diameter and 4mm thick is used as a strut with both end hinged. Find the collasping load.
Take $E = 2.1 \times 10^5 \ N/mm^2$

Ans

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(38^4 - 30^4)$$

$$= 62561.36 \ mm^4$$



Since both ends of the column are hinged
Effective length $= L = 4m = 4000 \ mm$

$$P_E = \frac{\pi^2 E I}{l^2}$$

$$= \frac{\pi^2 \times 2.1 \times 10^5 \times 62561.36}{(4000)^2}$$

$$= 8095.89 \ N = 8.095 \ kN$$

Q.2 A solid round bar 60mm in diameter and 2.5m long is used as a strut. one end of the strut is fixed while its other end is hinged. Find the safe compressive load for the strut using Euler's Formula. Take $E = 200 \ GN/m^2$ and take Factor of safety $= 3.0$

Ans Diameter of solid round bar,

$D = 60mm = 0.006 \ m$

modulus of Elasticity, $E = 200 \ GN/m^2$

$F.O.S = 3$

length of round bar $= l = 2.5 \ m$.

$L = l/\sqrt{2} = \frac{2.5}{\sqrt{2}} = 1.768$ (one end hinged other fixed)

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Euler's crippling load is given by :-

$$P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200 \times 10^9 \times \frac{\pi}{64} \times (60\ )^4}{(1.768)^2} \times 10^{-3}\ kN$$

$$= 401.124\ kN$$

Safe compressive load $= \frac{P}{F.O.S.}$

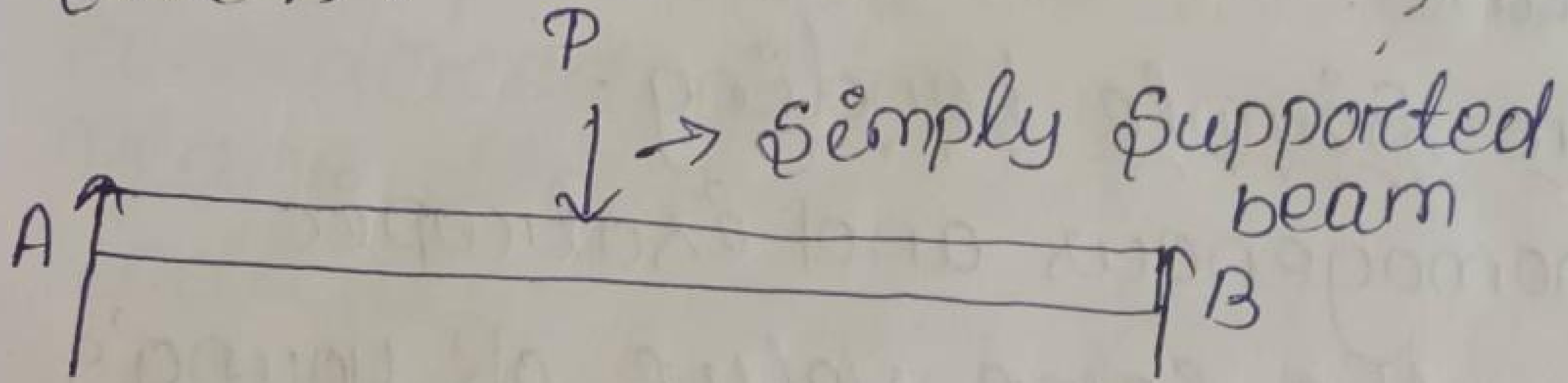
$$= \frac{401.124}{3}$$

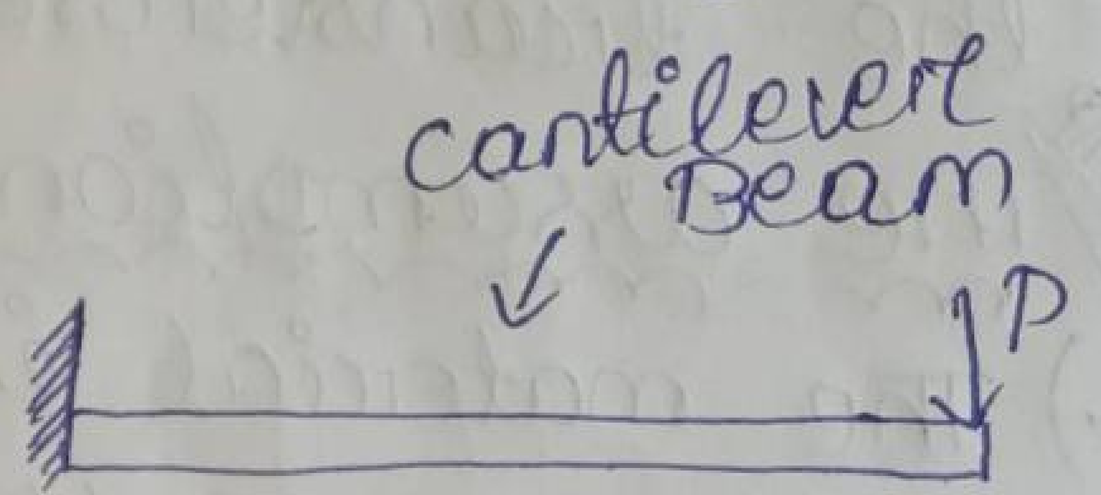
$$= 133.708\ kN$$

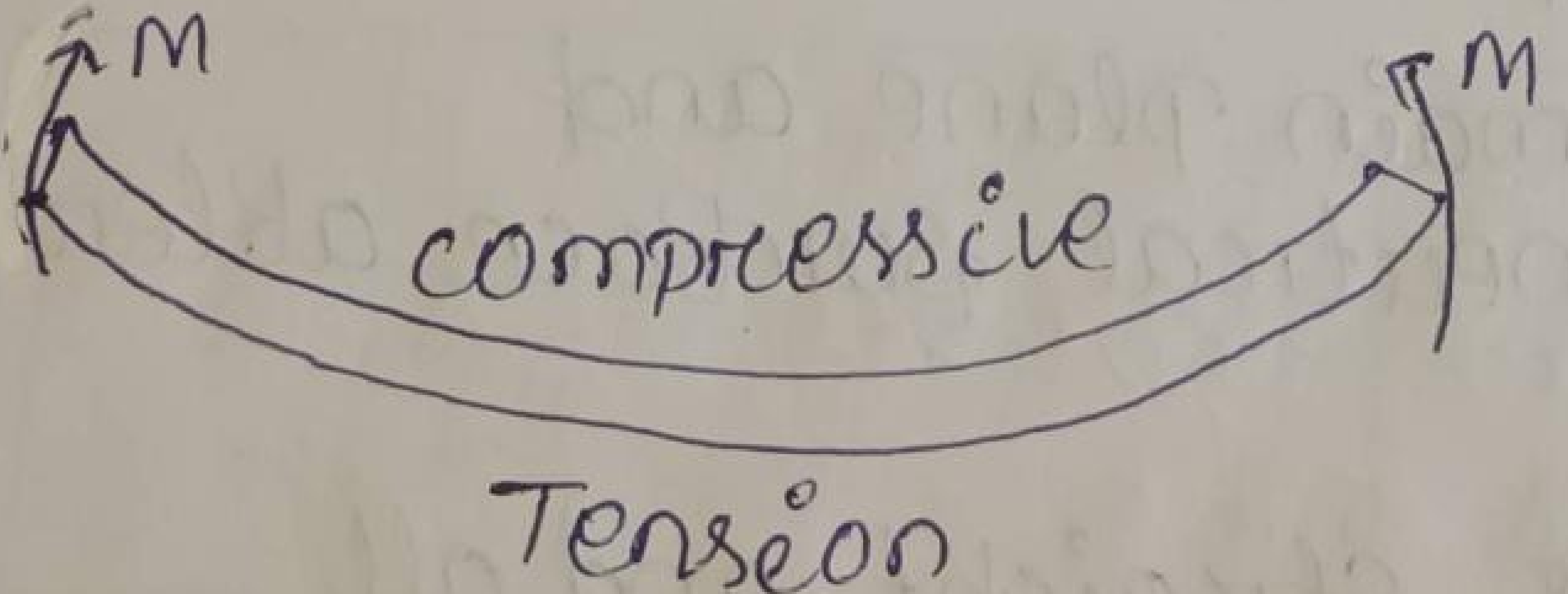
Bending Stress

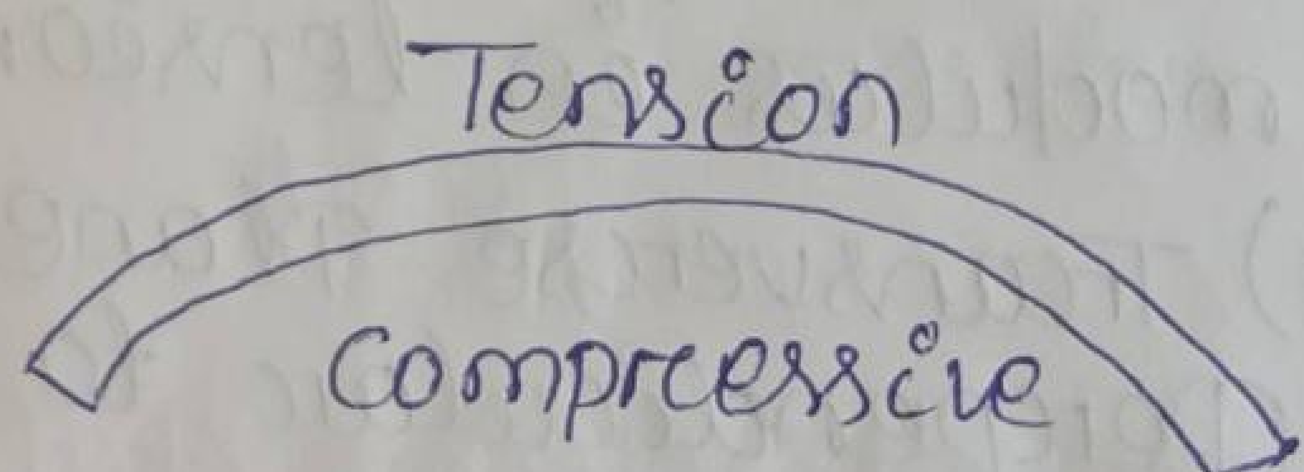
Pure bending:-
If a member is subjected to equal and opposite couples acting in the longitudinal plane, then the member is said to be pure bending.

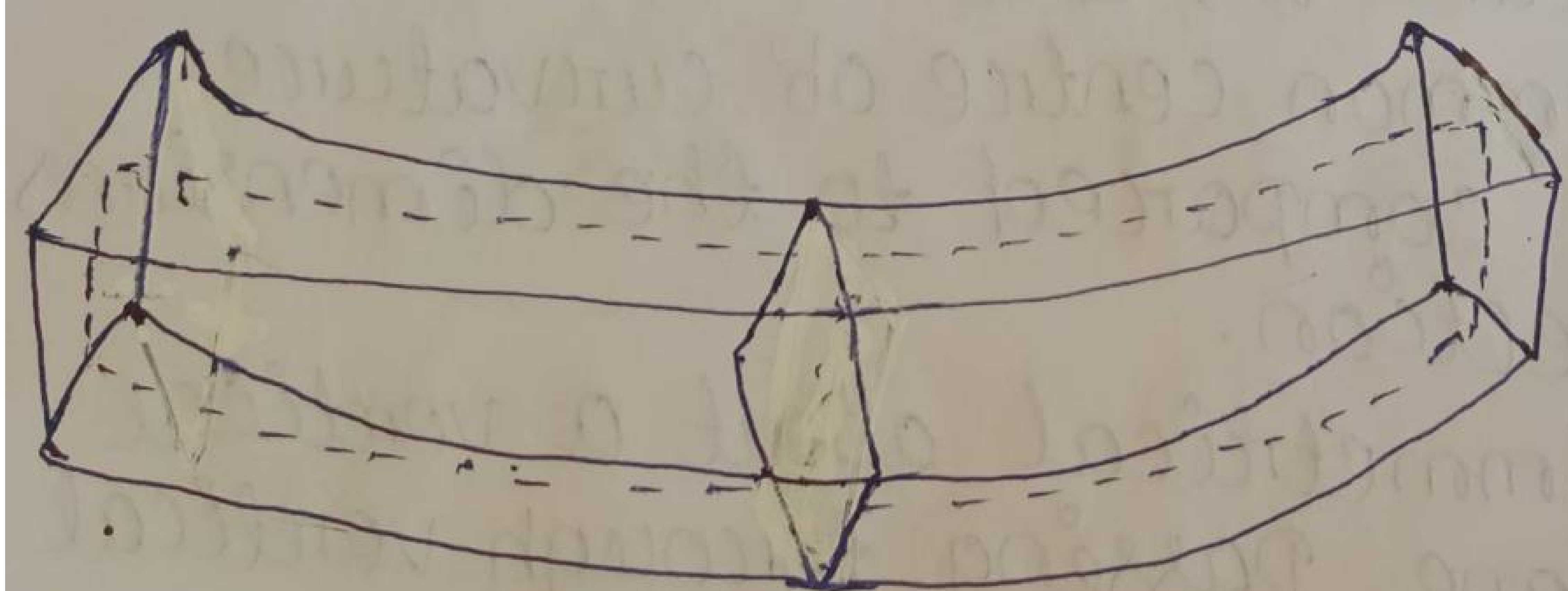
Bending Stress:-
If a constant bending moment no shear force acts on some length of a beam, the stresses set up on any cross-section on that part of the beam constitute a pure couple, the magnitude of which is equal to the bending moment. The internal stresses developed in the beam are known as Bending Stress.











Neutral layer :-

It is the layer in the beam in which longitudinal fibre do not change in length. In this layer, stress and strain is zero.

The line of intersection of neutral layer with the cross-section of the beam is known as neutral axis.

The theory of simple Bending :-

The following theory is applicable to the beam subjected to simple or pure bending when the cross-section is not subjected to a shear force. Since that will cause a distortion of the transverse plane.

Imp The assumption for simple bending :-

(1) The material is homogenous and isotropic

For example :- It has the same value of young's modulus in tension and compression.

(2) Transverse planes remain plane and perpendicular to the neutral surface after bending.

(3) Initially the beam is straight and all longitudinal filaments are bent into circular arcs with a common centre of curvature which is large compared to the dimensions of the cross-section.

(4) The beam is symmetrical about a vertical longitudinal plane passing through vertical axis of symmetry for horizontal beams.

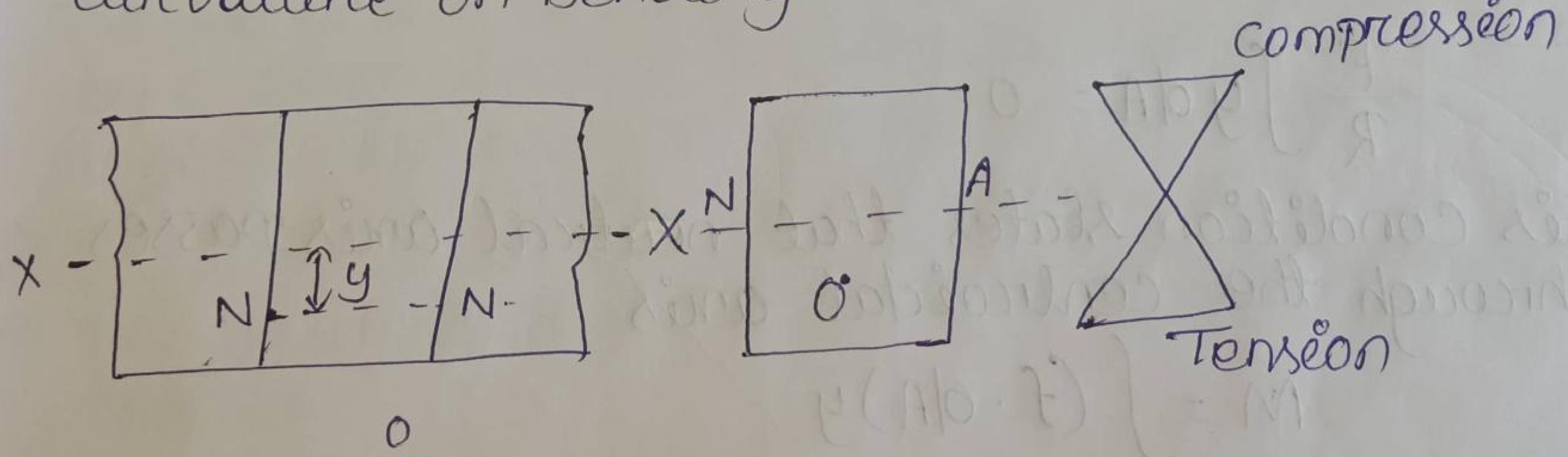
(5) The stress is purely longitudinal and the stress concentration effects near the concentrated loads are neglected.

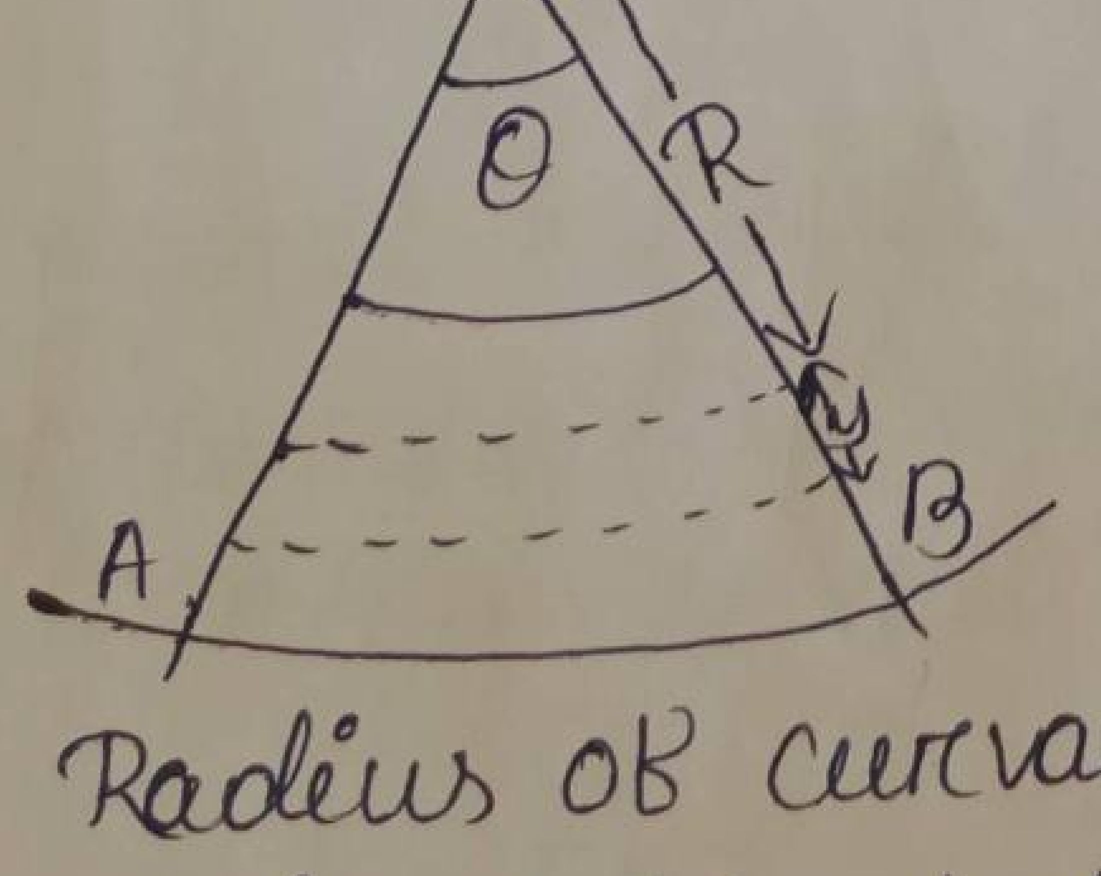
Bending equation for a beam :-

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

M = Bending moment of any cross-section of beam.
I = Moment of Inertia.
f, σ_b = Bending stress.
y = Centroidal axis/distance
E = Modulus of Elasticity
R = Radius of curvature.

Proof :-
consider a length of beam under the action of a bending moment 'M'. N-N is the original length considered of beam. The neutral surface is a Plane through : x-x. In the side view NA indicates neutral axis. 'O' is the of curvature on bending.





$l = \cancel{R\theta}R\theta$

R = Radius of curvature of the neutral surface
θ = angle subtended by the beam length at centre 'O'.
σ_b = longitudinal stress.

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A filament of original length NN at a distance Y from the neutral axis will be elongated to a length AB.

The Strain in AB = $\frac{AB - NN}{NN}$

$$\boxed{\frac{f}{E} = \frac{(R+y)\theta - R\theta}{R\theta}}$$

$$\boxed{\frac{f}{E} = \frac{y}{R}} \quad \text{——— (1)}$$

For pure bending :-

Net normal forces is zero.

$$\int f \cdot dA = 0$$

$$\int \frac{E}{R} y \, dA = 0$$

$$\frac{E}{R} \int y \, dA = 0$$

This condition states that neutral axis passes through the centroidal axis.

$$M = \int (f \cdot dA) y$$

$$= \int y \times \frac{E}{R} \times dA \times y$$

$$= \frac{E}{R} \int y^2 \, dA$$

$$M = \frac{E}{R} \times I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}} \quad \text{——— (ii)}$$

Equating (i) & (ii)

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

Section modulus :-

The ratio of I/y, where y is the furthest or the most distant point of the section from neutral axis is called section modulus.

It is denoted by 'Z'

$$Z = \frac{\text{moment of inertia about ~~about~~ neutral axis}}{\text{Distance of furthest point from neutral axis.}}$$

$$\boxed{Z = \frac{I}{y}}$$

V.V.I. Moment of resistance :-

The maximum bending moment which can be carried by a given section for a given maximum value of stress is known as moment of resistance.

V.V.I. Significance of section modulus :-

→ The strength of the beam depends on the section modulus.

$$Z = \frac{I}{y}$$

V.V.I. Flexural rigidity (F) :-

The product of modulus of elasticity and moment of inertia is called flexural rigidity.

$$F = E \times I$$

Maximum Bending Stress Formula (M_{max}):

Cantilever

Full U.D.L (M_{max}) = $\frac{wl^2}{2}$

Point load (M_{max}) = $P \times l$

Simply Supported:

Full U.D.L. (M_{max}) = $\frac{wl^2}{8}$

Point load (M_{max}) = $\frac{Pl}{4}$

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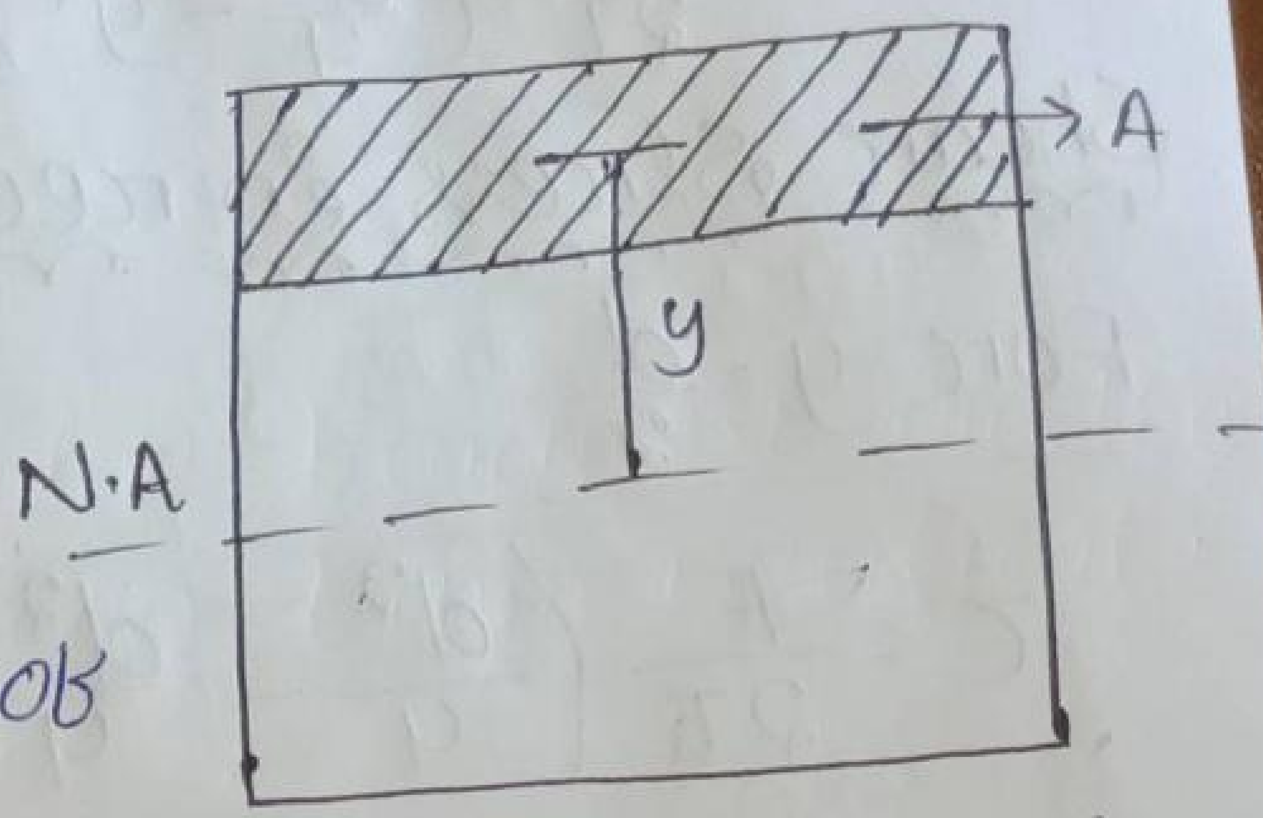
Sheare stress :

$$\tau = \frac{FA\bar{y}}{Ib}$$



τ = Sheare Stress

F = Sheare Force

A = Area above the neutral axis of the shaded portion.

$\bar{y}$ = distance betn centre of gravity of the section to the shaded area.

I = moment of inertia of the whole section above its neutral axis.

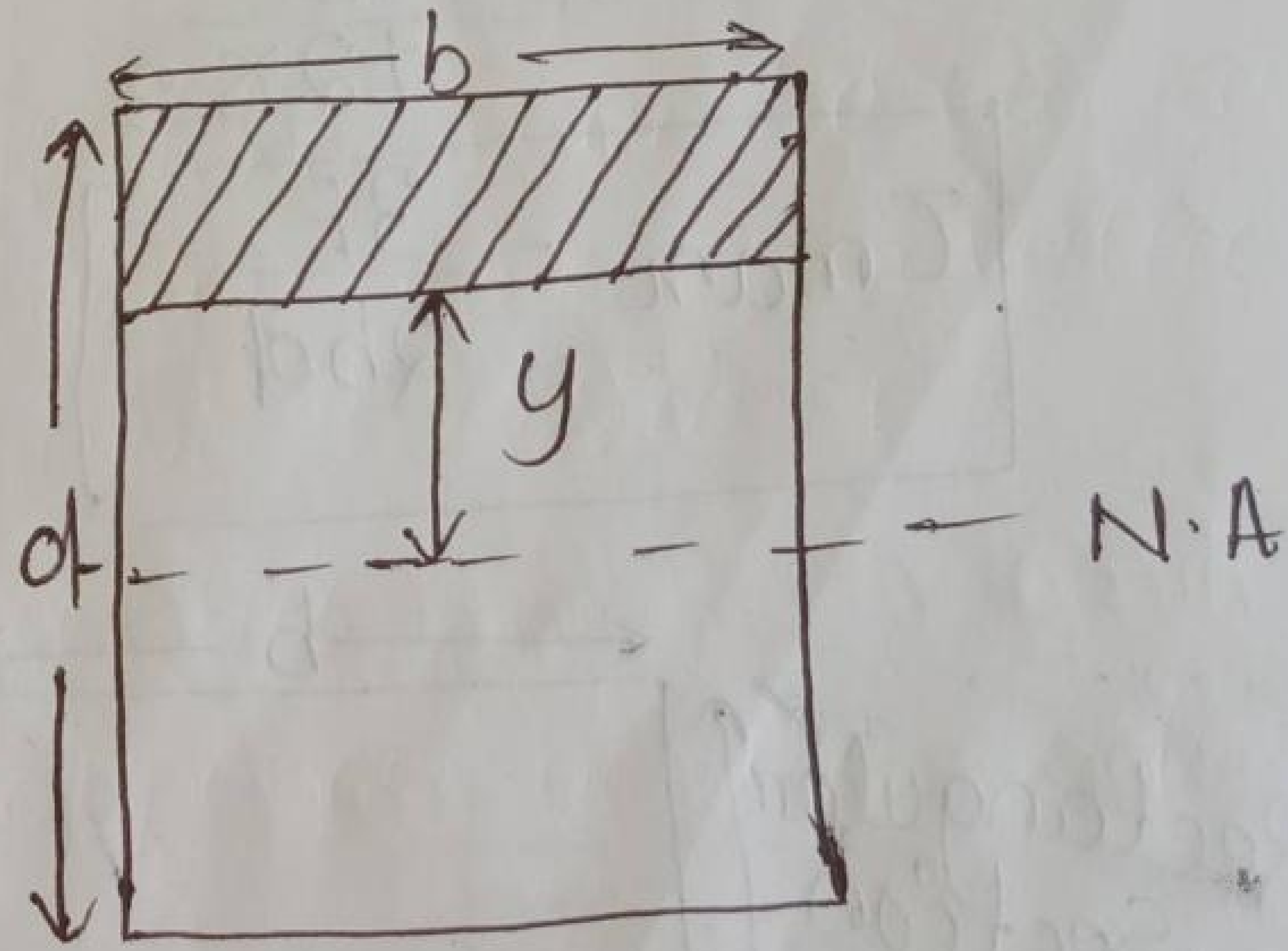
b = width at the section .

Rectangulare Section:

b = width of the beam

d = depth of the beam

y = distance between neutral axis to the section of the beam.



$$\tau = \frac{FA\bar{Y}}{Ib}$$

$$= \frac{F \times \left(\frac{d}{2} - y\right) \times \cancel{b} \times \frac{1}{2}\left(\frac{d}{2} + y\right)}{I\cancel{b}}$$

$$= \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

$$\bar{Y} = \frac{1}{2}\left(\frac{d}{2} - y\right) + y$$

$$= \frac{d}{4} - \frac{y}{2} + y$$

$$= \frac{d}{4} + \frac{y}{2}$$

$$= \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$A = \left(\frac{d}{2} - y\right) \times b$$

For Rectangular section:

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

Shear stress in rectangular section:

For $y = \frac{d}{2}$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - \frac{d^2}{4}\right)$$

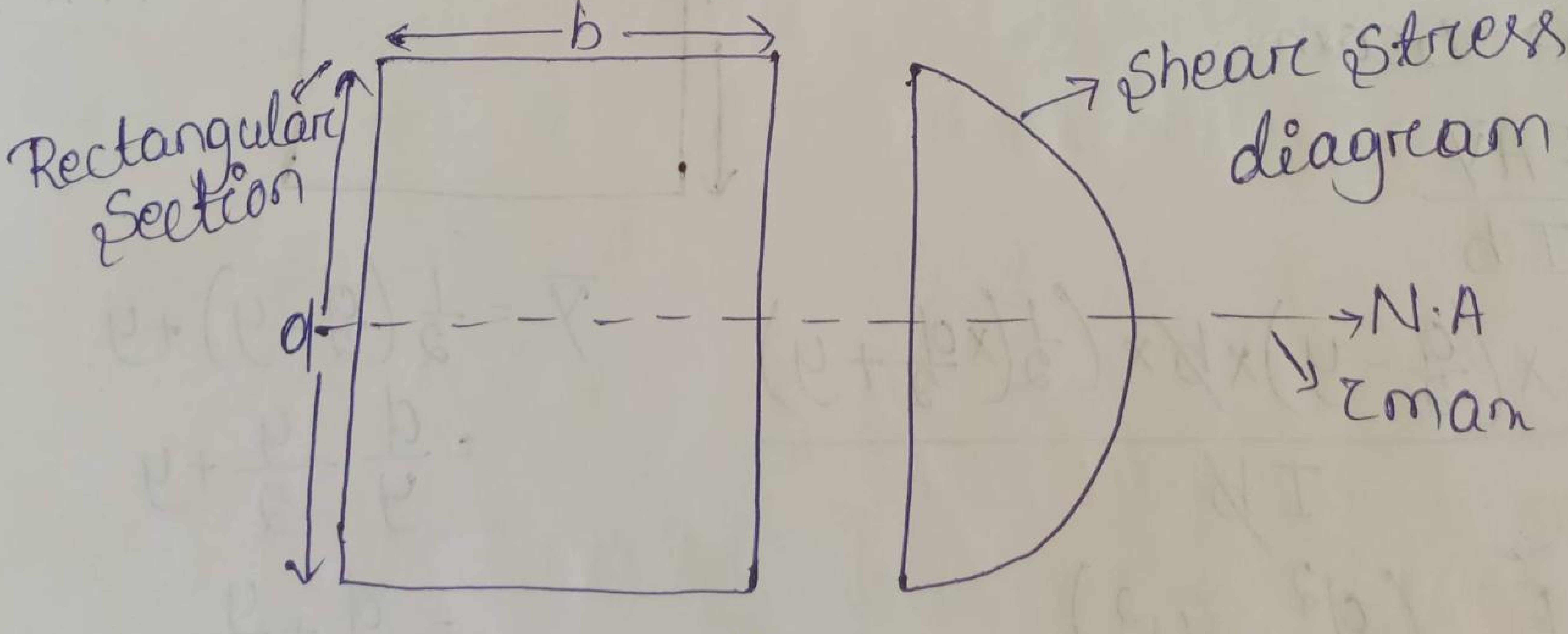
$$\tau = 0$$

For $y = 0$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4}\right)$$

$$= \frac{F}{\cancel{2} \times \frac{bd^{\cancel{3}}}{\cancel{12}\,3}} \times \frac{d^{\cancel{2}}}{\cancel{4}\,2}$$

$$\boxed{\tau_{max} = \frac{3F}{2bd}}$$

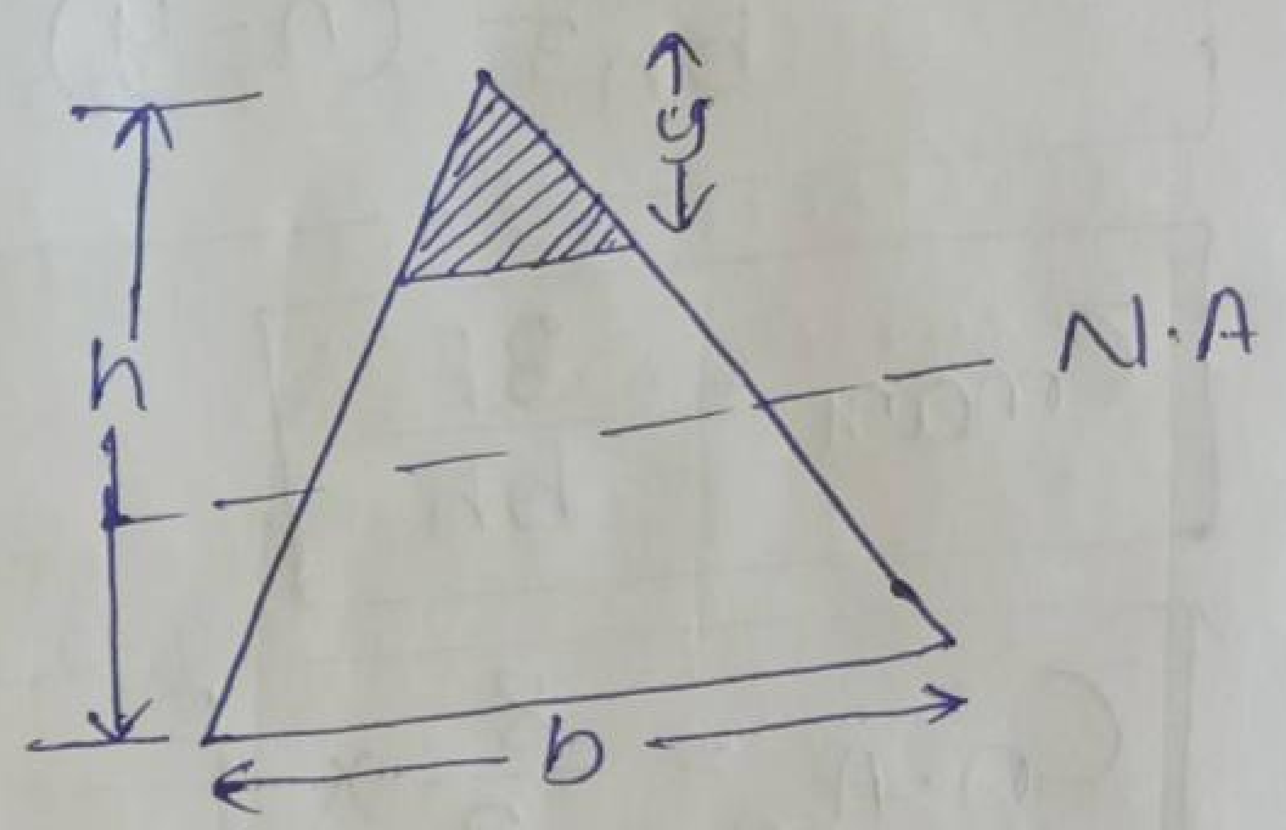


$$\tau_{max} = 1.5 \times \frac{F}{A} = N.A$$

Triangulare Section :-

$$\tau = \frac{FA\bar{y}}{Ib}$$

$$= \frac{F \times \frac{1}{2} \times \frac{by}{h} \times y \times \left(\frac{2h}{3} - \frac{2y}{3}\right)}{\frac{bh^3}{36} \times \frac{by}{h}}$$



$$\boxed{\tau = \frac{12F}{bh^3} y (h-y)}$$

Area of Shaded portion =

$$\frac{1}{2} \times \frac{by}{h} \times y$$

$$\frac{b'}{b} = \frac{y}{h}$$

$$\boxed{b' = \frac{y}{h} \times b}$$

$$\frac{d\tau}{dy} = \frac{12F}{bh^3} (h - 2y)$$

$$0 = \frac{12F}{bh^3} (h - 2y)$$

$$\boxed{y = \frac{h}{2}}$$

$$\tau_{max} = \frac{12F \times \frac{h}{2} \times (h - h/2)}{bh^3}$$

$$= \frac{3}{2} \times \frac{F}{\frac{1}{2} \times bh}$$

$$\boxed{\tau_{max} = 1.5\, \tau_{mean}}$$

$$\boxed{\text{Traingle centroid} = h/3}$$

$$\tau_{\text{neutral axis}} = \frac{12F}{bh^3} y (h-y)$$

$$= \frac{12F}{bh^3} \times \frac{2h}{3} \left(h - \frac{2h}{3}\right) \qquad y = \frac{2h}{3}$$

$$\boxed{\tau_{n.A} = \frac{8}{3} \times \frac{F}{bh}}$$

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$$\boxed{\tau = \frac{12F \times y}{bh^3}(h-y)}$$

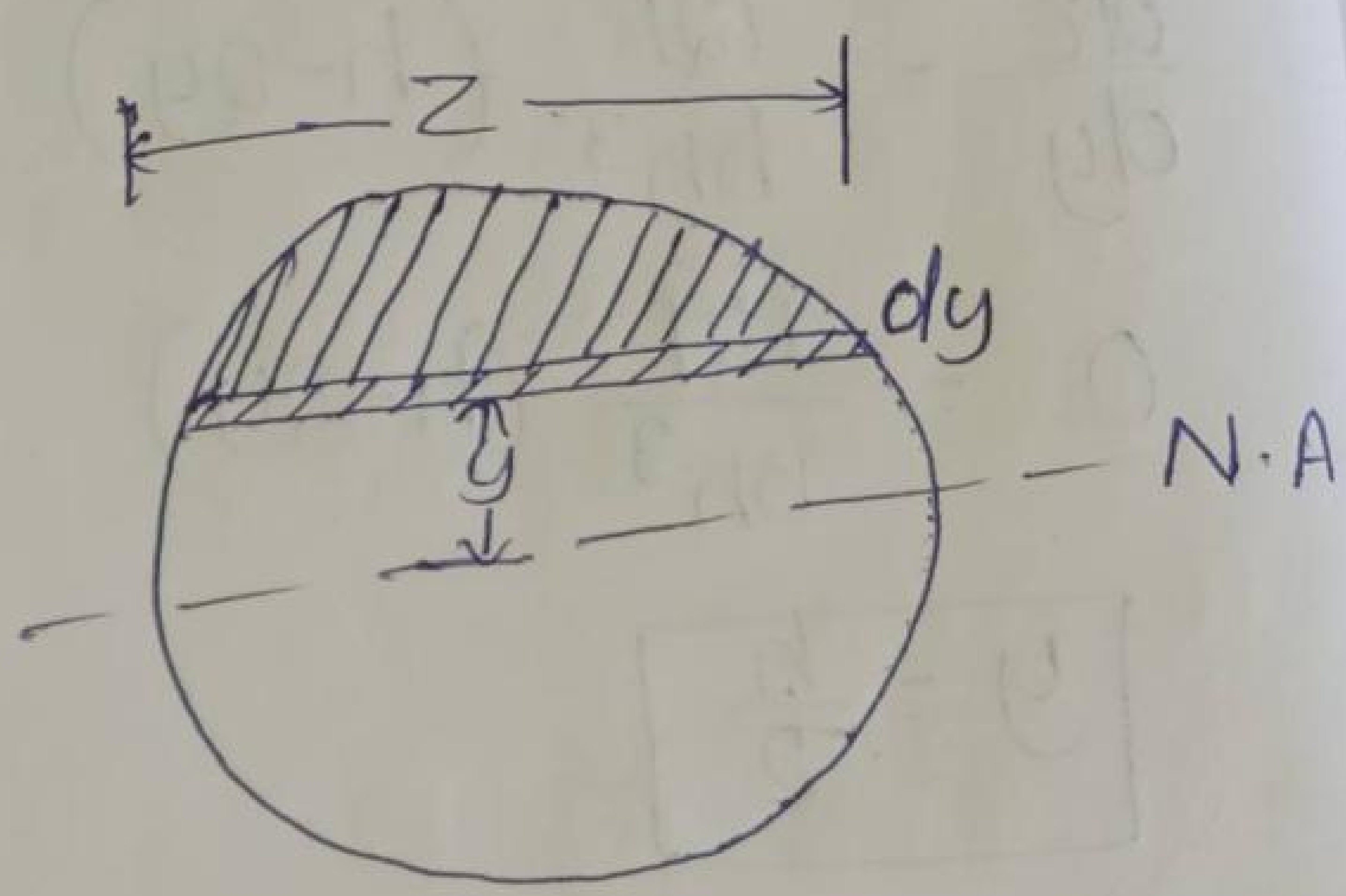
$$\boxed{\tau_{max} = \frac{3F}{bh}}$$

$$\boxed{\tau_{N.A} = \frac{8}{3} \times \frac{F}{bh}}$$

→ Triangular section

Circular Section:

$A\bar{y}$ = moment of shaded area about neutral axis



$$\tau = \frac{F \times A\bar{y}}{Ib}$$

$$\left(\frac{Z}{2}\right)^2 = r^2 - y^2$$

$$Z^2 = 4(r^2 - y^2)$$

Area of small strip $= Z \times dy$

moment of small strip about N.A

$$= A\bar{y} = Z\,dy \cdot y$$

$A\bar{y}$ for shaded portion

$$= \int_y^r Z \cdot dy \cdot y$$

$$Z^2 = 4(r^2 - y^2)$$

$$2Z \cdot dZ = -8y\,dy$$

$$y\,dy = \frac{-Z\,dZ}{4}$$

$$= -\frac{1}{4}\int_Z^0 Z \cdot Z \cdot dZ$$

$$= \frac{1}{4}\int_0^Z Z^2 \cdot dZ = \frac{Z^3}{12}$$

$$\tau = F \cdot \frac{1}{zI} \cdot \frac{z^3}{12} = F \times \frac{1}{I} \times \frac{z^2}{12} = \frac{F}{3I}(r^2 - y^2)$$

Shear stress is parabolic nature :

$$\tau_{max}(y=0) = \frac{F}{3I}(r^2) = \frac{F}{3 \times \left(\frac{\pi d^4}{64}\right)} \times \left(\frac{d^2}{4}\right)$$

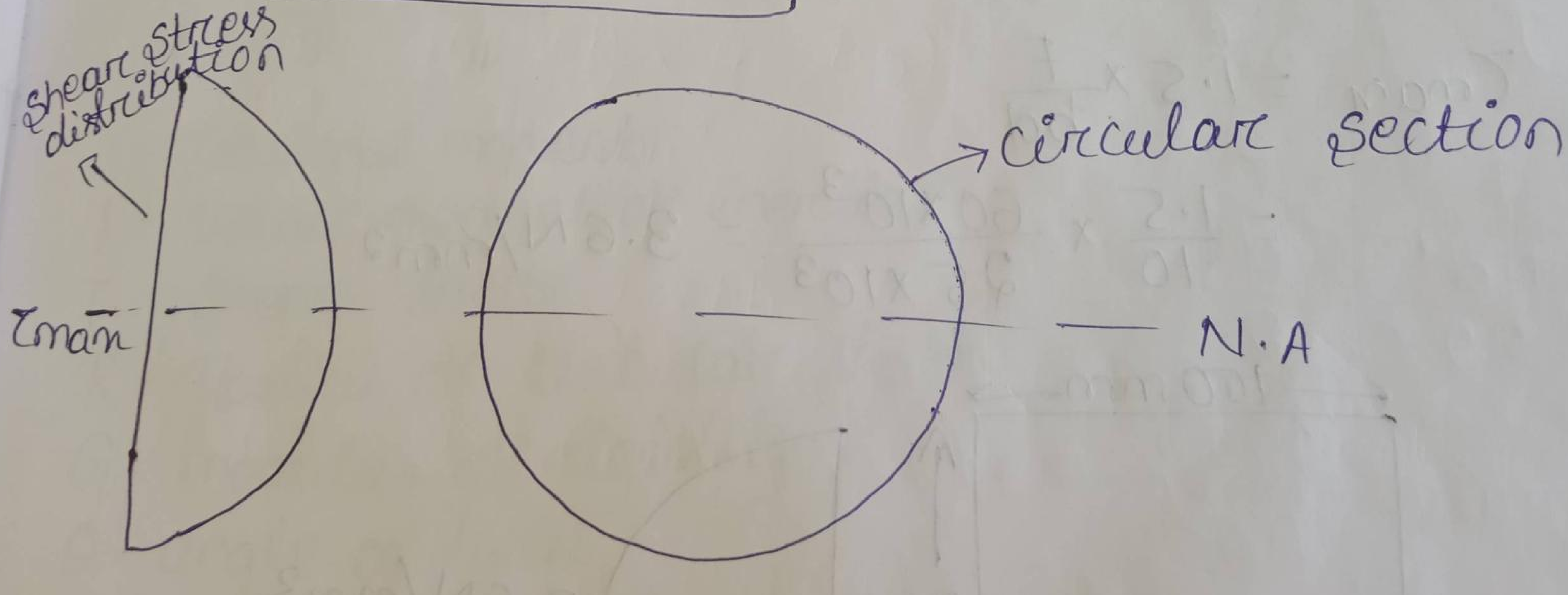
$$\boxed{\tau_{max} = \frac{4}{3}\tau_{av}}$$

Sheare Stress for circular section:

$$\boxed{\tau = \frac{F}{3I}(r^2 - y^2)}$$

$$\boxed{\tau_{max} = \frac{16F}{3\pi d^2} = \frac{4}{3}\frac{F}{\left(\frac{\pi d^2}{4}\right)}}$$

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{av}}$$



Q. A wooden beam 100mm wide, 250mm depth and 3m. long carrying a U.D.L 40KN/mt. Determine the maximum shear stress and draw the diagram of shear stress along the depth of the beam (Simply Supported beam).

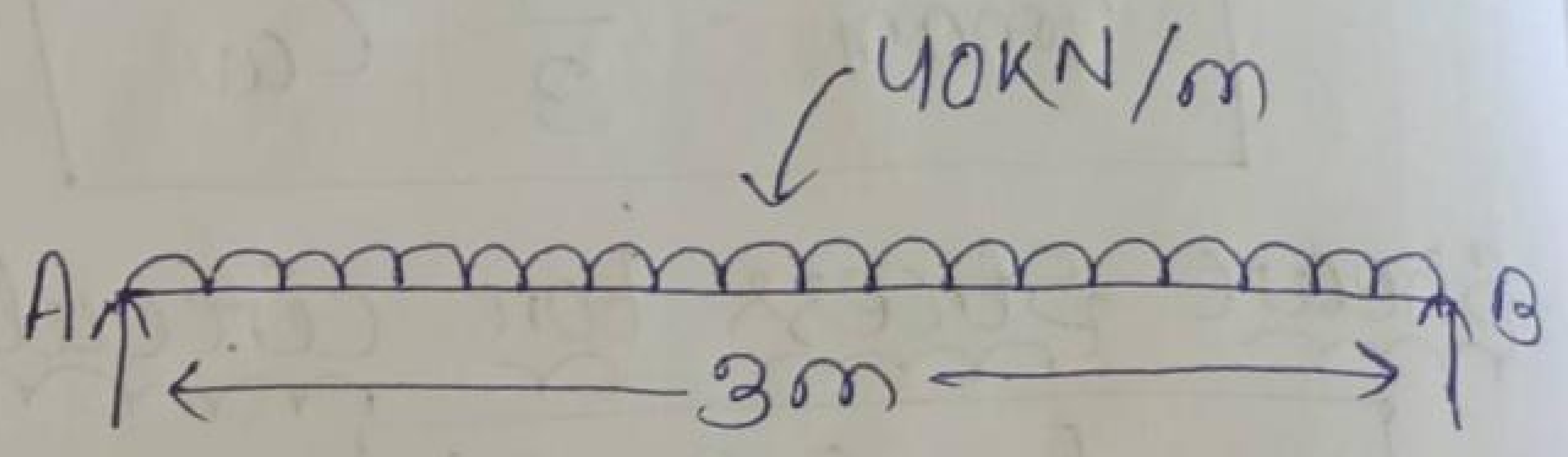
Ans

$b = 100\,mm$

$d = 250\,mm$

$L = 3\,m$

$W = 40\,KN/m$

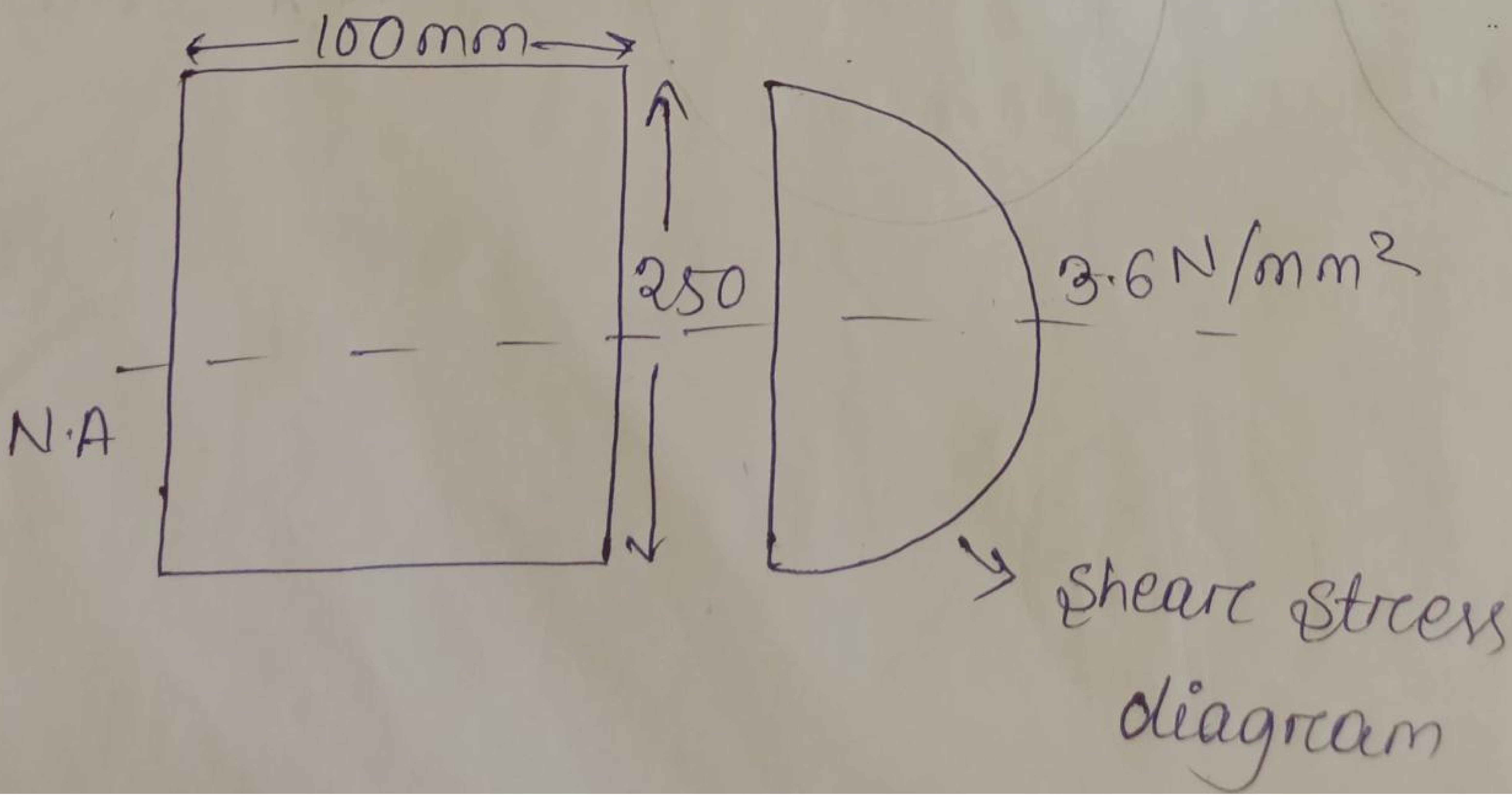


$$\text{max}^m \text{ Shear force} = \frac{W \times L}{2} = \frac{40 \times 3}{2} = 60\,KN$$

$$A = b \times d = 100 \times 250 = 25 \times 10^3\ mm^2$$

$$\tau_{max} = 1.5 \times \frac{F}{bd}$$

$$= \frac{1.5}{10} \times \frac{60 \times 10^3}{25 \times 10^3} = 3.6\,N/mm^2$$



Torque

Torsional moment:
It is a moment acting about the axis which is perpendicular to the plane of cross-section.

Assumption:

1. The material of the shaft is homogenous isotropic and perfectly elastic.
2. The material obey's Hooke's law and the stress remains within limit of proportionality.
3. The twisting couples acts in the transverse Planes only.
4. All radii remain straight after torsion.
5. Parallel planes normal to the axis do not warp or distort after torsion.

Torsional equation:-

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

T = Torsional moment
J = Polar moment of inertia.
τ = Shear stress
R = Radius of circular shaft
G = modulus of rigidity
θ = angle of twist
l = length of shaft

Polar moment of inertia:-

$$J = I_x + I_y$$

Polar sectional modulus:-

$$\frac{J}{R}$$

Torsional rigidity :-

The product of modulus of rigidity and Polar moment of inertia is known as Torsional rigidity (GJ).

Torsional moment of resistance :

The torque which can be carried by a given section of shaft for a given maximum value of shear stress is known as Torsional moment of resistance.

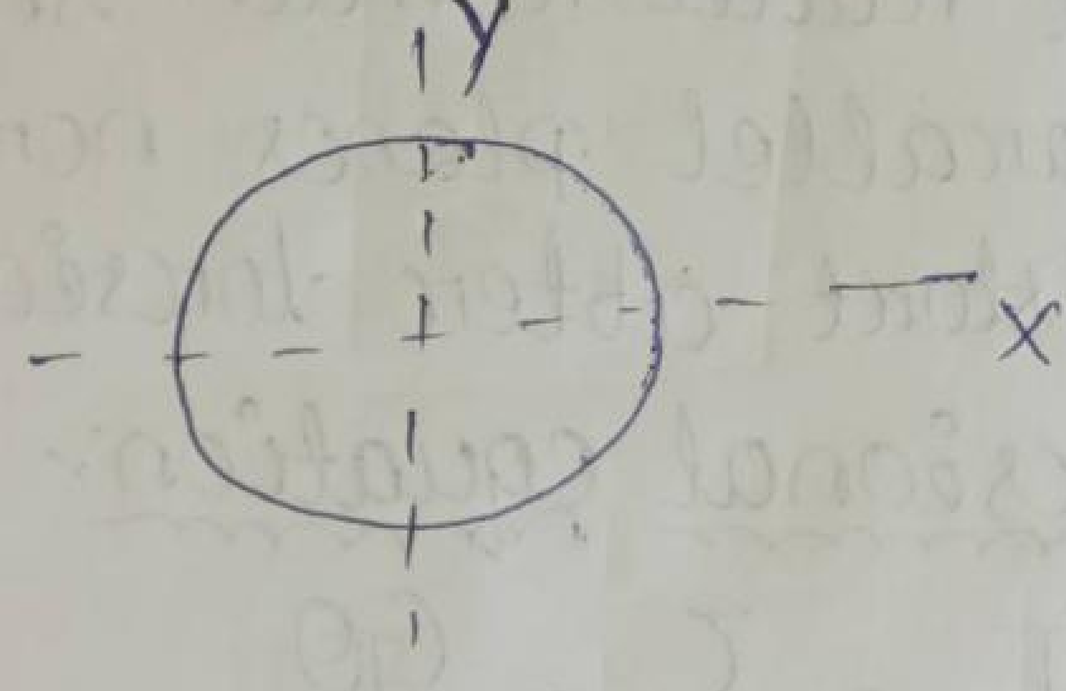
1. Solid Shaft :

$$J = I_x + I_y = 2I$$

$I_x = I_y = I$

$$= 2 \times \frac{\pi d^4}{64}$$

$$= \frac{\pi d^4}{32}$$



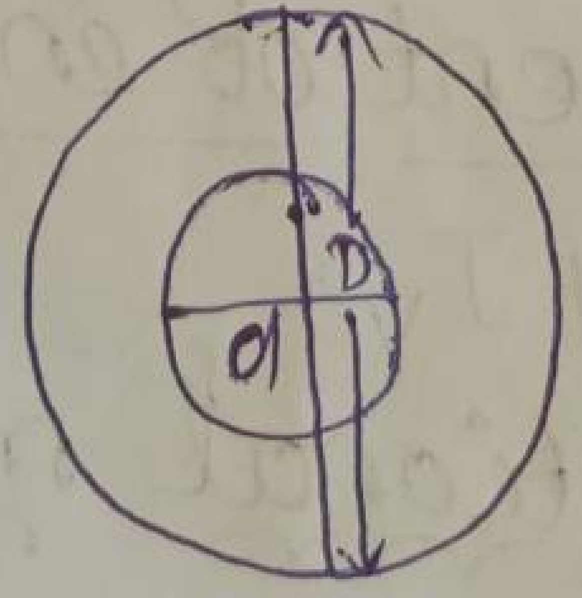
$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

$$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{\frac{d}{2}}$$

$$\Rightarrow \tau = \frac{16T}{\pi d^3} \longrightarrow \text{For solid shaft}$$

2. Hollow Shaft :

$$\frac{T}{J} = \frac{\tau}{R}$$



$$\Rightarrow \frac{T}{\frac{\pi(D^4 - d^4)}{32}} = \frac{\tau}{D/2}$$

$$\Rightarrow \tau = \frac{16TD}{\pi(D^4 - d^4)}$$

$$\boxed{\begin{aligned}\text{Solid shaft} &= \tau = \frac{16T}{\pi d^3} \\ \text{Hollow shaft} &= \tau = \frac{16TD}{\pi(D^4 - d^4)}\end{aligned}}$$

Power transmitted by torque :-

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

Work done = Force x distance

$$P = \frac{2\pi NT}{60} \text{ KW}$$

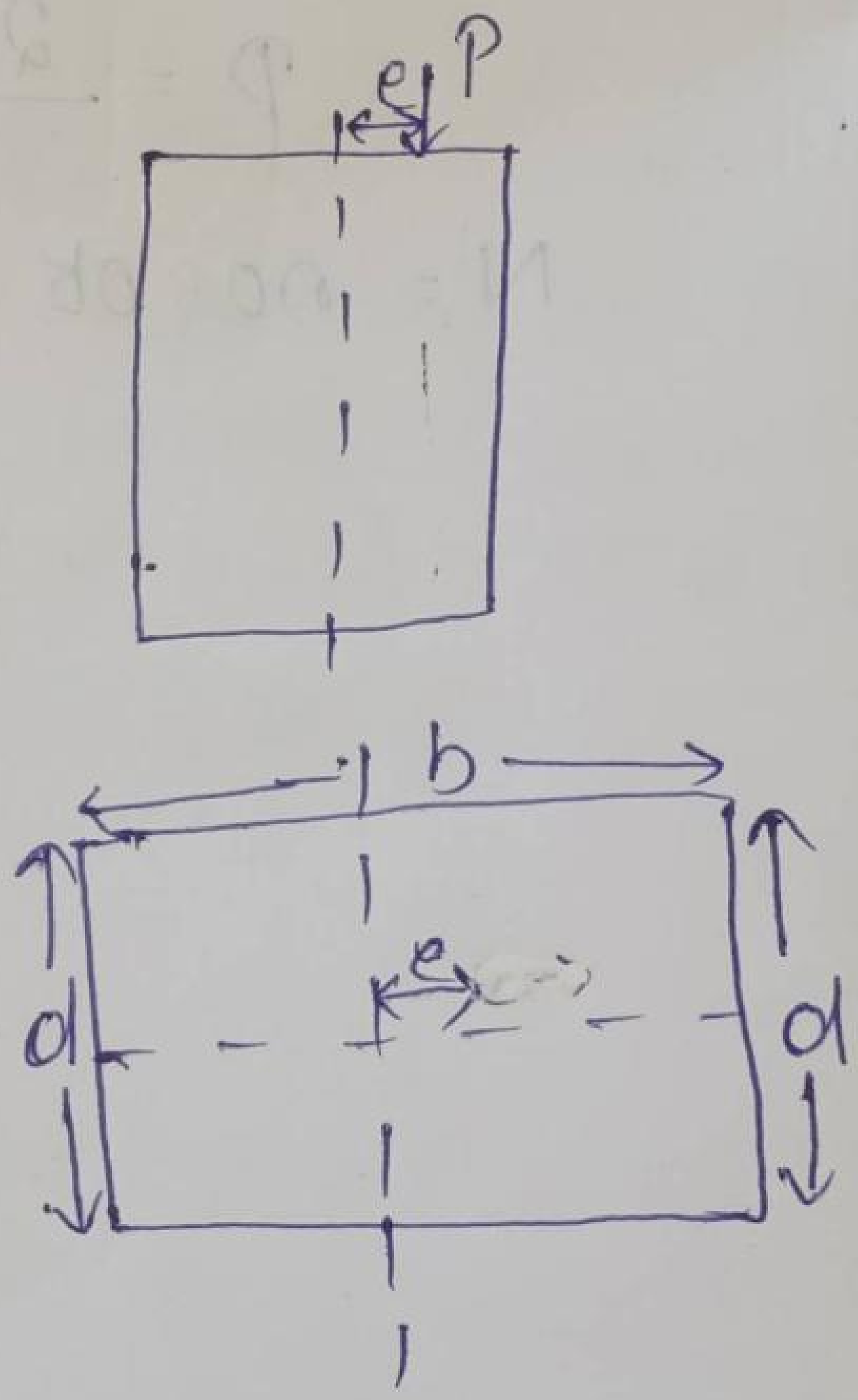
N = no. of revolution

Combined bending and direct stress

Symmetrical columns with eccentric loading about one axis

Consider a column ABCD subjected to an eccentric load about one axis (i.e. about Y-Y axis)

P = load acting on the column
e = Eccentricity of the load
b = width of the column section
d = Thickness of column



Plan

Area of column section

$$A = bd$$

moment of inertia of the column section about an axis through its centre of gravity and parallel to the axis about which the load is eccentric

$$I = \frac{db^3}{12}$$

Direct stress of column due to the load :-

$$\boxed{\sigma_0 = P/A}$$

moment due to load, $M = Pe$

Bending stress at any point of the column section at a distance y from y-y axis.

$$\sigma_b = \frac{My}{I} = \frac{M}{Z}$$

$$\sigma_b = \frac{M \times b/2}{\frac{db^3}{12}} = \frac{6 M \times \cancel{b}}{d \cancel{b}^3 b^2}$$

$$\boxed{\sigma_b = \frac{6 \times Pe}{db^2}}$$

Total stress at the extreme

$$\text{fibre} = \sigma_0 \pm \sigma_b$$

$$= \frac{P}{A} \pm \frac{M}{Z} \rightarrow \text{General equation}$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{6Pe}{db^2}}$$

$$\boxed{\sigma_{min} = \frac{P}{A} - \frac{6Pe}{db^2}}$$

Rectangular section for plane bisecting thickness.

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{Pe}{\left(\frac{\pi d^3}{32}\right)}}$$

$$\boxed{\sigma_{min} = \frac{P}{A} - \frac{Pe}{\frac{\pi d^3}{32}}}$$

Circular Section.

Rectangular Section

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

Circular Section

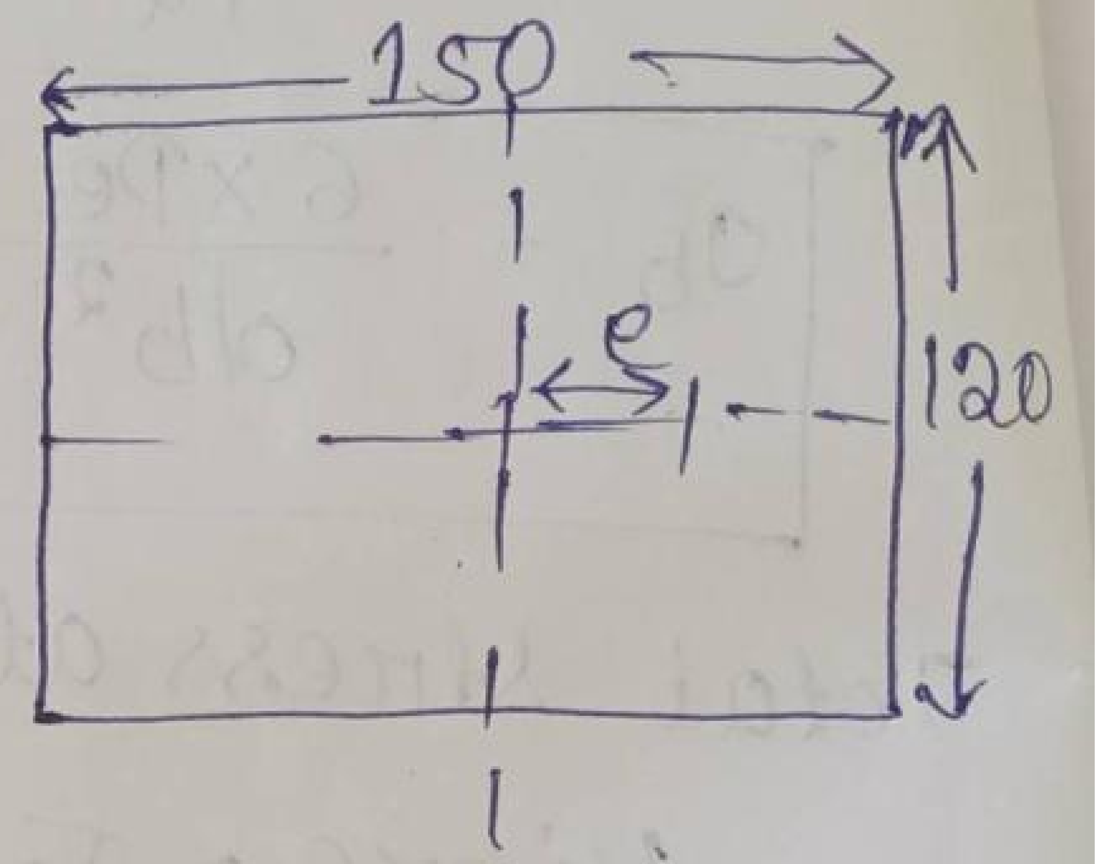
$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

Q. A Rectangulare strut is 150mm and 120mm thick. It carries a load of 180KN at an eccentricity of 10mm in a plane bisecting the thickness. Find the maximum and minimum stress in the section.

Ans

$A = b \times d = 150 \times 120$

$= 18,000 \, mm^2$



$P = 180 KN$

$b = 150 \, mm$

$d = 120 \, mm$

$e = 10 \, mm$

$$\sigma_{max} = \frac{P}{A} + \frac{6Pe}{b^2 d}$$

$$= \frac{180 \times 10^3}{150 \times 120} + \frac{6 \times 180 \times 10^3 \times 10}{150^2 \times 120}$$

$= 14 \, N/mm^2$ or $14 MPa$

$$\sigma_{min} = \frac{P}{A} - \frac{6Pe}{b^2 d}$$

$$= \frac{180 \times 10^3}{150 \times 120} - \frac{6 \times 180 \times 10^3 \times 10}{150^2 \times 120}$$

$= 6 \, N/mm^2 = 6 MPa$

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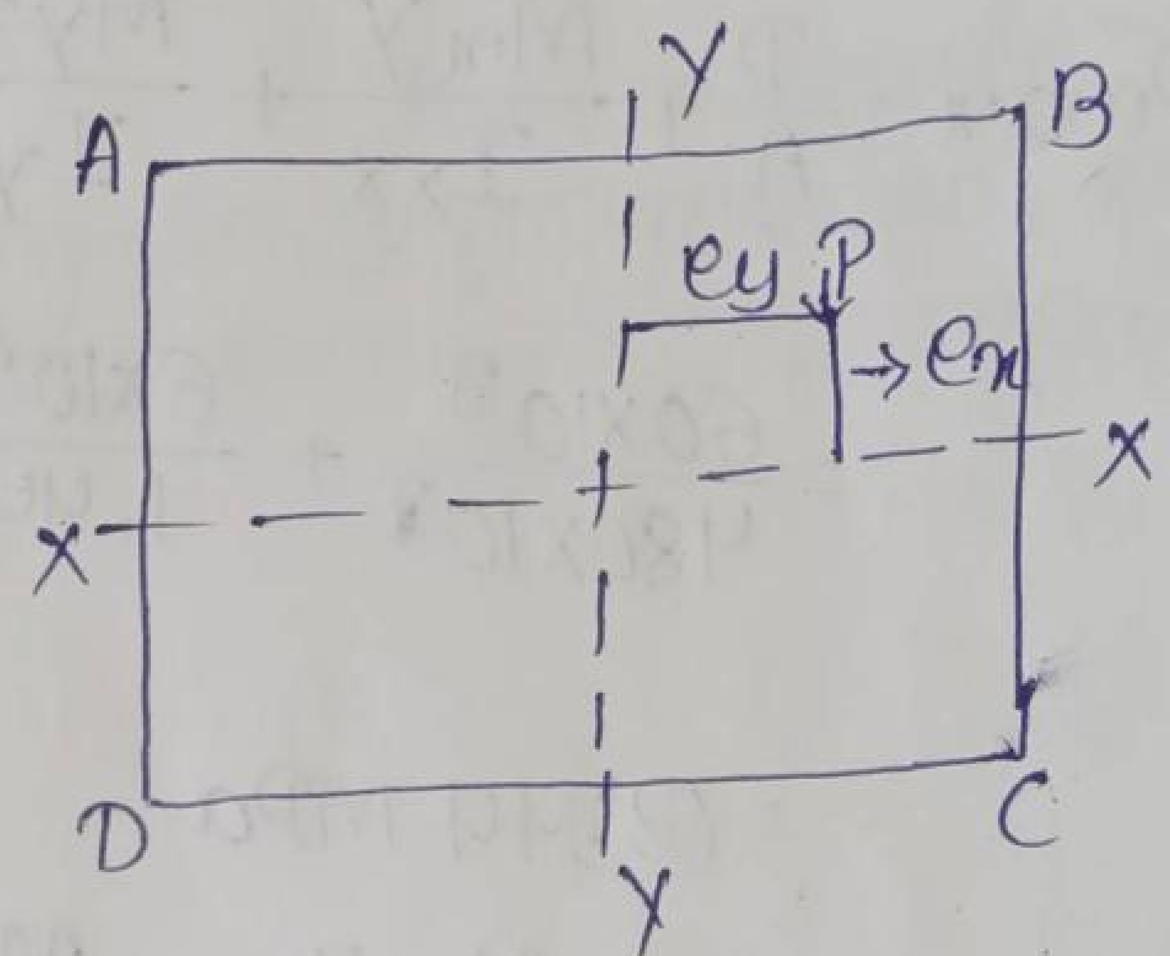
Symmetrical columns with eccentric loading about two axes

P = Load acting on the column

A = cross-sectional area of the column

e_x = Eccentricity of the load about x-x axis

e_y = Eccentricity of the load about y-y axis.



I_{xx} = moment of inertia about 'x' axis

I_{yy} = moment of inertia about 'y' axis

$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_{xx}} \pm \frac{M_y x}{I_{yy}}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

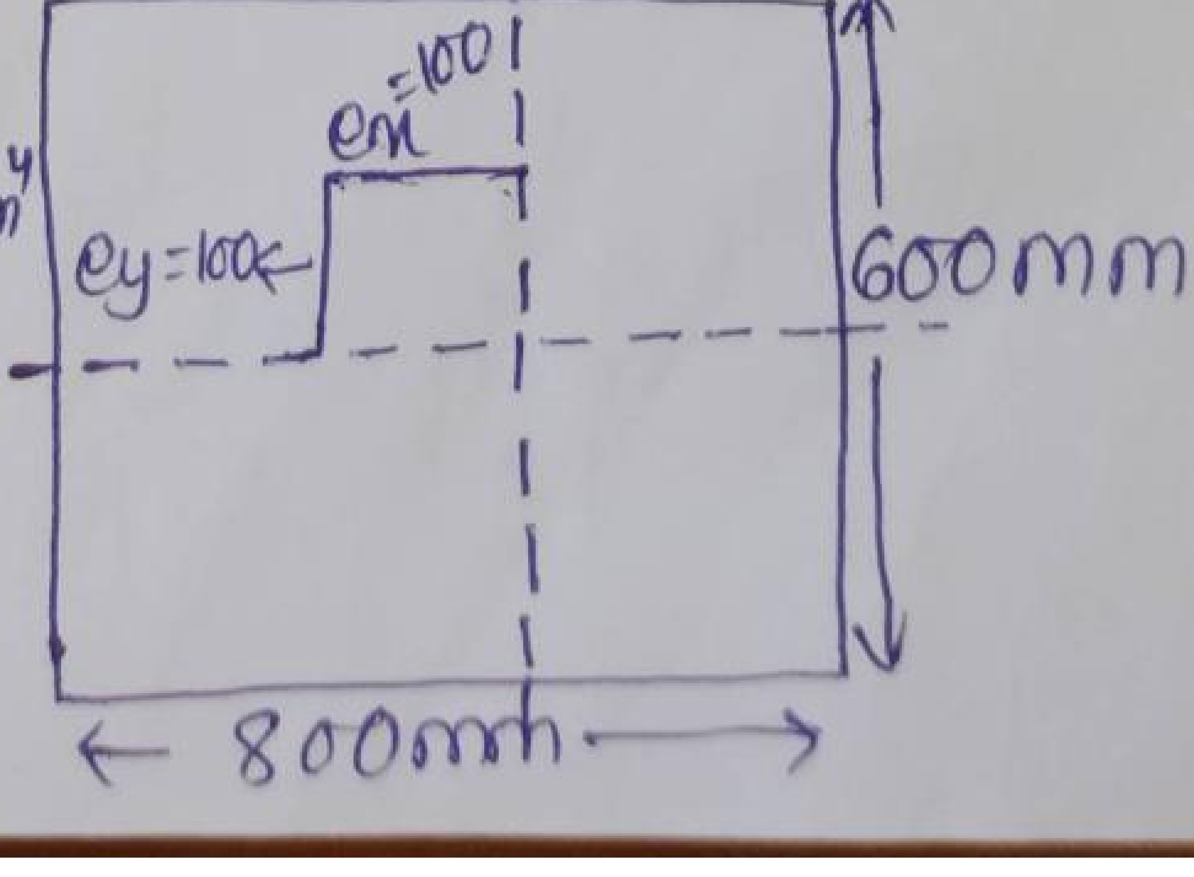
$$\sigma_{min} = \frac{P}{A} - \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$$

$M_x = P \times e_x$ $\quad y = d/2 \quad I_{xx} = bd^3/12$

$M_y = P \times e_y$ $\quad x = b/2 \quad I_{yy} = \frac{db^3}{12}$

Q.1 A column 800 mm × 600 mm is subjected to an eccentric load of 60KN as shown in figure. What is the maximum and minimum intensity of stress in the column.

Ans



$A = 600 \times 800 = 480 \times 10^3\ mm^2$

$I_{xx} = \frac{bd^3}{12} = \frac{800 \times 600^3}{12} = 1.44 \times 10^{10}\ mm^4$

$I_{yy} = \frac{db^3}{12} = \frac{600 \times 800^3}{12} = 2.56 \times 10^{10}\ mm^4$

$M_x = P \times e_x = 60 \times 10^3 \times 100 = 6 \times 10^6$ N-mm

$M_y = P \times e_y = 60 \times 10^3 \times 100 = 6 \times 10^6$ N-mm

$x = b/2 = \frac{800}{2} = 400$ mm

$y = d/2 = \frac{600}{2} = 300$ mm

$$\sigma_{max} = \frac{P}{A} + \frac{M_x Y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} + \frac{6 \times 10^6 \times 300}{1.44 \times 10^{10}} + \frac{6 \times 10^6 \times 400}{2.56 \times 10^{10}}$$

$$= 0.344 \text{ MPa}$$

$$\sigma_{min} = \frac{P}{A} - \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} - \frac{6 \times 10^6 \times 300}{1.44 \times 10^{10}} - \frac{6 \times 10^6 \times 400}{2.56 \times 10^{10}}$$

$$= 0.094 \text{ N/mm}^2$$

or

0.094 MPa

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Limits of eccentricity

When an eccentric load is acting on a column, it produces direct as well as bending stress. If the direct stress exceeds bending stress then there will be compressive stress will occur and if the bending stress exceeds direct stress then there will be tensile stress will occur.

As concrete is weak in tension and strong in compression, it is advisable to have less or zero tension. The condition for zero tension is:-

$$\sigma_{Direct} \geq \sigma_{bending}$$

$$P/A \geq \frac{Pe}{I/y}$$

$$\boxed{e \leq Z/A}$$

$Z = \frac{I}{y}$ → section modulus

(a) limit of eccentricity for a rectangular section:-

$$e \leq \frac{Z}{A}$$

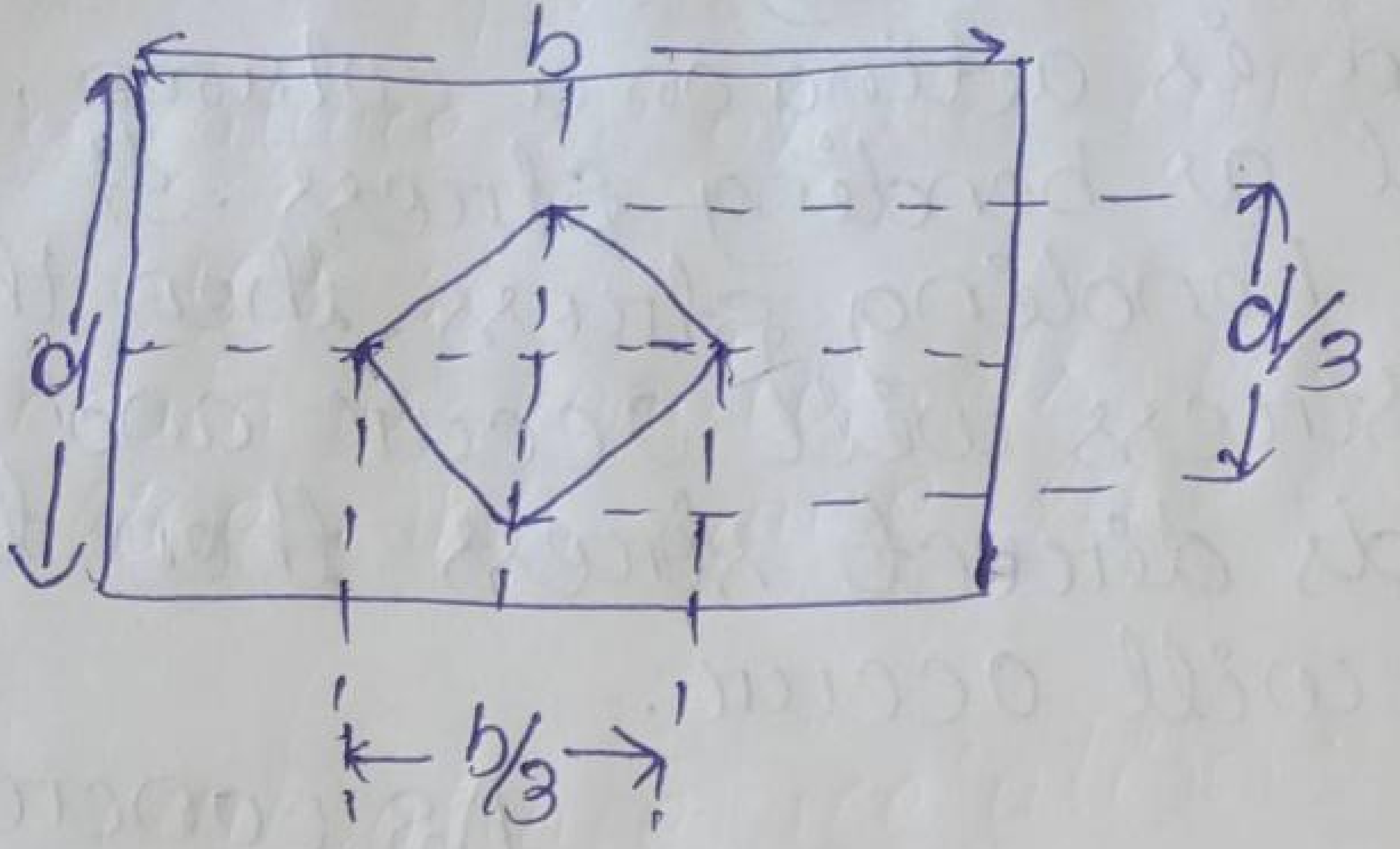
$$Z = \frac{I}{y}$$

$$I = \frac{bd^3}{12}, \quad y = d/2$$

$$Z = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$$

$$e \leq \frac{bd^2/6}{bd} \qquad A = bd$$

$$\boxed{e \leq d/6}$$



If the line of action of the load is within middle third as shown in figure, then the stress is compressive.

(b) limit of eccentricity of column section :-

$$e \leq \frac{Z}{A}$$

$$Z = I/y$$

$$Z = \frac{I}{y}$$

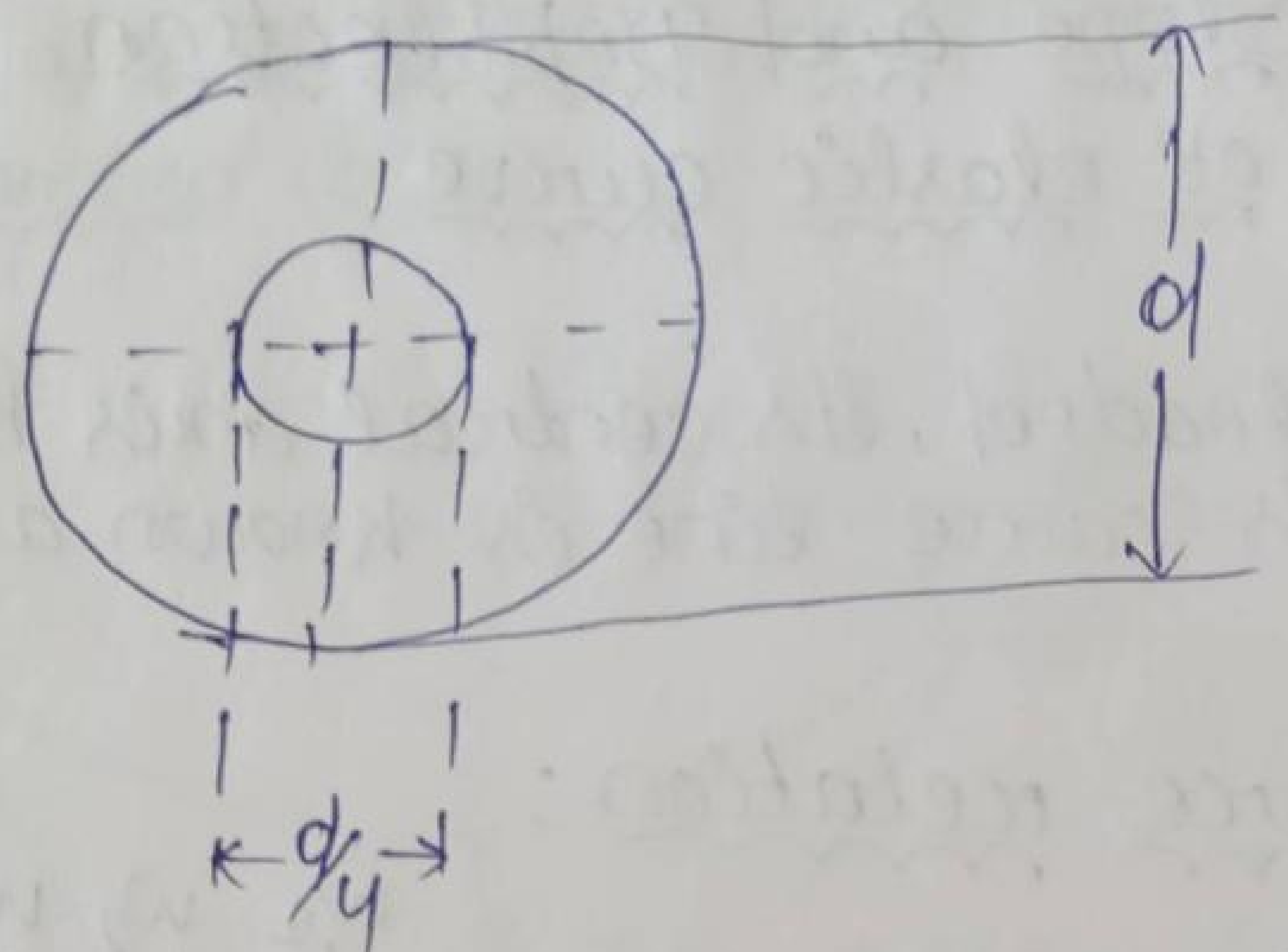
$$I = \frac{\pi d^4}{64} \quad y = d/2$$

$$Z = \frac{\pi d^4}{64} / d/2$$

$$= \frac{\pi d^3}{32}$$

$$e \leq \frac{\pi d^3}{32} / \frac{\pi}{4} d^2$$

$$\boxed{e \leq d/8}$$



Thus if the line of action of the load is within a circle of diameter equal to one-fourth of main circle, then the stress will be compressive.

(c) limit of eccentricity of hollow section

$$e \le \frac{Z}{A} \qquad Z = I/y$$

$$I = \frac{\pi(D^4 - d^4)}{64}$$

$$y = D/2$$

$$Z = \frac{\frac{\pi(D^4 - d^4)}{\cancel{64}\,32}}{D/\cancel{2}}$$

$$Z = \frac{\pi}{32D}(D^4 - d^4)$$

$$(D^4 - d^4) = (D^2 + d^2)(D^2 - d^2)$$

$$e \le \frac{\frac{\cancel{\pi}}{8\,\cancel{32}D}(D^4 - d^4)}{\frac{\cancel{\pi}}{\cancel{4}}(D^2 - D^4)}$$

$$\boxed{e \le \frac{D^2 + d^2}{8D}}$$

Slope and Deflection

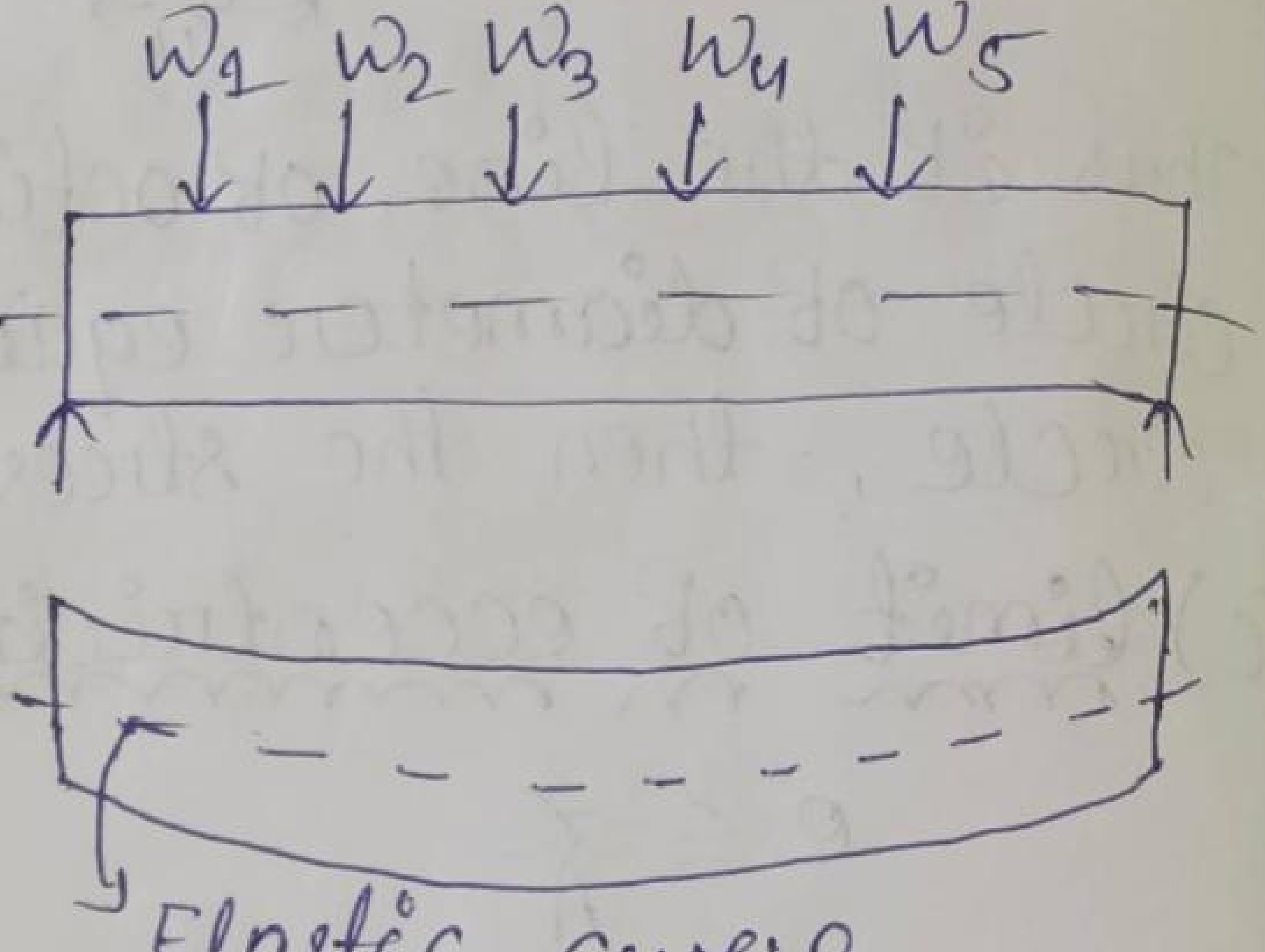
Shape and nature of elastic curve

Elastic curve:

When a beam is loaded, its central axis becomes a curved line. This curve line is known as Elastic curve.

Moment - curvature relation:

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$



Slope:

Slope is the angle formed by the tangent of the curve to the horizontal axis.

Deflection:

It is the translational movement of the beam from its original position is called deflection.

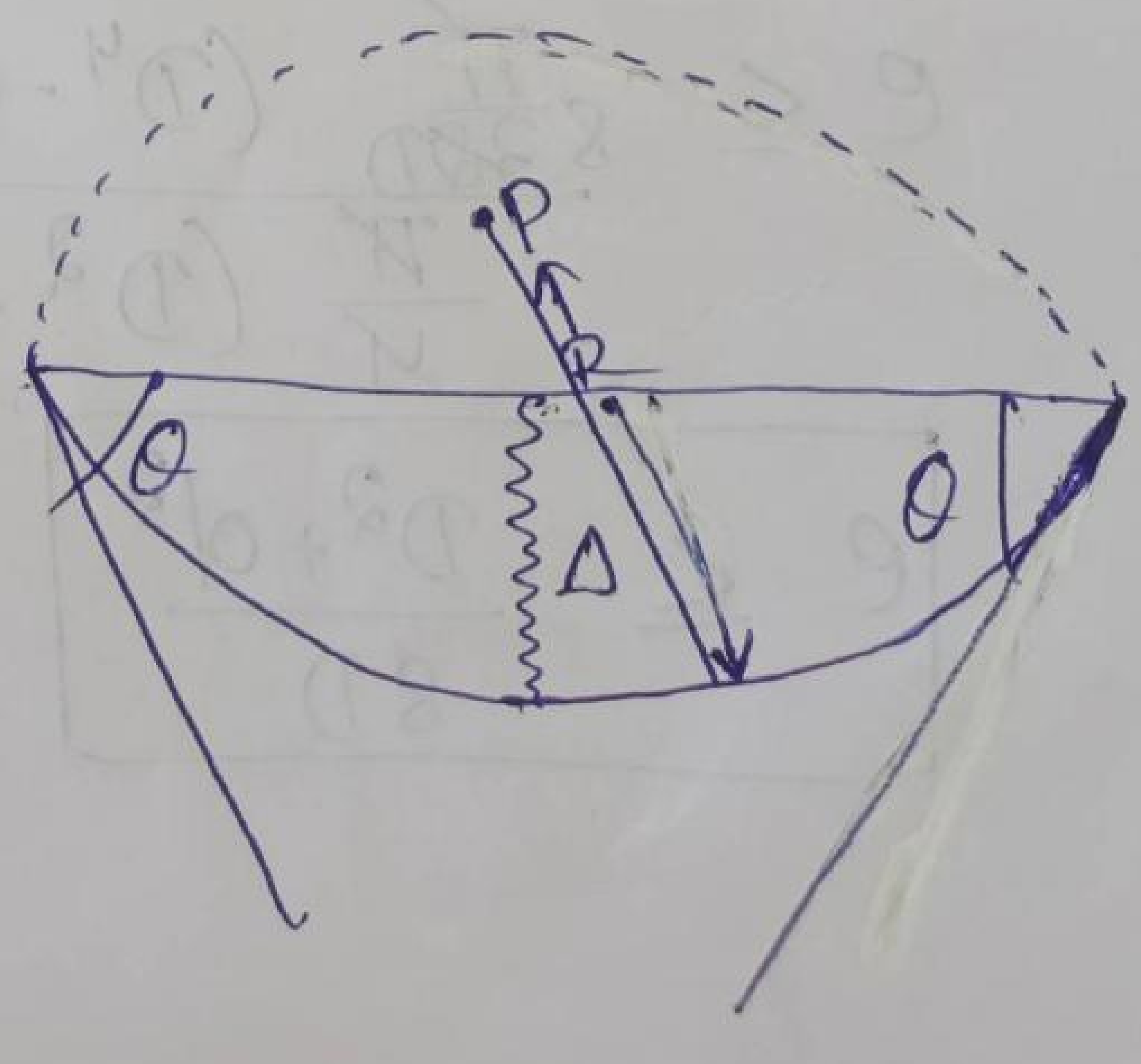
Radius of curvature:

Radius of curvature of a curve at a point is a measure of the radius of the circular arc.

θ = slope

Δ = deflection

R = Radius of curvature.



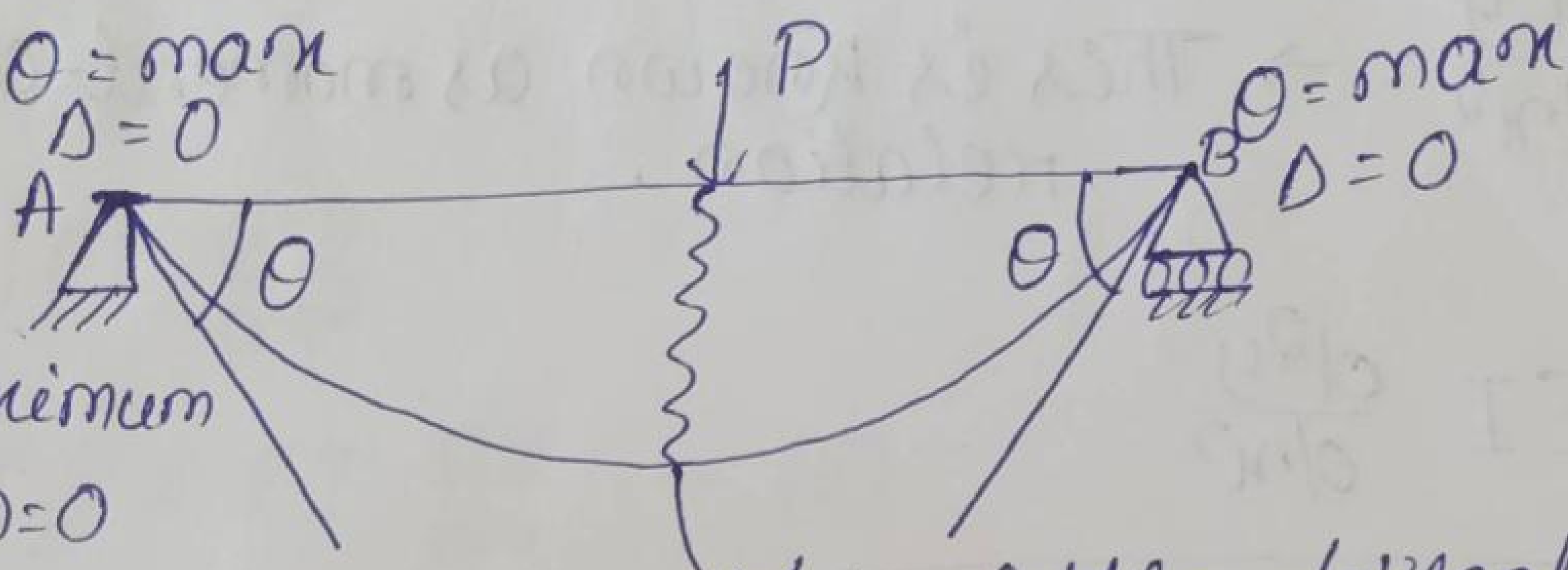
(Horizontal direction slope in maximum)

($M = max^m$)

Shape of elastic curve for different beam :

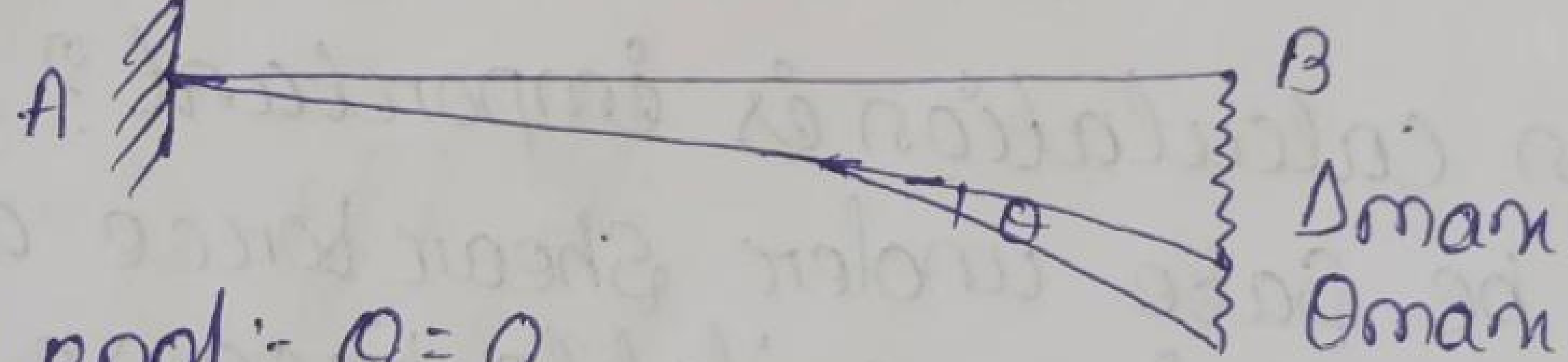
1. Simply supported beam

At ends
slope = maximum
Deflection = 0



→ In middle deflection is maximum slope is zero.

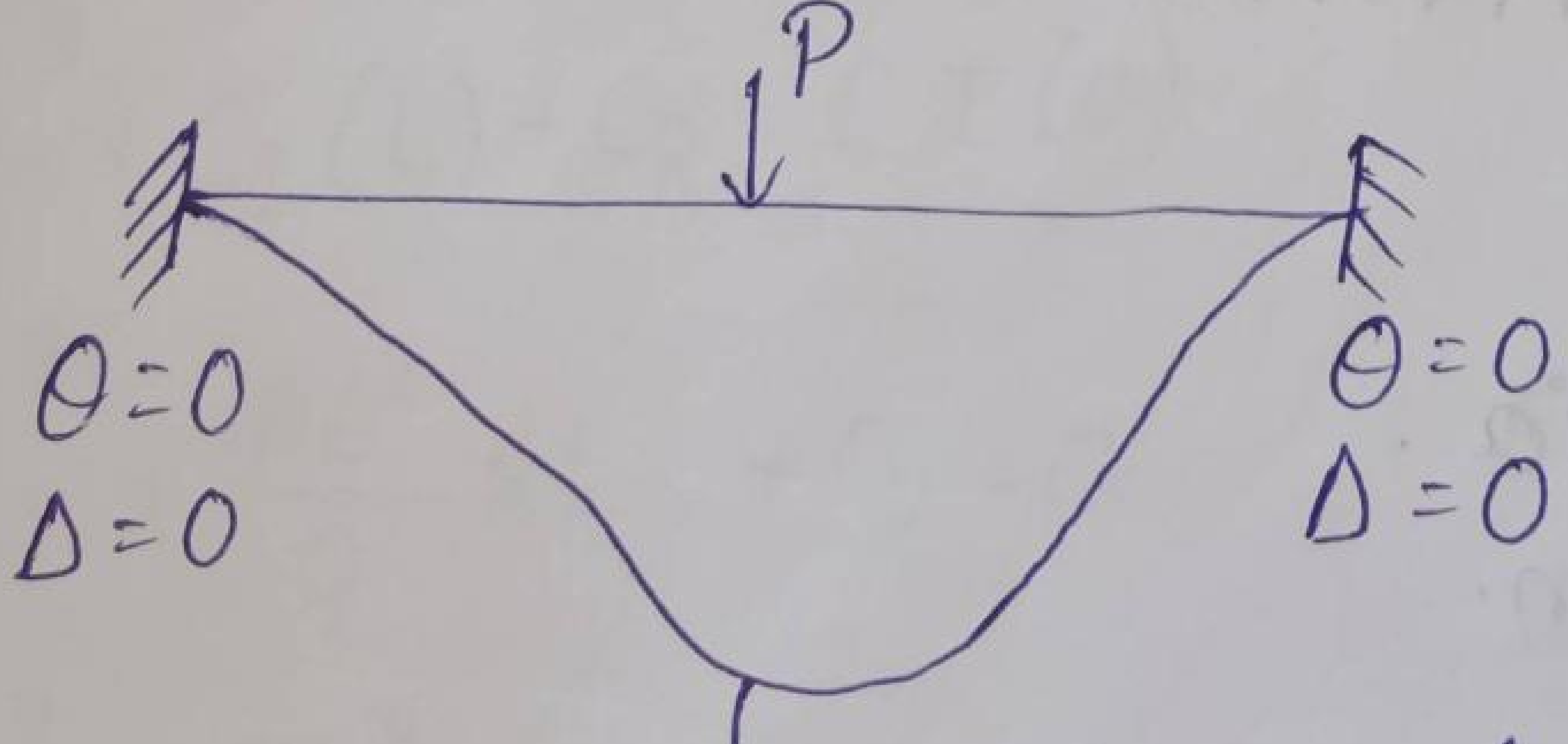
2. Cantilever beam :



Fixed end :- $\theta = 0$
$\Delta = 0$

Free end :- slope is maximum
deflection is maximum.

3. Fixed beam :



→ maximum deflection at midpoint.

Relation between slope, deflection and Radius of Curvature:

Imp

$$M = EI \frac{d^2y}{dx^2}$$ → This is known as moment-curvature relation.

$$\int M = \int EI \frac{d^2y}{dx^2}$$

$$\boxed{\int \frac{M}{EI} dx + C_1 = \frac{dy}{dx}}$$ → slope

$$\boxed{\int \left[\int \frac{M}{EI} dx \right] dx + C_1 x + C_2 = y}$$

→ Deflection

Q. Why deflection calculation is important?

Ans: A beam may be safe under shear force and bending moment but it is unsuitable because its deflection under the calculated safe load is excessive. So to control it, the deflection check is important.

Deflection is caused by loads, temperature, construction error, settlement.

Deflection of beam subjected to loads depend upon:

→ loading pattern.

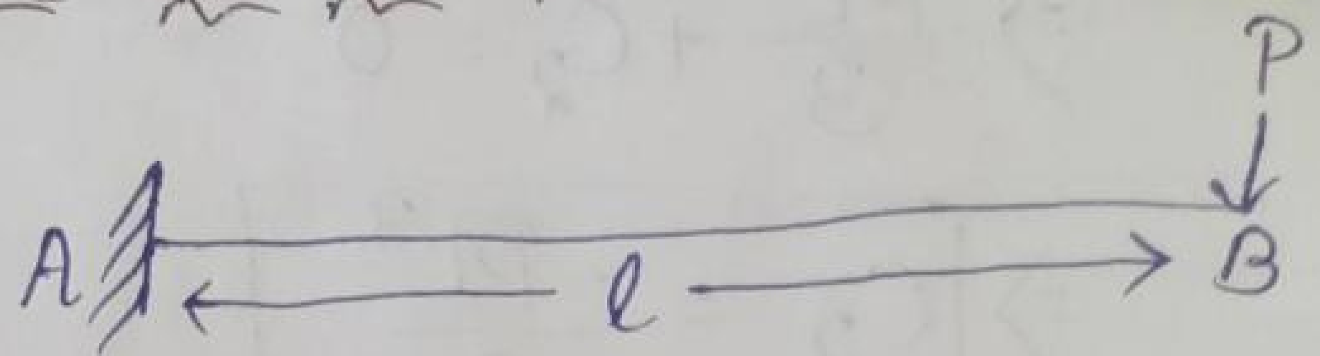
→ Elastic modulus

→ moment of inertia.

→ support condition.

→ length of beam.

Slope and deflection of cantilever due to point load:



$M = -P.x$

$M = EI \frac{d^2y}{dx^2}$

$\int -P.x = \int EI \frac{d^2y}{dx^2}$

$-\frac{Px^2}{2} + C_1 = EI \frac{dy}{dx}$ — (i)

$\int -\frac{Px^2}{2} + \int C_1 = \int EI \frac{dy}{dx}$

$-\frac{Px^3}{6} + C_1 x + C_2 = EI(y)$ — (ii)

At, $x = L$, $\Delta = 0$

At, $x = L$, $\Delta = 0$

(i) $-\frac{Px^2}{2} + C_1 = EI \frac{dy}{dx}$

$\Rightarrow -\frac{PL^2}{2} + C_1 = EI(0)$

$\Rightarrow C_1 = \frac{PL^2}{2}$

(ii) $-\frac{Px^3}{6} + C_1 x + C_2 = EI(y)$

$\Rightarrow -\frac{PL^3}{6} + C_1(L) + C_2 = EI(0)$

$\Rightarrow \frac{-PL^3}{6} + \frac{Pl^2}{2} \times L + C_2 = 0$

$\Rightarrow \frac{-PL^3}{6} + \frac{PL^3}{2} + C_2 = 0$

$\Rightarrow \frac{-PL^3 + 3PL^3}{6} + C_2 = 0$

$$\Rightarrow \frac{PL^3}{3} + C_2 = 0$$

$$\Rightarrow \boxed{C_2 = \frac{-PL^3}{3}}$$

maximum

- $$\frac{-Px^2}{2} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$(x=0)$

$$\Rightarrow \frac{-P \times 0^2}{2} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\Rightarrow C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{max} = \frac{PL^2}{2EI}}$$

- $$\frac{-Px^3}{6} + C_1 x + C_2 = EI(y)$$

$(x=0)$

$$\Rightarrow -\frac{P \times 0^3}{6} + C_1 \times 0 + C_2 = EI(y)$$

$$\Rightarrow C_2 = EI(y)$$

$$\Rightarrow EI(y) = C_2$$

$$\Rightarrow EI(y) = \frac{-PL^3}{3}$$

$$\Rightarrow \boxed{(y)_{max} = \frac{-PL^3}{3EI}}$$

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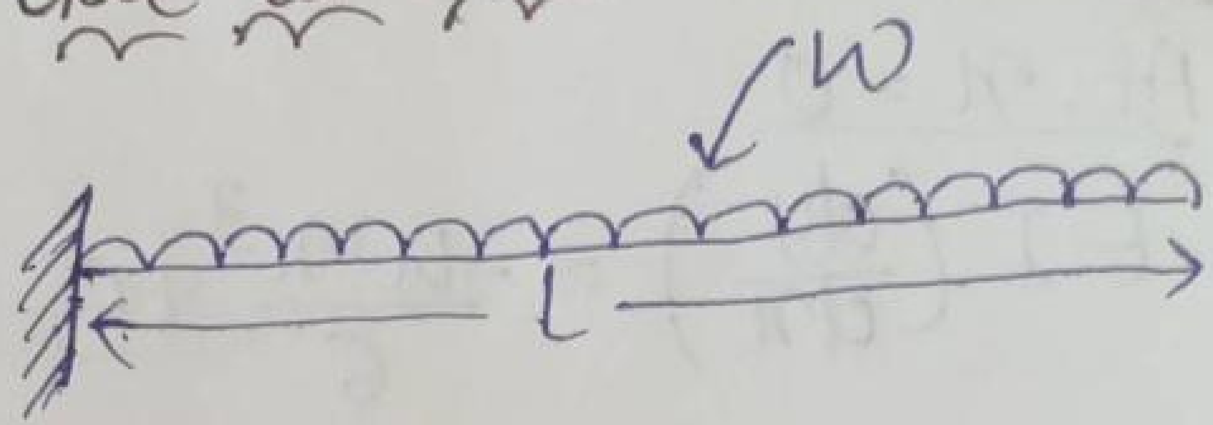
Slope and deflection of cantilever due to UDL :



$$M = -\frac{Wx^2}{2}$$

$$\int EI\frac{d^2y}{dx^2} = \int -\frac{Wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{-Wx^3}{6} + C_1 \quad -(i)$$

$$\int EI\frac{dy}{dx} = \int -\frac{Wx^3}{6} + \int C_1$$

$$EI(y) = \frac{-Wx^4}{24} + C_1x + C_2 \quad -(ii)$$

At $x = L$, $\theta = 0$
At $x = L$, $\Delta = 0$

$$EI\left(\frac{dy}{dx}\right) = -\frac{Wx^3}{6} + C_1$$

$$\Rightarrow EI(0) = \frac{-WL^3}{6} + C_1$$

$$\Rightarrow 0 = \frac{-WL^3}{6} + C_1$$

$$\Rightarrow \boxed{C_1 = \frac{WL^3}{6}}$$

$$EI(y) = \frac{-Wx^4}{24} + C_1x + C_2$$

$$\Rightarrow 0 = \frac{-WL^4}{24} + \frac{WL^3}{6} \times L + C_2$$

$$\Rightarrow 0 = \frac{-WL^4 + 4WL^4}{24} + C_2$$

$$\Rightarrow \boxed{C_2 = \frac{-WL^4}{8}}$$

Maximum

At, $x = 0$

$$EI\left(\frac{dy}{dx}\right) = -\frac{wx^3}{6} + C_1$$

$$\Rightarrow EI\left(\frac{dy}{dx}\right) = \frac{wL^3}{6}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{wL^3}{6EI}}$$

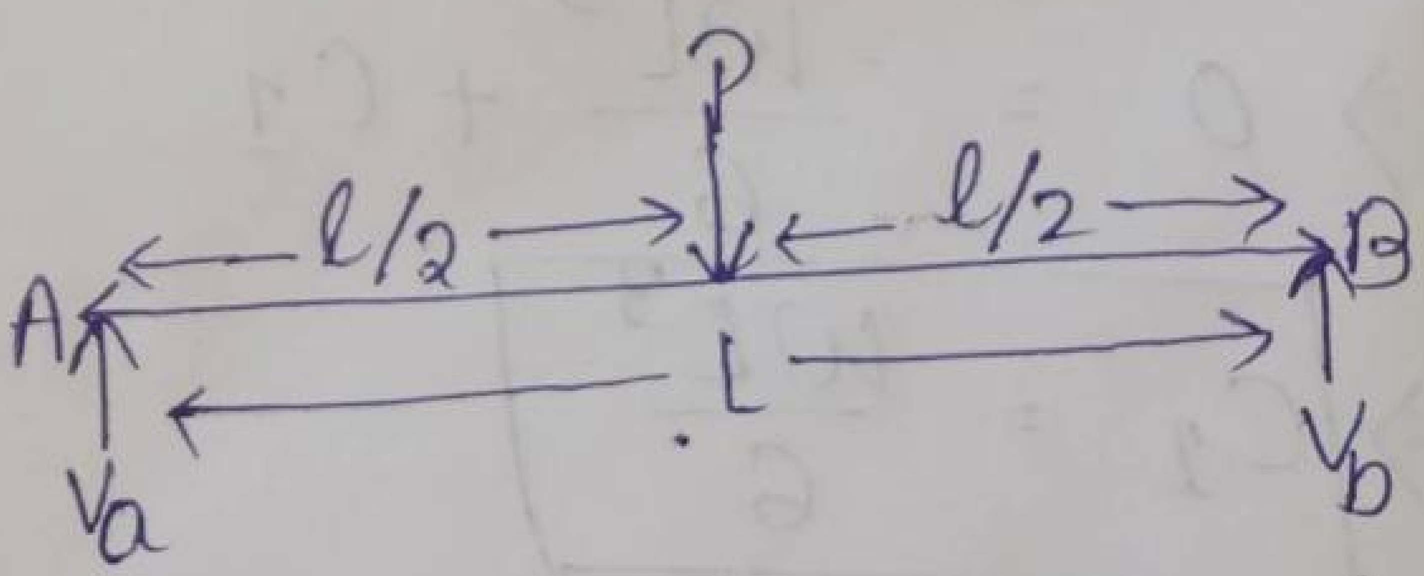
At, $x = 0$

$$EI(y) = -\frac{w\cancel{x^4}}{24} + C_1\cancel{x} + C_2$$

$$\Rightarrow EI(y) = -\frac{wL^4}{8}$$

$$\Rightarrow \boxed{(y)_{max} = \frac{-wL^4}{8EI}}$$

Slope and deflection of simply supported for point load:



$$\Sigma V = 0$$

$$\Rightarrow V_a + V_b = P$$

$$\Sigma M_B = 0$$

$$V_a \times L - P \times L/2 = 0$$

$$\Rightarrow \boxed{V_a = P/2} \qquad \boxed{V_b = P/2}$$

$$M = P/2\, x$$

$$\int \frac{P}{2} x = \int EI \frac{d^2y}{dx^2}$$

$$\frac{P}{2} \cdot \frac{x^2}{2} + C_1 = EI\left(\frac{dy}{dx}\right) \quad -(i) \rightarrow \text{Slope}$$

$$\int \frac{Px^2}{4} + \int C_1 = \int EI\left(\frac{dy}{dx}\right)$$

$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y) \quad -(ii) \rightarrow \text{Deflection}$$

At, $x = 0$, $y = 0$

At, $x = L/2$, $\frac{dy}{dx} = 0$

$$EI\left(\frac{dy}{dx}\right) = \frac{Px^2}{4} + C_1$$

$$0 = \frac{P(L/2)^2}{4} + C_1$$

$$\boxed{C_1 = \frac{-PL^2}{16}}$$

$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y)$$

$$0 + 0 + C_2 = EI(0)$$

$$\boxed{C_2 = 0}$$

maximum

$$EI\frac{dy}{dx} = \frac{Px^2}{4} + C_1$$

At, $x = 0$

$$EI\left(\frac{dy}{dx}\right) = \frac{Px^2}{4} + C_1$$

$$EI\left(\frac{dy}{dx}\right) = 0 + C_1$$

$$\boxed{\left(\frac{dy}{dx}\right)_{max} = \frac{-PL^2}{16EI}}$$

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At, $x = L/2$

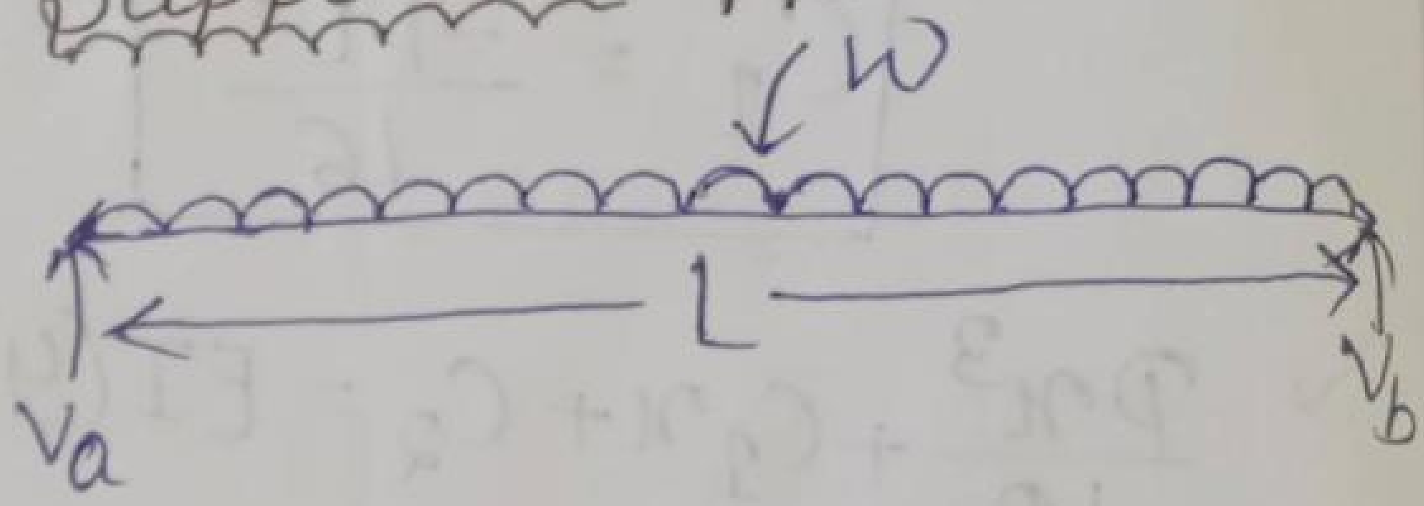
$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y)$$

$$\Rightarrow \frac{P(L/2)^3}{12} + \frac{-PL^2}{16} \times \frac{L}{2} + 0 = EI(y)$$

$$\Rightarrow \frac{PL^3}{96} - \frac{PL^3}{32} = EI(y)$$

$$\Rightarrow \boxed{(y)_{max} = \frac{-PL^3}{48EI}}$$

Slope and deflection of simply supported for UDL:



$\Sigma V = 0$

$V_a + V_b = wL$

$\Sigma M_A = 0$

$V_a \times L - w \times L \times \frac{L}{2} = 0$

$$\boxed{V_a = \frac{wL}{2}, \quad V_b = \frac{wL}{2}}$$

$$M = \frac{wL}{2}x - \frac{wx^2}{2}$$

$$\int \frac{wL}{2}x - \int \frac{wx^2}{2} = \int EI\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{wL}{2} \times \frac{x^2}{2} - \frac{wx^3}{6} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\int \frac{wLx^2}{4} - \int \frac{wx^3}{6} + \int C_1 = \int EI\left(\frac{dy}{dx}\right)$$

$$\frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2 = EI(y)$$

At, $x=0, y=0$

$x=L/2, \frac{dy}{dx}=0$

$$\frac{wL \times (L/2)^2}{4} - \frac{w(L/2)^3}{6} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\frac{wL^3}{16} - \frac{wL^3}{48} + C_1 = EI(0)$$

$$\frac{2wL^3}{48} + C_1 = 0$$

$$\boxed{C_1 = \frac{-wL^3}{24}}$$

$$\frac{wLx^3}{12} - \frac{wx^4}{24} + C_1x + C_2 = EI(y)$$

$$\boxed{C_2 = 0}$$

<u>maximum</u>

At, $x=0$

$$EI\left(\frac{dy}{dx}\right) = \frac{wLx^2}{4} - \frac{wLx^3}{6} + C_1$$

$$EI\left(\frac{dy}{dx}\right) = \frac{-wL^3}{24}$$

$$\boxed{\left(\frac{dy}{dx}\right)_{max} = \frac{-wL^3}{24EI}}$$

At, $x = L/2$

$$EI(y) = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$$

$$EI(y) = \frac{wL \times (L/2)^3}{12} - \frac{w \times (L/2)^4}{24} + C_1(L/2) + 0$$

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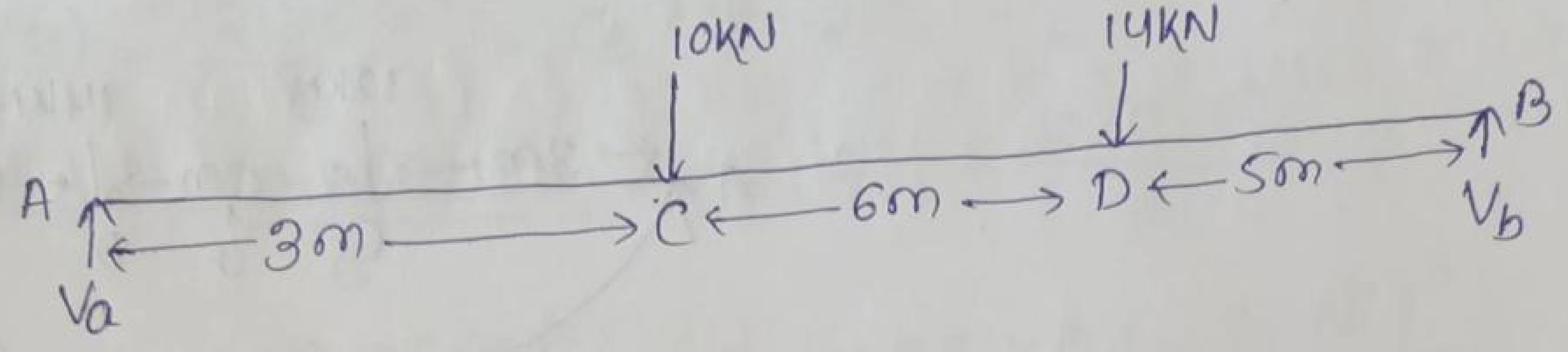
$$EI(y) = \frac{WL^4}{96} - \frac{WL^4}{384} - \frac{WL^4}{48}$$

$$EI(y) = \frac{-5WL^4}{384}$$

$$\boxed{y = \frac{-5WL^4}{384EI}}$$

SL. NO	slope	deflection
cantilever Point load	$\frac{PL^2}{2EI}$	$\frac{PL^3}{3EI}$
cantilever U.D.L	$\frac{WL^3}{6EI}$	$\frac{WL^4}{8EI}$
Simply supported Point load	$\frac{PL^2}{16EI}$	$\frac{PL^3}{48EI}$
Simply supported (UDL)	$\frac{WL^3}{24EI}$	$\frac{5WL^4}{384EI}$

Macaulay's method



Ans $\Sigma V = 0$

$\Rightarrow V_a + V_b = 24\,kN$

$\Sigma M_B = 0$

$\Rightarrow V_a \times 14 - 10\times 11 - 14\times 5 = 0$

$\Rightarrow V_a \times 14 = 180$

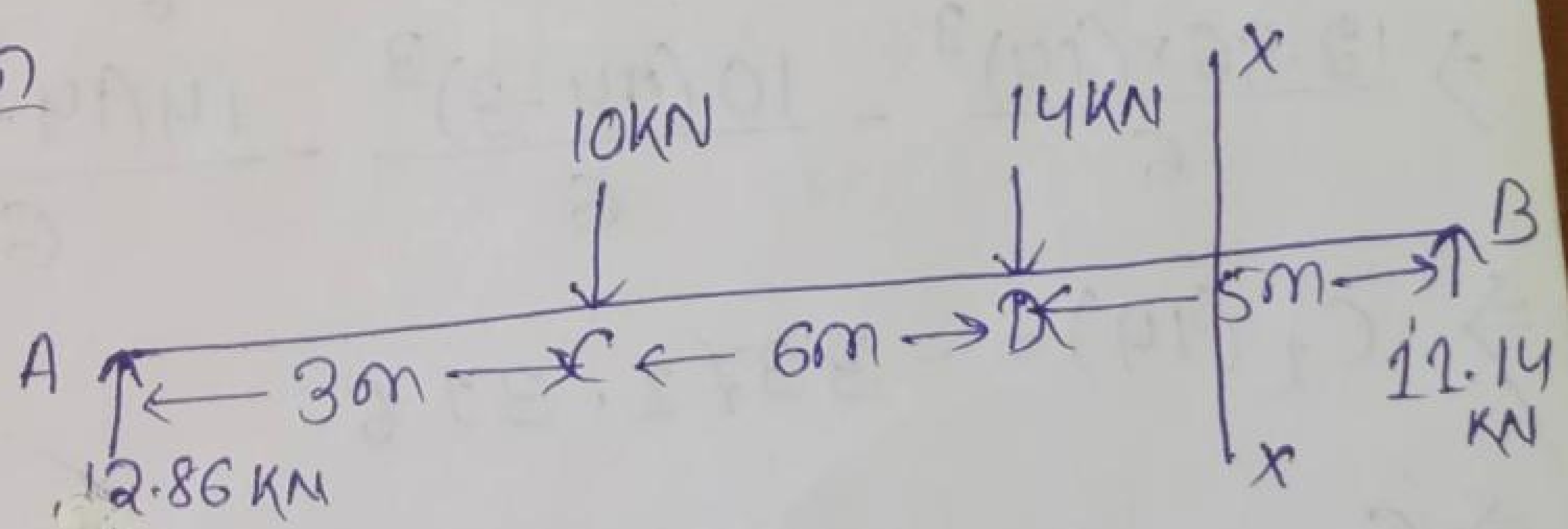
$\Rightarrow V_a = \frac{180}{14}$

$\Rightarrow V_a = 12.86\,kN$.

$V_b = 24 - 12.86 = 11.14\,kN$

Moment curvature relation

$$M = EI\frac{d^2y}{dx^2}$$



$$M = 12.86x - 10(x-3) - 14(x-9)$$

$$\Rightarrow 12.86x - 10(x-3) - 14(x-9) = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \int 12.86x - \int 10(x-3) - \int 14(x-9) = \int EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{12.86x^2}{2} - \frac{10(x-3)^2}{2} - \frac{14(x-9)^2}{2} + C_1 = EI\frac{dy}{dx} \quad -(1)$$

$$\Rightarrow \int\frac{12.86x^2}{2} - \int\frac{10(x-3)^2}{2} - \int\frac{14(x-9)^2}{2} + \int C_1 = \int EI\frac{dy}{dx}$$

$$\Rightarrow \frac{12.86x^3}{6} - \frac{10(x-3)^3}{6} - \frac{14(x-9)^3}{3} + C_1x + C_2 = EI(y) \quad -(2$$

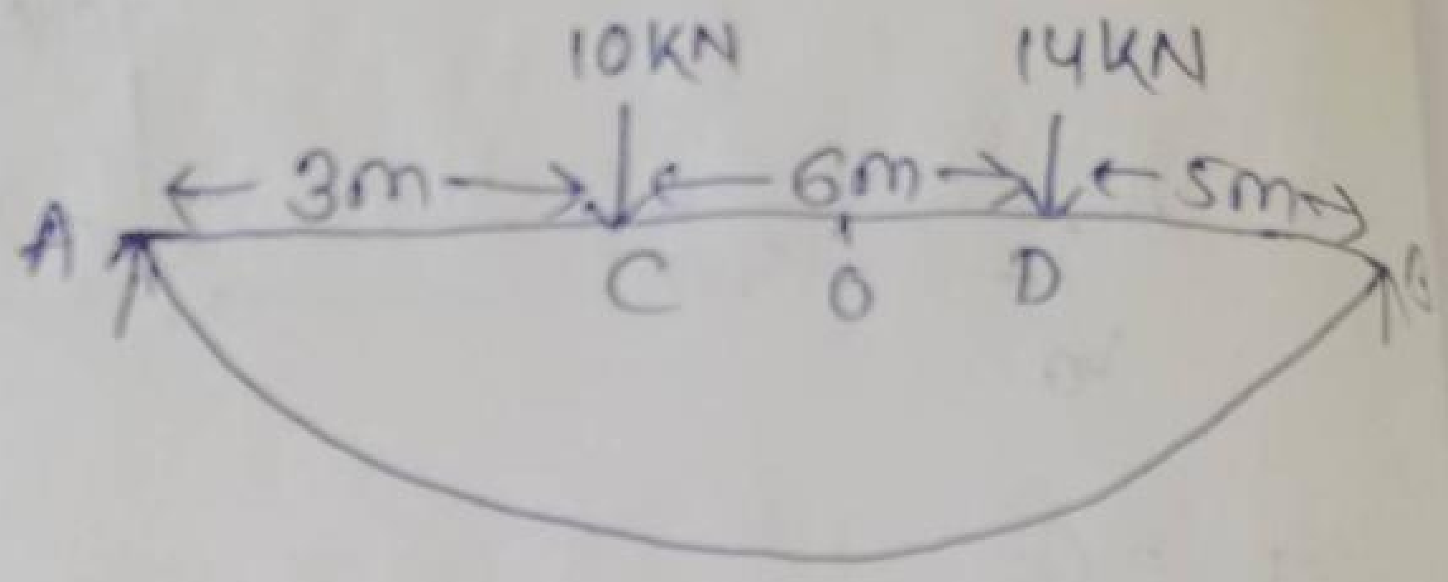
Now, the boundary condition

$x = 0$

$y = 0$

$x = L$

$y = 0$



$x = 0, \quad y = 0$

$$\frac{12.86x^3}{6} + C_1 x + C_2 = EI(y)$$

$$\Rightarrow 0 + 0 + C_2 = EI(0)$$

$$\Rightarrow C_2 = 0$$

$x = 14, \; y = 0$

$$\frac{12.86x^3}{6} - \frac{10(x-3)^3}{6} - \frac{14(x-9)^3}{6} + C_1 x + C_2 = EI(y)$$

$$\Rightarrow \frac{12.86 \times (14)^3}{6} - \frac{10(14-3)^3}{6} - \frac{14(14-9)^3}{6} + C_1 \times 14 + 0 = EI(0)$$

$$\Rightarrow C_1(14) = -3371.316$$

$$\Rightarrow C_1 = -240.81$$

Deflection at C, $x = 3m$

$$\frac{12.86x^3}{6} + C_1 x + \cancel{C_2}^{0} = EI(y)$$

$$\Rightarrow \frac{12.86(3)^3}{6} + (-240.81) \times 3 = EI(y)$$

$$\Rightarrow -664.56 = EI(y)$$

$$\Rightarrow \boxed{y_C = \frac{-664.56}{EI}}$$

Deflection at D $x = 9m$

$$EI(y) = \frac{12.86x^3}{6} + C_1 x + C_2 - \frac{10(x-3)^3}{6}$$

$$EI(y) = \frac{12.86(9)^3}{6} + C_1(9) - \frac{10(9-3)^3}{6}$$

$$= \frac{12.86(9)^3}{6} - 240.81 \times 9 - \frac{10(6)^3}{6}$$

$$= -964.8$$

$$\boxed{y_D = \frac{-964.8}{EI}}$$

Deflection at mid $x = 7m$

$$EI(y) = \frac{12.86x^3}{6} + C_1 x + C_2 - \frac{10(x-3)^3}{6}$$

$$EI(y) = \frac{12.86 \times (7)^3}{6} - 240.81(7) + 0 - \frac{10(7-3)^3}{6}$$

$$EI(y) = -1057.17$$

$$\boxed{y_D = \frac{-1057.17}{EI}}$$

Indeterminate beam

Statically determinate structure:

The structure having unknown force is less than or equal to available equilibrium equation is called statically determinate structure.

Statically indeterminate structure:

The structure having unknown force more than available equilibrium equation is called statically indeterminate structure.

Principle of superposition:

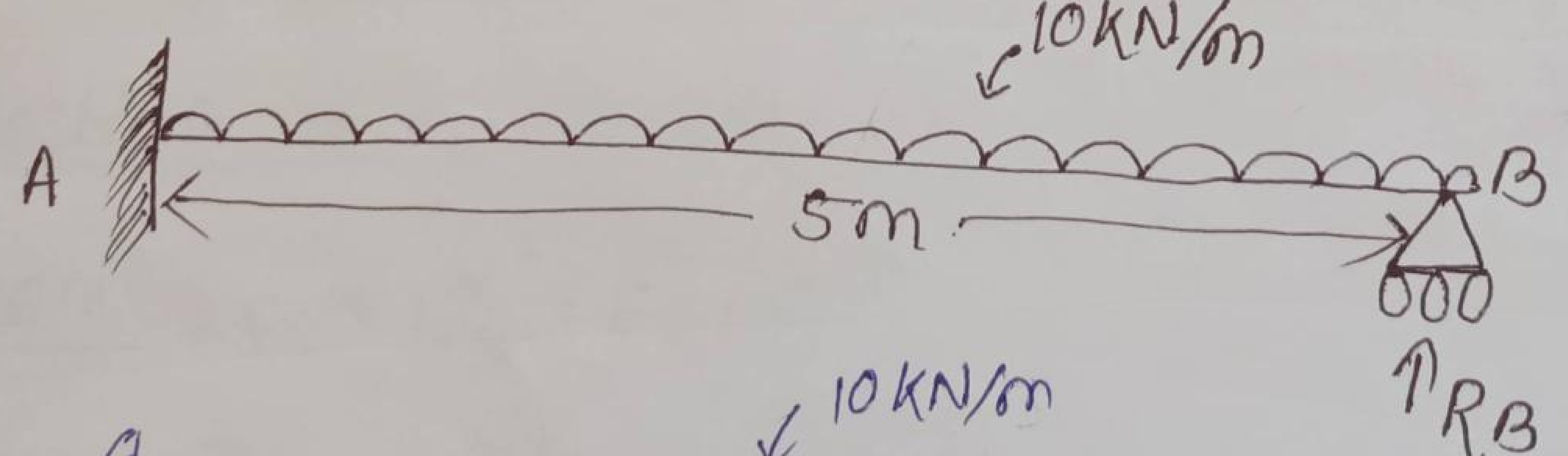
It simply states that on a linear elastic structure the combined effect of several load action simultaneously equal to algebric sum of the effect of each load acting individually.

Method of consistent deformation:

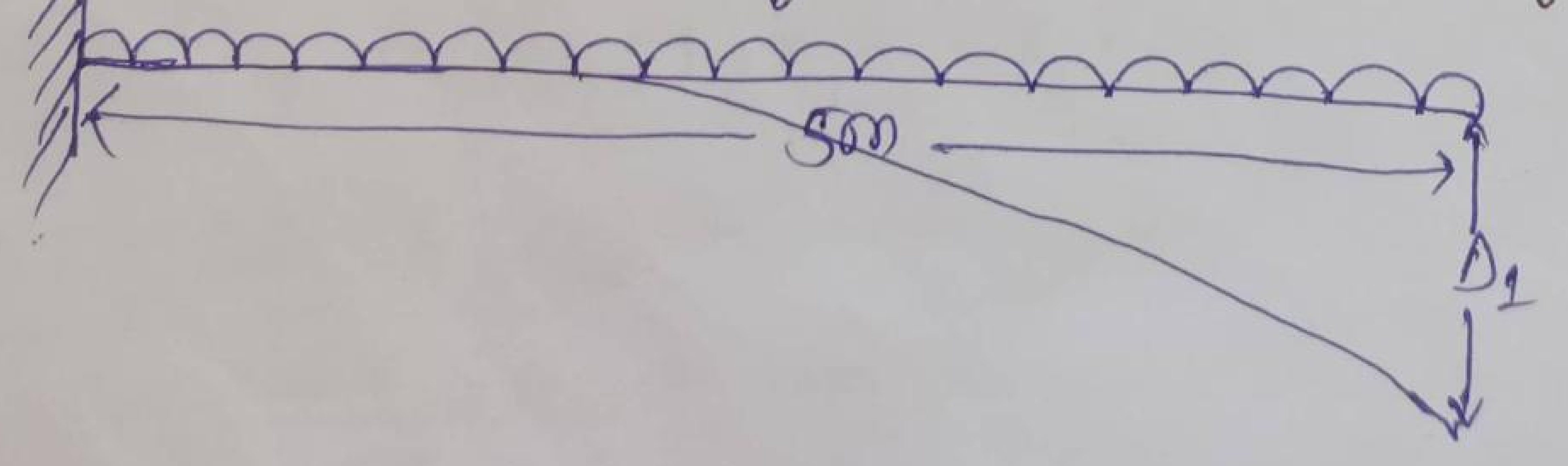
It is a force method which is used to analyse indeterminate beam with degree of indeterminancy 1 or 2.

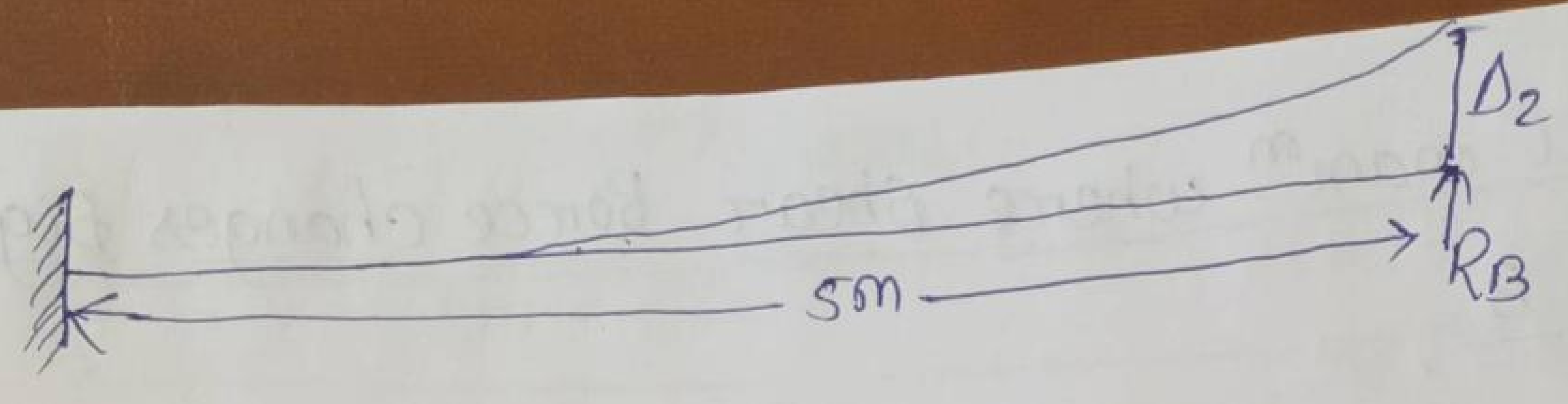
Propped cantilever beam:

(1)



Ans





$$\Delta_1 - \Delta_2 = 0$$

$$\Delta_1 = \Delta_2$$

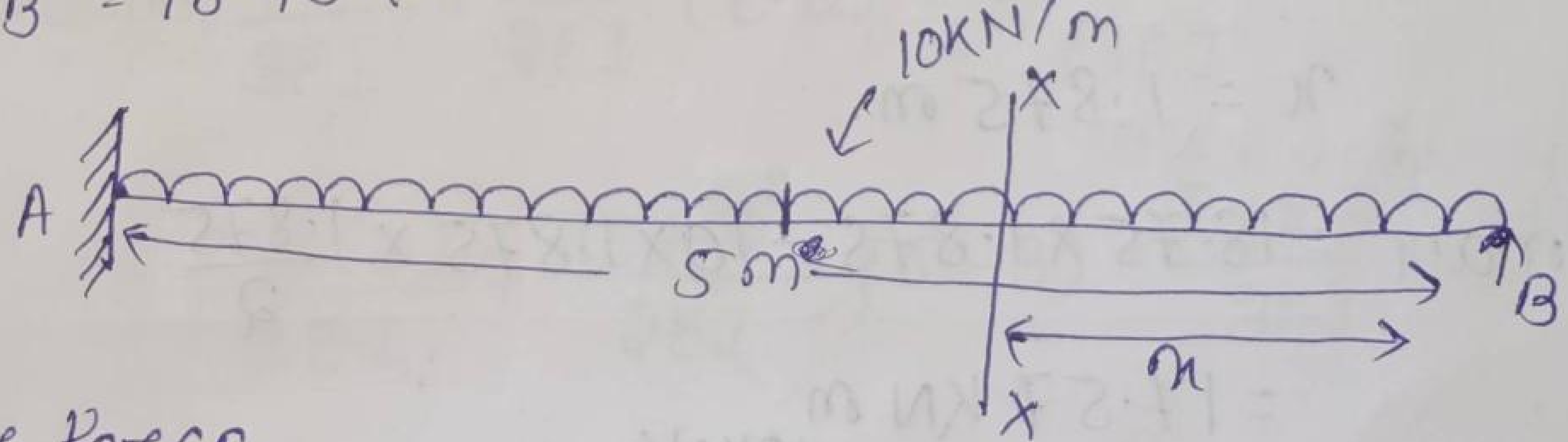
$$\Rightarrow \frac{wl^4}{8\cancel{EI}} = \frac{R_B \times l^3}{3\cancel{EI}}$$

$$\Rightarrow R_B = \frac{3wl}{8}$$

$$\Rightarrow R_B = \frac{3}{\cancel{8}_4} \times \cancel{10}^5 \times 5$$

$$\Rightarrow R_B = \frac{75}{4}$$

$$\Rightarrow R_B = 18.75\,KN$$



Shear Force

Betⁿ A and B

$$(S.F)_{xx} = -18.75 + 10x$$

At B

$$(S.F)_B = -18.75 + 10\times 0 \qquad x = 0$$

$$= -18.75\,KN$$

At A

$$(S.F)_A = -18.75 + 10 \times 5 \qquad x = 5$$

$$= -31.25\,KN$$

B.M will max^m where shear force changes sign.

Bet^n A & B

$(B.M)_{xx} = 18.75x - 10 \times x \times \frac{x}{2}$

At B $x = 0$

$(B.M)_B = 0$

At A $x = 5$

$(B.M)_A = 18.75 \times 5 - 10 \times 5 \times \frac{5}{2}$

$= -31.25$ KN-m

$\frac{d}{dx}(B.M_{xx}) = 18.75 - 10x = 0$

$18.75 - 10x = 0$

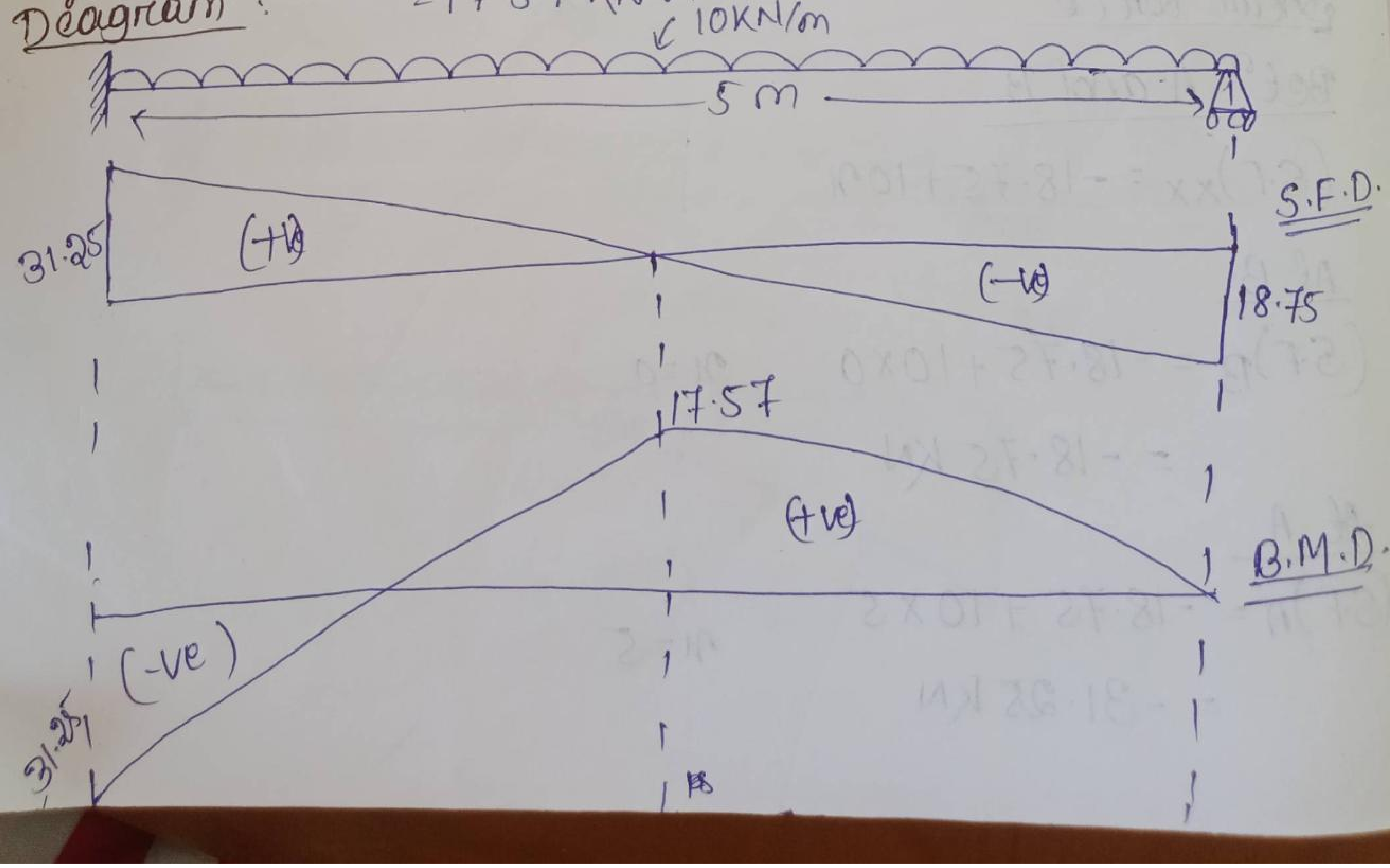
$x = \frac{18.75}{10}$

$x = 1.875$ m

$(B.M)_{max} = 18.75 \times 1.875 - 10 \times 1.875 \times \frac{1.875}{2}$

$= 17.57$ KN m

Diagram :



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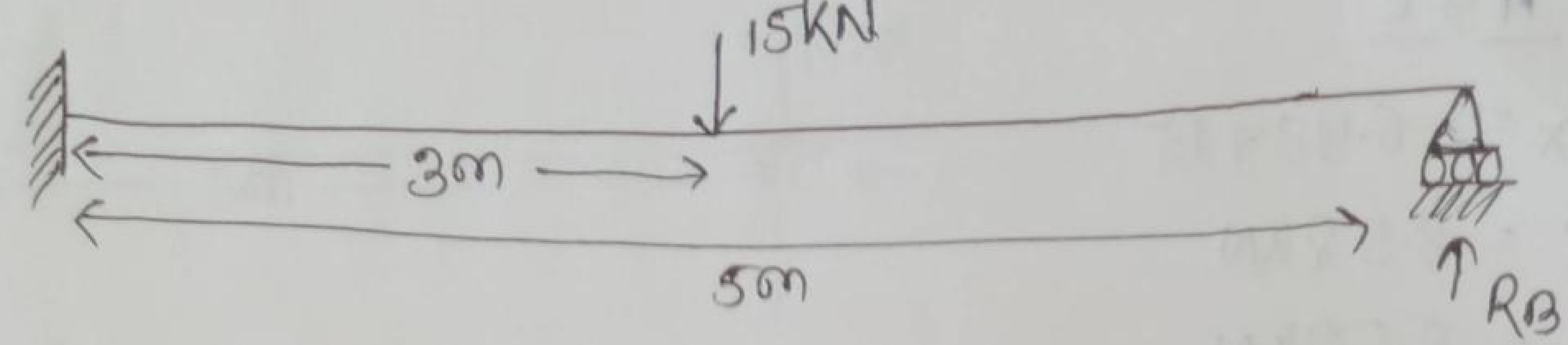
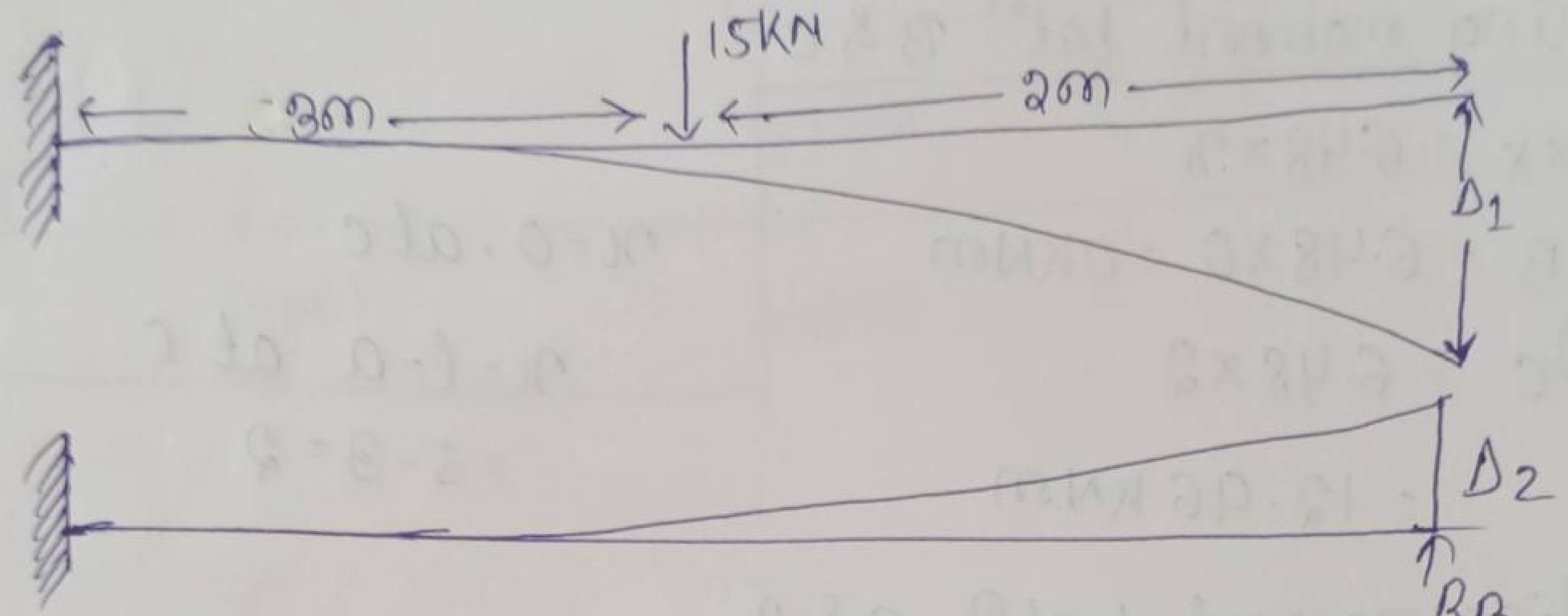


Ans



$$\Delta_1 - \Delta_2 = 0$$

$$\Delta_1 = \Delta_2$$

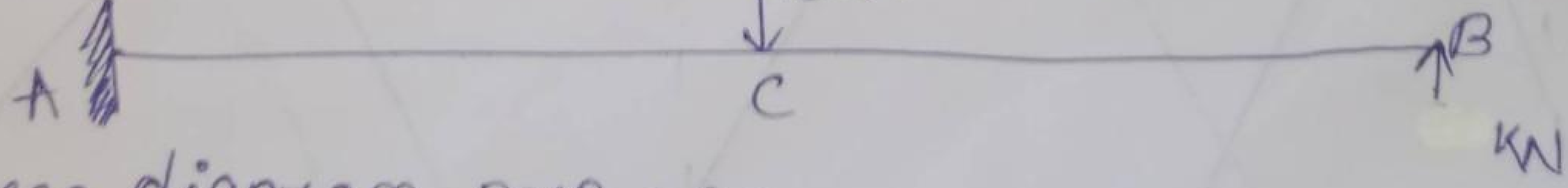
$$\frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(l-a) = \frac{R_B \times (5)^3}{3EI}$$

$$\Rightarrow \frac{15 \times (3)^3}{3\cancel{EI}} + \frac{15 \times 3^2}{2\cancel{EI}}(5-3) = \frac{R_B \times 5^3}{3\cancel{EI}}$$

$$\Rightarrow 270 = \frac{RB \times 5^3}{3}$$

$$\Rightarrow R_B \times 5^3 = 270 \times 3$$

$$\Rightarrow R_B = \frac{270 \times 3}{5^3} = 6.48\ KN$$



Shearforce diagram Betn C&B

$(S.F)_{xx} = -6.48\ KN$

$(S.F)_B = -6.48 KN$

$(S.F)_C = -6.48 KN$

Betⁿ A & C

$(S.F)_{xx} = -6.48 + 15$

$(S.F)_C = 8.52 KN$

$(S.F)_A = 8.52 KN$

Bending moment betⁿ B & C

$(B.M)_{xx} = 6.48 \times x$

$(B.M)_B = 6.48 \times 0 = 0 KNm$ $\quad x = 0,$ at C

$(B.M)_C = 6.48 \times 2$ $\quad x = l - a$ at C

$= 12.96 KNm$ $\quad = 5 - 3 = 2$

Bending moment betⁿ C & A

$(B.M)_{xx} = 6.48 \times x - 15(x-2)$

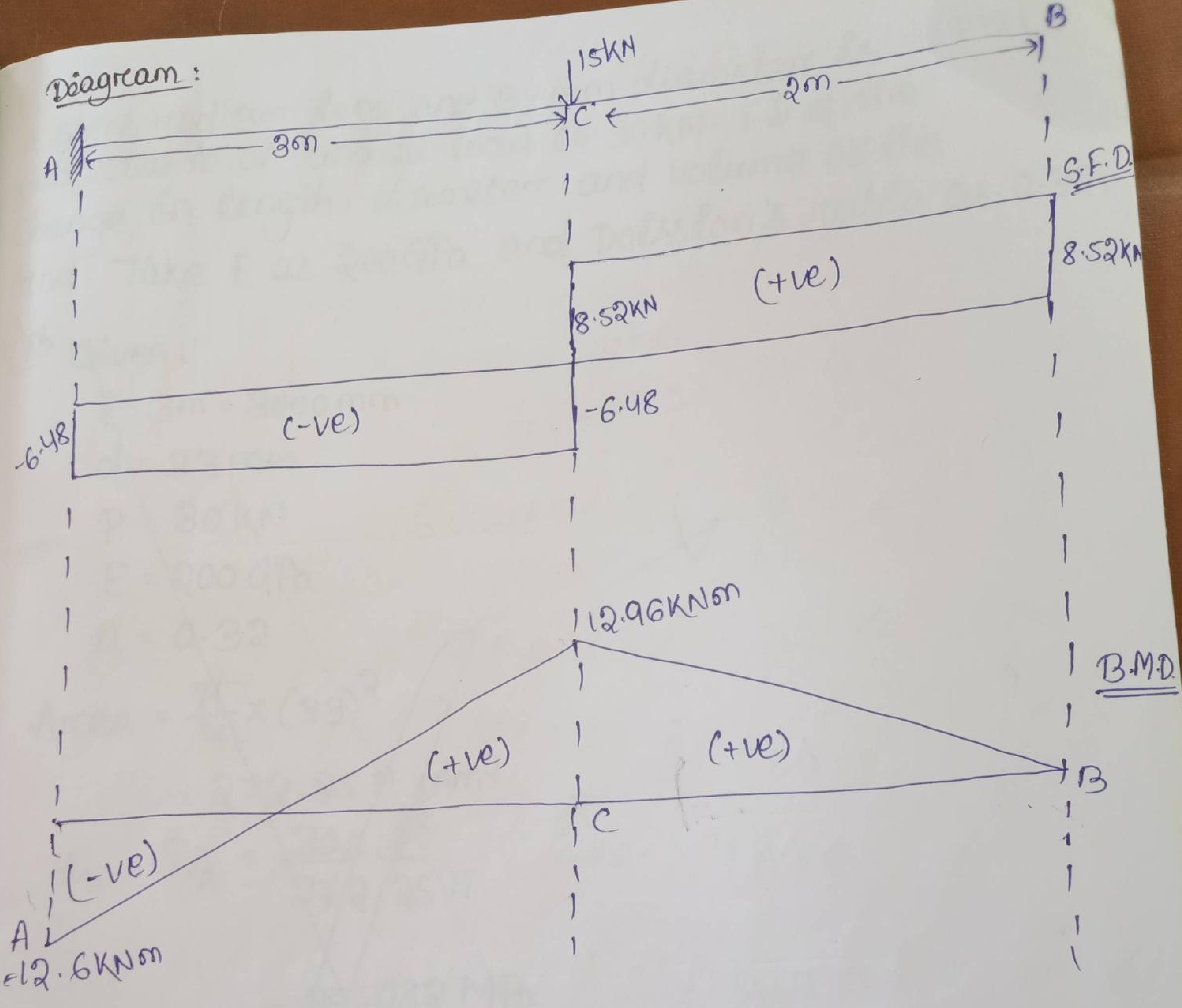
$(B.M)_C = 6.48 \times 2 - 15(2-2)$

$= 12.96 KNm$

$(B.M)_A = 6.48 \times 5 - 15(5-2)$

$= -12.6 KNm$.

Diagram :



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